PHYSICS 200 MIDTERM #1

October 5, 2007

Questions 1-8 : Multiple choice / short answer - 1 point each

Question 9: Show your work - 7 points total

Name: MARK VAN RAAMSDONK

Student Number: 58301912

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$t' = \gamma(t - \tfrac{v}{c^2}x) \qquad t = \gamma(t' + \tfrac{v}{c^2}x')$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$e^{i\pi} = -1$$

$$\Delta T = \frac{v^2}{c^2}\frac{L_1 + L_2}{c}$$

$$\vec{p} = \gamma m \vec{v}$$

$$f' = f\gamma\left[1 - \cos\theta\,\frac{v}{c}\right]$$

$$I = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2 \cdot c^2$$

$$d\tau = dt\sqrt{1 - \frac{v^2}{c^2}}$$

$$E = mc^2$$

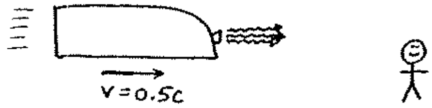
1 light year = $c \times 1$ year

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$c \approx 3\times10^8$ m/s

POSSIBLY USEFUL FORMULAE

1

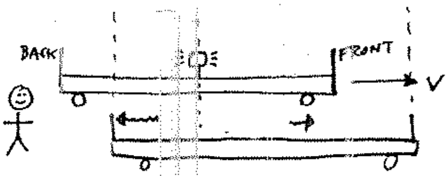


2ND POSTULATE! The speed of light is c in all frames of reference independent of the motion of the source.

A train moving at 0.5c emits light (at speed c in the train's frame) from its front headlight. What speed does a fixed observer beside the track measure for the light?

$\boxed{\text{Answer: } c}$

2



A flash of light is emitted from the center of a moving train. According to a fixed observer near the track, the light reaches the back of the train

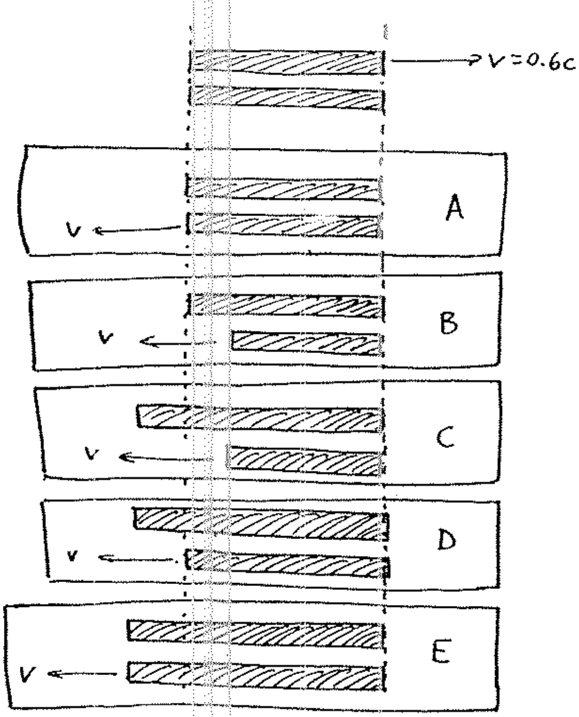
a) at the same time as
b) before
c) after

the light reaches the front of the train.

The observer measures light to travel at speed c in her own frame. But the ~~when the light reaches the back of the train the distance it has travelled is less than~~ point where light hits the back is closer to the source than the point where light hits the front, in this frame.

$\boxed{\text{Answer: } b}$

3



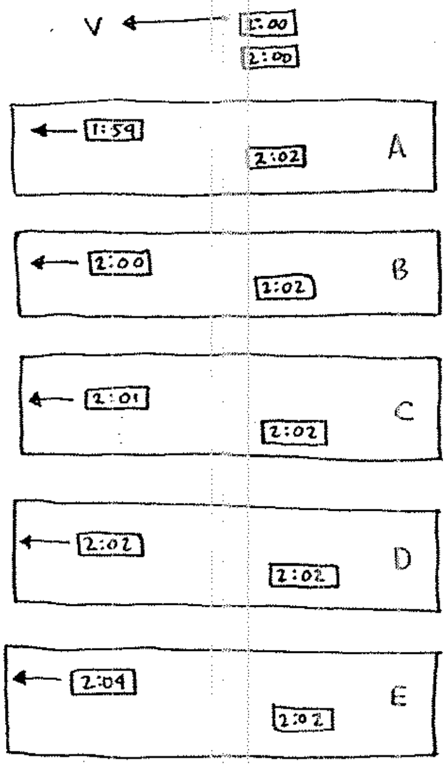
The top picture shows two rods, as observed in the frame of the lower rod. Which of the remaining pictures represents an observation of the same rods in the frame of the upper rod?

$\boxed{\text{Answer: } C}$

In the top picture, the upper rod appears shorter than it actually is due to length contraction (moving objects are observed to be shorter than they actually are in their direction of motion).

So the upper rod will be longer as observed in its own frame. The lower rod is moving in this frame, so it appears shorter than in the upper picture. Thus the answer must be C.

(4)



Two identical clocks are set to the same time as one passes the other at high velocity (as shown in the top figure). Which of the other figures represents a possible observation of the clocks at some later time in the frame of the fixed clock.

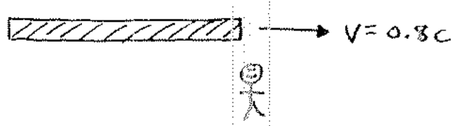
(Assume the readings on the clocks are exact).

The moving clock should be observed to run slow, so after some time, less time will have passed on it than on the fixed clock. But the amount of time passed on the moving clock must still be observed as positive, so the answer is C

Answer: C

---

(5)



A meter stick travels at 0.8c relative to some fixed observer. How long does the observer measure for the time it takes the stick to pass?

A) $\frac{1\,m}{c}$

B) $\frac{0.8\,m}{c}$

C) $\frac{0.6\,m}{c}$

D) $\frac{1.25\,m}{c}$

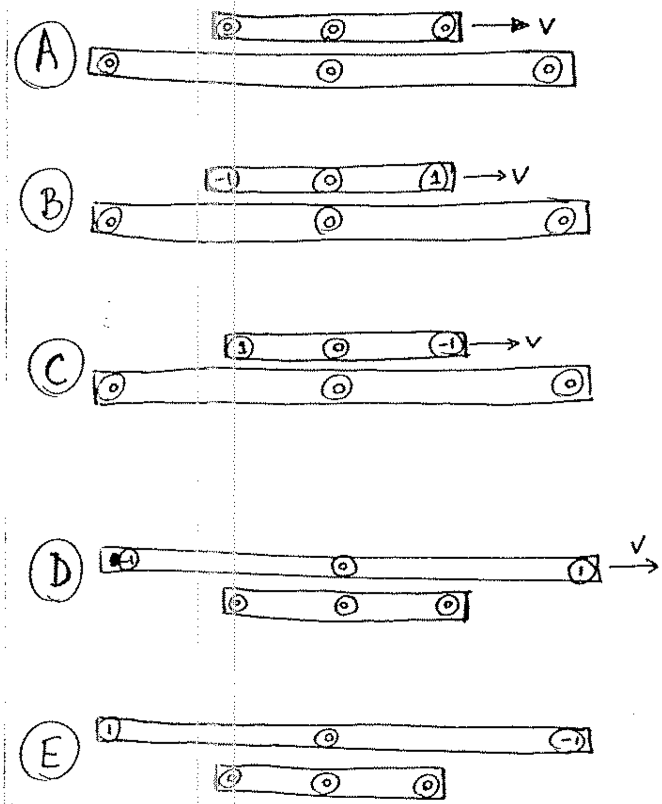
E) $\frac{0.75\,m}{c}$

Answer: E

The observed length of the meter stick is $0.6\,m = \frac{1\,m}{\gamma}$ due to length contraction.

So from the time the front passes the observer, the back must travel 0.6m at 0.8c, so the time for the stick to pass is $\frac{0.6\,m}{0.8\,c} = \frac{0.75\,m}{c}$ in the observer's frame.

8



Two identical meter sticks, each with synchronized clocks at the ends and in the middle, pass each other at $v = \sqrt{\frac{3}{4}}c$. Which picture represents a possible observation of the system at some instant in the frame of the lower meter stick. Numbers in the circles represent the clock readings (in some units).

$\boxed{\text{Answer: } C}$

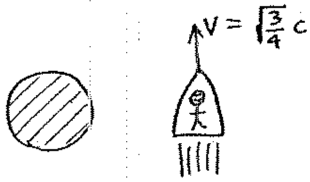
The moving meter stick will appear shorter, by length contraction. The times in the moving frame are related to the times and positions in the fixed frame by

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

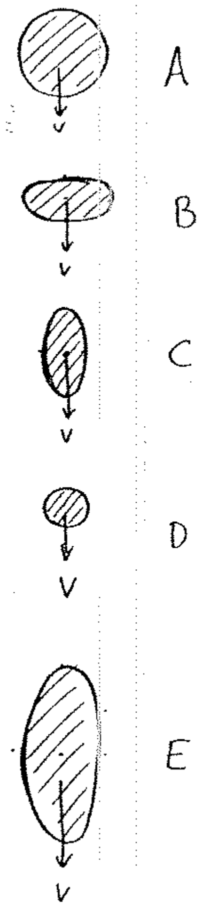
$$= -\gamma\frac{v}{c^2}\cdot x$$

So for larger x, t' should be smaller, as in picture C.

(6)



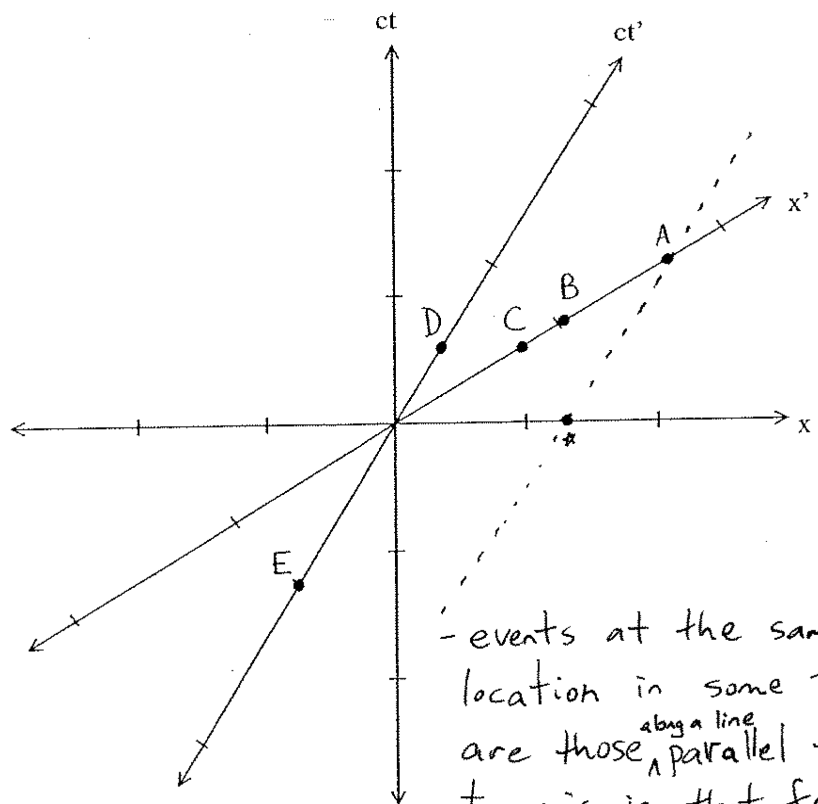
Which of the diagrams below best represent the shape of the shaded object in the top picture, as observed by the person in the rocket in the top picture.



Answer: B

This has length contraction in the direction of motion, but not in the transverse direction.

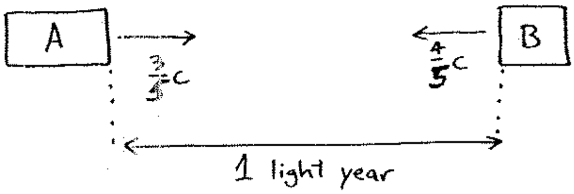
(7) According to an observer in the frame using x' and t' coordinates, which event is at the same location as the event marked ★ ?



- events at the same location in some frame are those along a line parallel to the t axis in that frame.

Answer: A

9



Two ships approach each other at high velocity, as shown. The ships are initially 1 light year apart.

a) What is the speed of ship B as measured by ship A? (2 points)

We have: $u' = \dfrac{u - v}{1 - \frac{uv}{c^2}}$ where $v = \frac{3}{5}c$ and $u = -\frac{4}{5}c$.

This gives:
$$u' = \frac{-\frac{7}{5}c}{1 + \frac{12}{25}} = -\frac{35}{37}c$$

Thus A observes the speed of B's ship to be $\frac{35}{37}c$.

b) Assuming the clock on ship A reads $t' = 0$ in the picture above, what does the clock read when the two ships meet? (3 points)

$\boxed{\text{SOLUTION 1}}$ In the original frame, the trajectories of A and B are

$$x_A(t) = \tfrac{3}{5}ct \qquad x_B(t) = 1\,\ell yr - \tfrac{4}{5}ct$$

The ships meet when $x_A(t) = x_B(t)$ or

$$\tfrac{3}{5}ct = 1\,\ell yr - \tfrac{4}{5}ct$$
$$\Rightarrow \tfrac{7}{5}c \cdot t = 1\,\text{year} \cdot c$$
$$\Rightarrow t = \tfrac{5}{7}\text{ years}$$

A's clock appears to run slower during this time by a factor of $\gamma_A = \frac{5}{4}$, so the time on A's clock when the ships meet is

$$\boxed{t' = \tfrac{5}{7}\text{ years} \times \tfrac{4}{5} = \tfrac{4}{7}\text{ years}}$$

We could have also calculated that $x_A = x_B = \frac{3}{7}\,1yr$ and used $t' = \gamma(x_0 - v_0 t)$

$\boxed{\text{SOLUTION 2}}$

The trajectory of B in A's frame can be obtained from

$$x' = \gamma\left(x_B(t) - v_A t\right) = \gamma\left(1\,\ell yr - \tfrac{4}{5}ct - v_A t\right) \quad (1)$$
$$t' = \gamma\left(t - \tfrac{v}{c^2}x_B\right) = \gamma\left(t - \tfrac{v}{c^2}1\,\ell yr + \tfrac{v}{c^2}\cdot\tfrac{4}{5}ct\right) \quad (2)$$

by eliminating t. We want to know t' when $x' = 0$, so we find

(1) $\Rightarrow t = \frac{5}{7}$ years and then

(2) $\Rightarrow t' = \gamma_A\left(\frac{5}{7}yrs - \frac{3}{5}\cdot 1yr + \frac{3}{5}\cdot\frac{4}{5}\cdot\frac{5}{7}yrs\right) = \frac{5}{4}\cdot\frac{16}{35}yrs = \frac{4}{7}$ years

c) How far away does A observe B to be when A's clock reads $t'=0$? (2 points)

FIRST WAY

Using solution ②, we find that $t'=0$ gives (from eqn ②)

$$0 = \gamma\left(t - \frac{v}{c^2}\cdot 1\,\text{lyr} + \frac{v}{c^2}\cdot\frac{4}{5}ct\right)$$

$$\Rightarrow t = \frac{15}{37}\,\text{yrs}$$

Then, from equation ①

$$x' = \gamma\left(\frac{15}{37}\,\text{yrs}\left(4 - \frac{7}{5}c\right) + 1\,\text{lyr}\right)$$

$$= \frac{5}{4} \times \frac{16}{37}\,\text{lyr}$$

$$\Rightarrow \boxed{x' = \frac{20}{37}\,\text{lyr}}$$

SECOND WAY

We know that A and B are at the same place when A's clock reads $t' = \frac{4}{7}$ years, and that A observes B's velocity to be $u' = -\frac{35}{37}c$, so A sees B at a distance

$$D = |\Delta t'||u'|$$

$$= \frac{4}{7}\,\text{years} \times \frac{35}{37}c$$

$$\Rightarrow \boxed{D = \frac{20}{37}\,\text{lyr}}$$