CHAPTER 51

ENERGY FROM THE NUCLEUS

In a system of interacting particles, we can extract useful energy when the system moves to a lower energy state (that is, a more tightly bound state). In an atomic system, we can extract this energy through chemical reactions, such as burning. In a nuclear system, we can extract energy in a variety of ways. For example, the energy released in radioactive decay has been used to provide electrical power to cardiac pacemakers and to space probes.

In this chapter we consider the two primary methods that are used to extract energy from the nucleus and convert it to useful purposes. In nuclear fission, *a heavy nucleus is split into two fragments. In* nuclear fusion, *two light nuclei are combined into a heavier nucleus. Figure 50-6 showed that either of these processes can result in more tightly bound nuclei and therefore can release excess nuclear binding energy to be converted into other forms of energy. Reactors based on nuclear fission today provide a significant share of the world's electrical power. Research and engineering are actively under way to develop reactors based on nuclear fusion.*

51-1 THE ATOM AND THE NUCLEUS

When we get energy from coal by burning it in a furnace, we are tinkering with *atoms* of carbon and oxygen, rearranging their outer *electrons* in more stable combinations. When we get energy from uranium by consuming it in a nuclear reactor, we are tinkering with its *nucleus*, rearranging its *nucleons* in more stable combinations.

Electrons are held in atoms by the Coulomb force, and it takes a few electron volts to remove one of the outer electrons. On the other hand, nucleons are held in nuclei by the strong nuclear force, and it takes a few *million* electron volts to pull one of *them* out. This factor is also reflected in our ability to extract about a million times more energy from a kilogram of uranium than from a kilogram of coal.

In both the atomic and nuclear cases, the appearance of energy is accompanied by a decrease in the rest energy of the fuel. The only difference between consuming uranium and burning coal is that, in the former case, a much larger fraction of the available rest energy (again, by a factor of several million) is converted to other forms of energy.

We must be clear about whether our concern is for the quantity of energy or for the rate at which the energy is delivered—that is, the *power.* In the nuclear case, will the kilogram of uranium burn slowly in a power reactor or explosively in a bomb? In the atomic case, are we thinking about exploding a stick of dynamite or digesting a jelly doughnut? (Surprisingly, the energy release is greater in the second case than in the first!)

Table 51-1 shows how much energy can be extracted from 1 kg of matter by doing various things to it. Instead of reporting the energy directly, we measure it by showing how long the extracted energy could operate a 100-W lightbulb. Row 5, the total mutual annihilation of matter and antimatter, is the ultimate in extracting energy from matter. When you have used up all the available mass you can do no more. (However, no one has yet figured out an economi-

TABLE 51-1 Energy from 1 kg of Matter

Form of Matter	*Process*	*Time*[a]
Water	A 50-m waterfall	5 s
Coal	Burning	8 h
^{235}U	Fission	3×10^4 y
Hot deuterium gas	Fusion	3×10^4 y
Matter and antimatter	Annihilation	3×10^7 y

[a] These numbers show how long the energy generated could power a 100-W lightbulb.

cal way to produce and store 1 kg of antimatter to use for energy production.)

Keep in mind that the comparisons of Table 51-1 are on a per-unit-mass basis. Kilogram for kilogram we get several million times more energy from uranium than we do from coal or from falling water. On the other hand, there is a lot of coal in the Earth's crust and a lot of water backed up behind the Bonneville Dam on the Columbia River.

51-2 NUCLEAR FISSION: THE BASIC PROCESS

In 1932, the English physicist James Chadwick discovered the neutron. A few years later Enrico Fermi and his collaborators in Rome discovered that, if various elements are bombarded by these new projectiles, new radioactive elements are produced. Fermi had predicted that the neutron, being uncharged, would be a useful nuclear projectile; unlike the proton or the α particle, it experiences no repulsive Coulomb force when it approaches a nuclear surface. Because there is no Coulomb barrier for it, the slowest neutron can penetrate and interact with even the most massive, highly charged nucleus. *Thermal neutrons*, which are neutrons in equilibrium with matter at room temperature, are convenient and effective bombarding particles. At 300 K, the average kinetic energy of such neutrons is

$$K_{av} = \tfrac{3}{2}kT = \tfrac{3}{2}(8.62 \times 10^{-5}\ \text{eV/K})(300\ \text{K}) = 0.04\ \text{eV}.$$

In 1939, the German chemists Otto Hahn and Fritz Strassmann, following work initiated by Fermi and his collaborators, bombarded uranium with thermal neutrons. They found by chemical analysis that after the bombardment a number of new radioactive elements were present, among them one whose chemical properties were remarkably similar to those of barium. Repeated tests finally convinced these chemists that this "new" element was not new at all; it really *was* barium. How could this middle-mass element ($Z = 56$) be produced by bombarding uranium ($Z = 92$) with neutrons?

The riddle was solved within a few weeks by physicists Lise Meitner and her nephew Otto Frisch. They showed that a uranium nucleus, having absorbed a neutron, could split, with the release of energy, into two roughly equal parts, one of which might well be barium. They named this process *nuclear fission*.*

The fission of ^{235}U by thermal neutrons, a process of great practical importance, can be represented by

$$^{235}U + n \rightarrow X + Y + bn. \qquad (51\text{-}1)$$

Here X and Y stand for *fission fragments*, middle-mass nuclei that are usually highly radioactive. The factor b, which has the average value 2.47 for fission events of this type, is the number of neutrons released in such events.

Figure 51-1 shows two visible tracks made by fission fragments X and Y in a process such as is represented by Eq. 51-1. Two details are immediately apparent: (1) The two tracks do not have the same length. This occurs because the two fragments do not have the same mass; when the uranium nucleus splits, the two pieces tend to have different masses. Figure 51-2 shows the mass distribution; as you can see, there is a high probability that one fragment will have a mass number of about 95 and the other about 140. You can also see that the chance of an even split is only about 1 in 10^4. (2) The second apparent feature of Fig. 51-1 is that the two tracks are not quite exactly back to back. Can you explain this?

A heavy nucleus such as ^{235}U has a neutron-to-proton ratio of about 1.6. On the average, we expect that the fragments X and Y should have about the same neutron-to-proton ratio. A glance at Fig. 50-4 shows that, for example, a stable nucleus with mass number 95 has about 40 protons and 55 neutrons, for a neutron-to-proton ratio of about 1.4. If the $A = 95$ fragment formed in fission has a neutron-

*See "The Discovery of Fission," by Otto Frisch and John Wheeler, *Physics Today*, November 1967, p. 43. Also see *Lise Meitner: A Life in Physics* by Ruth Lewin Sime (University of California Press, 1996).

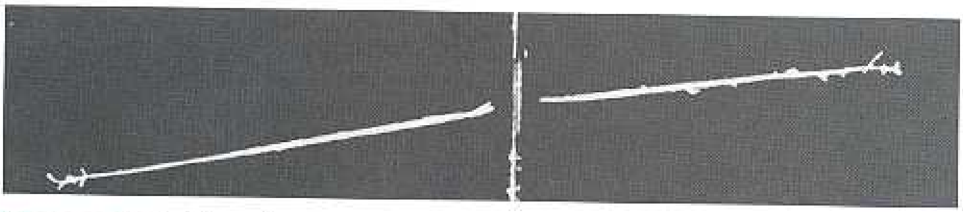

FIGURE 51-1. When a fast charged particle passes through a cloud chamber, it leaves a track of liquid droplets. The two back-to-back tracks represent fission fragments, produced by a fission event that took place in a thin vertical uranium foil in the center of the chamber.

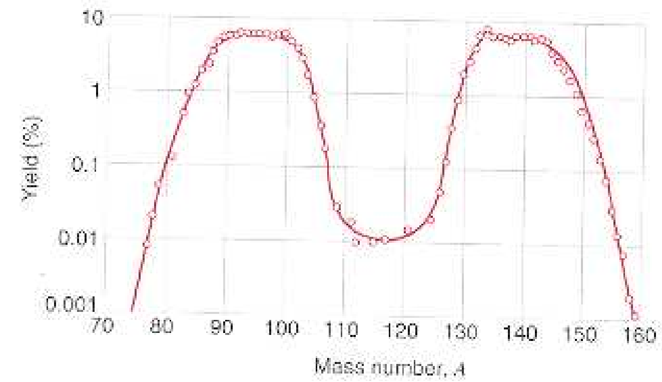

FIGURE 51-2. The distribution in mass of the fission fragments X and Y (see Eq. 51-1) from the fission of ^{235}U by thermal neutrons. Note that the vertical scale is logarithmic.

to-proton ratio of 1.6, it will be far from the line of stable nuclei. This explains why the fission fragments are radioactive. Usually through successive beta decays (and sometimes by neutron emission), they try to relieve their neutron excess and move toward the region of stable nuclei.

Let us consider a specific example of a fission process. When ^{235}U captures a neutron, it forms for an instant an unstable, highly excited nucleus of ^{236}U that can then fission, perhaps as follows:

$$^{235}U + n \rightarrow {}^{236}U \rightarrow {}^{140}Xe + {}^{94}Sr + 2n. \quad (51\text{-}2)$$

The fission fragments ^{140}Xe and ^{94}Sr decay until each reaches a stable end product, as follows:

$$^{140}Xe \xrightarrow[14\,s]{} {}^{140}Cs \xrightarrow[65\,s]{} {}^{140}Ba \xrightarrow[13\,d]{} {}^{140}La \xrightarrow[40\,h]{} {}^{140}Ce \text{ (stable)}$$

$$^{94}Sr \xrightarrow[75\,s]{} {}^{94}Y \xrightarrow[19\,min]{} {}^{94}Zr \text{ (stable)}$$

The decays are β^- events, the half-lives being indicated at each stage. As for all beta decays, the mass numbers (140 and 94) remain unchanged as the decays continue.

The disintegration energy Q for fission is very much larger than for chemical processes. We can support this by a rough calculation. From the binding energy curve of Fig. 50-6, we see that for heavy nuclides ($A = 240$, say) the binding energy per nucleon is about 7.6 MeV. In the intermediate range ($A = 120$, say), it is about 8.5 MeV. The difference in total binding energy between a single nucleus ($A = 240$) and two fragments (assumed equal) into which it may be split is then

$$Q = 2(8.5\text{ MeV})\frac{A}{2} - (7.6\text{ MeV})A \approx 200\text{ MeV}.$$

Sample Problem 51-1 shows a more careful calculation, which agrees very well with this rough estimate.

SAMPLE PROBLEM 51-1. Calculate the disintegration energy Q for the fission event of Eq. 51-2, taking into account the decay of the fission fragments. Needed atomic masses are

^{235}U	235.043923 u	^{140}Ce	139.905434 u
n	1.008665 u	^{94}Zr	93.906316 u.

Solution If we replace the fission fragments in Eq. 51-2 by their stable end products, we see that the overall transformation is

$$^{235}U \rightarrow {}^{140}Ce + {}^{94}Zr + n.$$

The single neutron comes about because the (initiating) neutron on the left side of Eq. 51-2 cancels one of two neutrons on the right side of that equation.

The mass difference $\Delta m = m_i - m_f$ for this reaction is

$$\begin{aligned}\Delta m &= 235.043923\text{ u} - (139.905434\text{ u} + 93.906316\text{ u} \\ &\quad + 1.008665\text{ u}) \\ &= 0.223508\text{ u},\end{aligned}$$

and the corresponding energy is

$$Q = \Delta m\, c^2 = (0.223508\text{ u})(931.5\text{ MeV/u}) = 208.2\text{ MeV},$$

in good agreement with our previous rough estimate of 200 MeV.

About 80% of the disintegration energy is in the form of the kinetic energy of the two fragments, the remainder going to the neutron and the radioactive decay products.

If the fission event takes place in a bulk solid, most of the disintegration energy appears as an increase in the internal energy of the solid, which shows a corresponding rise in temperature. Five percent or so of the disintegration energy, however, is associated with neutrinos that are emitted during the beta decay of the primary fission fragments. This energy is carried out of the system and does not contribute to the increase in its internal energy.

51-3 THEORY OF NUCLEAR FISSION

Soon after the discovery of fission, Niels Bohr and John Wheeler developed a theory, based on the analogy between a nucleus and a charged liquid drop, that explained its main features.

Figure 51-3 suggests how the fission process proceeds. When a heavy nucleus such as ^{235}U absorbs a slow neutron, as in Fig. 51-3*a*, that neutron falls into the potential well associated with the strong nuclear forces that act in the nuclear interior. Its potential energy is then transformed into internal excitation energy, as Fig. 51-3*b* suggests. As we show in Sample Problem 51-2, the resulting excitation energy of ^{236}U is considerable, about 6.5 MeV.

Figure 51-3*c* shows that the nucleus, behaving like an energetically oscillating, charged liquid drop, will sooner or later develop a short "neck" and will begin to separate into two charged "globs." If conditions are right, the electrostatic repulsion between these two globs will force them apart, breaking the neck. The two fragments, each still carrying some residual excitation energy, then fly apart. Fission has occurred.

So far this model gives a good qualitative picture of the fission process. It remains to be seen, however, whether it can answer a hard question: Why are some heavy nuclides (^{235}U and ^{239}Pu, say) readily fissionable by slow neutrons but other, equally heavy, nuclides (^{238}U and ^{243}Am, say) are not?

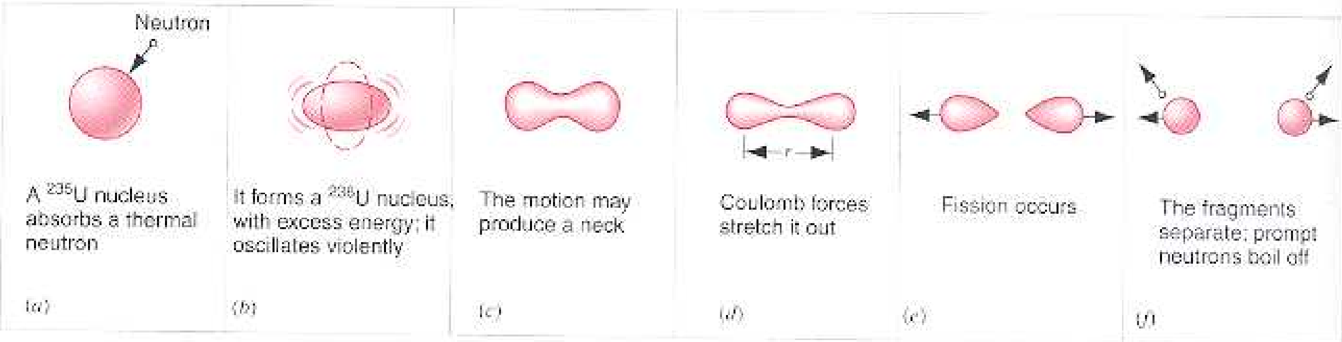

FIGURE 51-3. The stages in a fission process, according to the liquid-drop fission model.

Bohr and Wheeler were able to answer this question. Figure 51-4 shows the potential energy curve for the fission process that they derived from their model. The horizontal axis displays the *distortion parameter r*, which is a rough measure of the extent to which the oscillating nucleus departs from a spherical shape. Figure 51-3*d* suggests how this parameter is defined before fission occurs. When the fragments are far apart, this parameter is simply the distance between their centers.

The energy interval between the initial state and the final state of the fissioning nucleus—that is, the disintegration energy Q—is displayed in Fig. 51-4. The central feature of that figure, however, is that the potential energy curve passes through a maximum at a certain value of r. There is a *potential barrier* of height E_b that must be surmounted (or

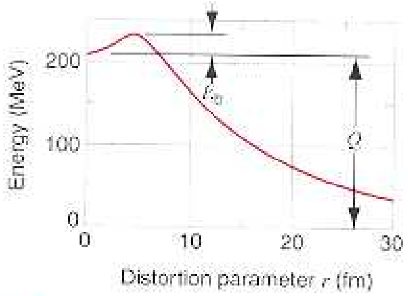

FIGURE 51-4. The potential energy at various stages in the fission process, showing the disintegration energy Q and the barrier height E_b.

tunneled) before fission can occur. This reminds us of alpha decay (see Fig. 50-9), which also is a process that is inhibited by a potential barrier. We see then that fission will occur *only* if the absorbed neutron provides an excitation energy E_n great enough to overcome the barrier or to have a reasonable probability of tunneling through it.

Table 51-2 shows a test of fissionability by thermal neutrons applied to four heavy nuclides, chosen from dozens of candidates that might have been considered. For each nuclide both the barrier height E_b and the excitation energy E_n are given. E_b was calculated from the theory of Bohr and Wheeler, and E_n was computed (as in Sample Problem 51-2) from the known masses.

For ^{235}U and ^{239}Pu we see that $E_n > E_b$. This means that fission by absorbing a thermal neutron is predicted to occur for these nuclides. This is confirmed by noting, in the table, the large measured cross sections (that is, the reaction probabilities) for the process.

For the other two nuclides (^{238}U and ^{243}Am), we have $E_n < E_b$, so that there is not enough energy to surmount the barrier or to tunnel through it effectively. The excited nucleus (Fig. 51-3*b*) prefers to get rid of its excitation energy by emitting a gamma ray instead of by breaking into two large fragments. Table 51-2 shows, as we expect, that the cross sections for thermal neutron fission in these cases are exceedingly small. These nuclides *can* be made to fission, however, if they absorb a substantially energetic (rather than a thermal) neutron. For ^{238}U, for example, the absorbed neutron must have an energy of at least 1.3 MeV for the fission process to "go" with reasonable probability.

TABLE 51-2 Test of the Fissionability of Four Nuclides

Target Nuclide	*Nuclide Being Fissioned*	E_b *(MeV)*	E_n *(MeV)*	$E_n - E_b$ *(MeV)*	*Fission Cross Section*[a] *(barns)*
^{235}U	^{236}U	5.2	6.5	+1.3	584
^{238}U	^{239}U	5.7	4.8	−0.6	5×10^{-6}
^{239}Pu	^{240}Pu	4.8	6.4	+1.6	742
^{243}Am	^{244}Am	5.8	5.5	−0.3	<0.08

[a] The cross section is a measure of the probability for a nuclear reaction to occur. The cross section is measured in units of barns, where 1 barn $= 10^{-28}\ m^2$.

SAMPLE PROBLEM 51-2. Consider a ^{236}U nucleus in its ground state. How much energy is required to remove a neutron from it, leaving a ^{235}U nucleus behind? The needed atomic masses are

^{235}U 235.043923 u; n 1.008665 u; ^{236}U 236.045562 u.

Solution The increase in mass of the system as the neutron is pulled out is

$$\Delta m = 1.008665 \text{ u} + 235.043923 \text{ u} - 236.045562 \text{ u} = 0.007026 \text{ u}.$$

This means that an energy equal to

$$E_n = \Delta m\, c^2 = (0.007026 \text{ u})(931.5 \text{ MeV/u}) = 6.545 \text{ MeV}$$

must be expended. This, by definition, is the binding energy of the neutron in the ^{236}U nucleus.

When a ^{235}U nucleus absorbs a thermal neutron, 6.545 MeV is the amount of excitation energy that the thermal neutron brings into the ^{236}U nucleus. In effect, the ^{236}U nucleus is formed in an excited state 6.545 MeV above the ground state. The excited nucleus can get rid of this energy either by emitting gamma rays (which leaves a ^{236}U nucleus in its ground state) or by fission (see Eq. 51-1). It turns out that fission is about six times more likely than gamma-ray emission.

51-4 NUCLEAR REACTORS: THE BASIC PRINCIPLES

Energy releases per atom in individual nuclear events such as alpha emission are roughly a million times larger than those of chemical events. To make large-scale use of nuclear energy, we must arrange for one nuclear event to trigger another until the process spreads throughout bulk matter like a flame through a burning log. The fact that more neutrons are generated in fission than are consumed (see Eq. 51-1) raises just this possibility: the neutrons that are produced can cause fission in nearby nuclei and in this way a chain of fission events can propagate itself. Such a process is called a *chain reaction*. It can either be rapid and uncontrolled as in a nuclear bomb or controlled as in a nuclear reactor.

Suppose that we wish to design a nuclear reactor based, as most present reactors are, on the fission of ^{235}U by slow neutrons. The fuel in such reactors is almost always artificially "enriched," so that ^{235}U makes up a few percent of the uranium nuclei rather than the 0.7% that occurs in natural uranium; the remaining 99.3% of natural uranium is ^{238}U, which is not fissionable by thermal neutrons. Although on the average 2.47 neutrons are produced in ^{235}U fission for every thermal neutron consumed, there are serious difficulties in making a chain reaction "go." Here are three of the difficulties, together with their solutions:

1. *The neutron leakage problem.* A certain percentage of the neutrons produced will simply leak out of the reactor core and be lost to the chain reaction. If too many do so, the reactor will not work. Leakage is a surface effect, its magnitude proportional to the *square* of a typical reactor core dimension (surface area $= 4\pi r^2$ for a sphere). Neutron production, however, is a volume effect, proportional to the *cube* of a typical dimension (volume $= \frac{4}{3}\pi r^3$ for a sphere). The fraction of neutrons lost by leakage can be made as small as we wish by making the reactor core large enough, thereby decreasing its surface-to-volume ratio ($= 3/r$ for a sphere).

2. *The neutron energy problem.* Fission produces *fast* neutrons, with kinetic energies of about 2 MeV, but fission is induced most effectively by *slow* neutrons. The fast neutrons can be slowed down by mixing the uranium fuel with a substance that has these properties: (*a*) it is effective in causing neutrons to lose kinetic energy by collisions and (*b*) it does not absorb neutrons excessively, thereby removing them from the fission chain. Such a substance is called a *moderator*. Most power reactors in this country are moderated by water, in which the hydrogen nuclei (protons) are the effective moderating element.

3. *The neutron capture problem.* Neutrons may be captured by nuclei in ways that do not result in fission. The most common possibility is capture followed by the emission of a gamma ray. In particular, as the fast (MeV) neutrons generated in the fission processes are slowed down in the moderator to thermal equilibrium (0.04 eV), they must pass through an energy interval (1 − 100 eV) in which they are particularly susceptible to nonfission capture by ^{238}U.

To minimize such *resonance capture*, as it is called, the uranium fuel and the moderator (water, say) are not intimately mixed but are "clumped," remaining in close contact with each other but occupying different regions of the reactor volume. The hope is that a fast fission neutron, produced in a uranium "clump" (which might be a fuel rod), will with high probability find itself in the moderator as it passes through the "dangerous" resonance energy range. Once it has reached thermal energies, it will very likely wander back into a clump of fuel and produce a fission event. The task for reactor designers is to produce the most effective geometrical arrangement of fuel and moderator.

Figure 51-5 shows the neutron balance in a typical power reactor operating with a steady output. Let us trace the behavior of a sample of 1000 thermal neutrons in the reactor core. They produce 1330 neutrons by fission in the ^{235}U fuel and 40 more by fast fission in the ^{238}U, making a total of 370 new neutrons, all of them fast. Exactly this same number of neutrons is then lost to the chain by leakage from the core and by nonfission capture, leaving 1000 thermal neutrons to continue the chain. What has been gained in this cycle, of course, is that each of the 370 neutrons produced by fission has deposited about 200 MeV of energy in the reactor core, heating it up.

An important reactor parameter is the *multiplication factor k*, the ratio of the number of neutrons present at the beginning of a particular generation to the number present at the beginning of the next generation. For the situation of

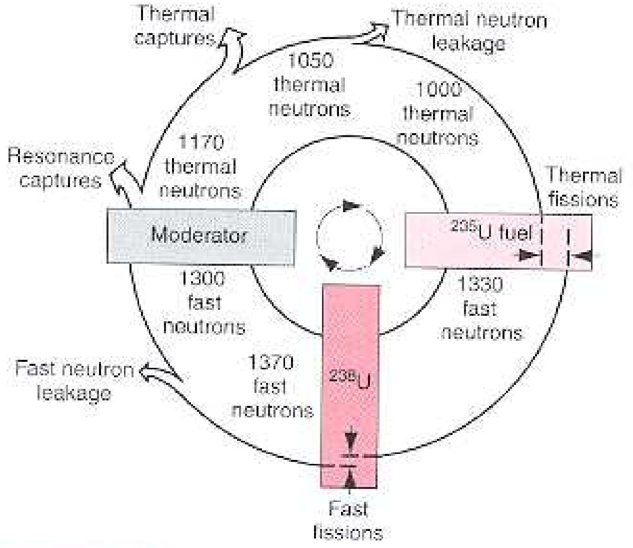

FIGURE 51-5. A generation of 1000 thermal neutrons is followed through various stages in a reactor. At a steady operating level, the loss of neutrons due to captures (in the fuel, moderator, and structural elements) and leakage through the surface is exactly balanced by the production of neutrons in the fission processes.

Fig. 51-5, the multiplication factor is exactly 1. For $k = 1$, the operation of the reactor is said to be exactly *critical*, which is what we wish it to be for steady power production. Reactors are designed so that they are inherently *supercritical* ($k > 1$); the multiplication factor is then adjusted to critical operation ($k = 1$) by inserting *control rods* into the reactor core. These rods, containing a material such as cadmium that absorbs neutrons readily, can then be withdrawn as needed to compensate for the tendency of reactors to go subcritical as (neutron-absorbing) fission products build up in the core during continued operation.

If you pulled out one of the control rods, how fast would the reactor power level increase? This *response time* is controlled by the fascinating circumstance that a small fraction of the neutrons generated by fission is not emitted promptly from the newly formed fission fragments but is emitted from these fragments later, as they decay by beta emission. Of the 370 "new" neutrons analyzed in Fig. 51-5, for example, about 16 are delayed, being emitted from fragments following beta decays whose half-lives range from 0.2 to 55 s. These delayed neutrons are few in number but they serve the useful purpose of slowing down the reactor response time to match human reaction times.

Figure 51-6 shows the broad outlines of an electric power plant based on a *pressurized-water reactor* (PWR), a type in common use in the United States. In such a reactor, water is used both as the moderator and as the heat transfer medium. In the *primary loop*, water at high temperature and pressure (possibly 600 K and 150 atm) circulates through the reactor vessel and transfers heat from the reactor core to the steam generator, which provides high-pressure steam to operate the turbine that drives the generator. To complete the *secondary loop*, low-pressure steam from the turbine is condensed to water and forced back into the steam generator by a pump. To give some idea of scale, a typical reactor vessel for a 1000-MW (electric) plant may be 10 m high and weigh 450 tons. Water flows through the primary loop at a rate of about 300,000 gal/min.

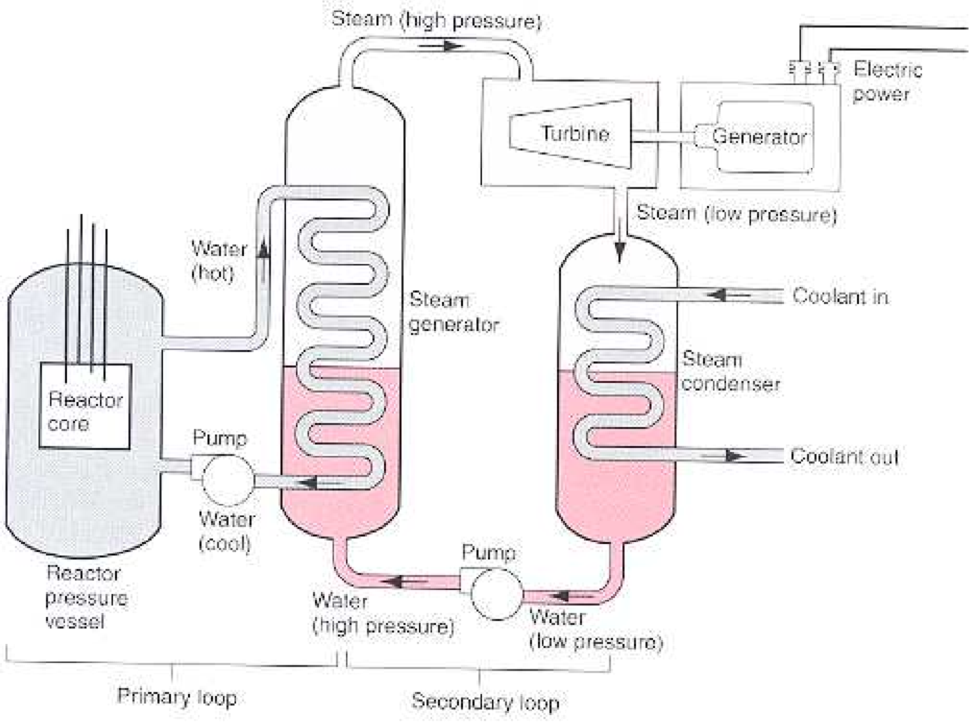

FIGURE 51-6. A simplified layout of a nuclear power plant based on a pressurized-water reactor.

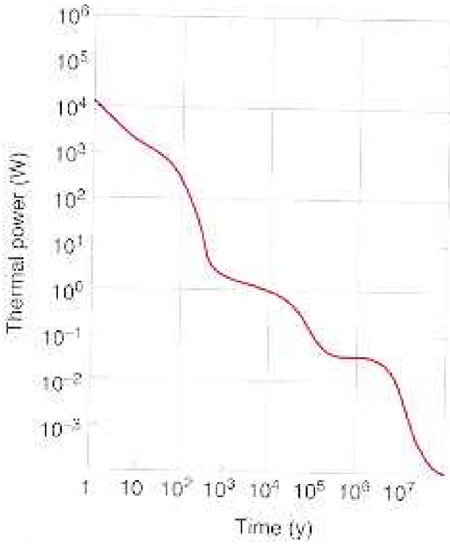

FIGURE 51-7. Thermal power released by the radioactive wastes of one year's operation of a typical large nuclear power plant, as a function of time after the fuel is removed. The curve represents the effect of many radionuclides with a range of half-lives. Note that both scales are logarithmic.

An unavoidable feature of reactor operation is the accumulation of radioactive wastes, including both fission products and heavy "transuranic" nuclides such as plutonium and americium. One measure of their radioactivity is the rate at which they release energy in thermal form. Figure 51-7 shows the variation with time of the thermal power generated by such wastes from one year's operation of a typical large nuclear plant. Note that both scales are logarithmic. The total activity of the waste 10 years after its removal from the reactor is about 3×10^7 Ci.

SAMPLE PROBLEM 51-3. A large electric generating station is powered by a pressurized-water nuclear reactor. The thermal power in the reactor core is 3400 MW, and 1100 MW of electricity is generated. The fuel consists of 86,000 kg of uranium, in the form of 110 tons of uranium oxide, distributed among 57,000 fuel rods. The uranium is enriched to 3.0% ^{235}U. (*a*) What is the plant efficiency? (*b*) At what rate R do fission events occur in the reactor core? (*c*) At what rate is the ^{235}U fuel disappearing? Assume conditions at start-up? (*d*) At this rate of fuel consumption, how long would the fuel supply last? (*e*) At what rate is mass being lost in the reactor core?

Solution (*a*) The efficiency e is the ratio between the power output (in the form of electric energy) to the power input (in the form of thermal energy), or

$$e = \frac{\text{electric output}}{\text{thermal input}} = \frac{1100\text{ MW}}{3400\text{ MW}} = 0.32 \text{ or } 32\%.$$

As for all power plants, whether based on fossil fuel or nuclear fuel, the efficiency is controlled by the second law of thermodynamics. In this plant, 3400 MW − 1100 MW or 2300 MW of power must be discharged as thermal energy to the environment.
(*b*) If P (= 3400 MW) is the thermal power in the core and Q (= 200 MeV) is the average energy released per fission event, then, in steady-state operation,

$$R = \frac{P}{Q} = \frac{3.4 \times 10^9\text{ J/s}}{(200\text{ MeV/fission})(1.6 \times 10^{-13}\text{ J/MeV})}$$
$$= 1.06 \times 10^{20}\text{ fissions/s}.$$

(*c*) ^{235}U disappears by fission at the rate calculated in (*b*). It is also consumed by (nonfission) neutron capture at a rate about one-fourth as large. The total ^{235}U consumption rate is then $(1.25)(1.06 \times 10^{20}\text{ s}^{-1})$ or $1.33 \times 10^{20}\text{ s}^{-1}$. We recast this as a mass rate as follows:

$$\frac{dM}{dt} = (1.33 \times 10^{20}\text{ s}^{-1})\left(\frac{0.235\text{ kg/mol}}{6.02 \times 10^{23}\text{ atoms/mol}}\right)$$
$$= 5.19 \times 10^{-5}\text{ kg/s} = 4.5\text{ kg/d}.$$

(*d*) From the data given, we can calculate that, at start-up, about (0.03)(86,000 kg) or 2600 kg of ^{235}U were present. Thus a somewhat simplistic answer would be

$$T = \frac{2600\text{ kg}}{4.5\text{ kg/d}} = 580\text{ d}.$$

In practice, the fuel rods are replaced (often in batches) before their ^{235}U content is entirely consumed.
(*e*) From Einstein's $E = \Delta m\,c^2$ relation, we can write

$$\frac{dM}{dt} = \frac{dE/dt}{c^2} = \frac{3.4 \times 10^9\text{ W}}{(3.00 \times 10^8\text{ m/s})^2}$$
$$= 3.8 \times 10^{-8}\text{ kg/s} = 3.3\text{ g/d}.$$

The mass loss rate is about the mass of one penny every day! This mass loss rate (reduction in rest energy) is quite a different quantity than the fuel consumption rate (loss of ^{235}U) calculated in part (*c*).

51-5 A NATURAL REACTOR

On December 2, 1942, when the reactor assembled by Enrico Fermi and his associates first went critical, they had every right to expect that they had put into operation the first fission reactor that had ever existed on this planet. About 30 years later it was discovered that, if they did in fact think that, they were wrong.

Some two billion years ago, in a uranium deposit now being mined in Gabon, West Africa, a natural fission reactor went into operation and ran for perhaps several hundred thousand years before shutting itself off.

The story of this discovery is fascinating at the level of the best detective thriller. More important, it provides a first-class example of the nature of the scientific evidence needed to back up what may seem at first to be an improbable claim. It set a high standard for all who speculate about past events. We consider here only two points.*

*For the complete story, see "A Natural Fission Reactor," by George A. Cowan, *Scientific American*, July 1976, p. 36.

1. *Was there enough fuel?* The fuel for a uranium-based fission reactor must be the easily fissionable isotope ^{235}U, which constitutes only 0.72% of natural uranium. This isotopic ratio has been measured not only for terrestrial samples but also in Moon rocks and in meteorites, in which the same value is always found. The initial clue to the discovery in Gabon was that the uranium from this deposit was deficient in ^{235}U, some samples having an abundance as low as 0.44%. Investigation led to the speculation that this deficit in ^{235}U could be accounted for if, at some time in the past, this isotope was partially consumed by the operation of a natural fission reactor.

The serious problem remains that, with an isotopic abundance of only 0.72%, a reactor can be assembled (as Fermi and his team learned) only with the greatest of difficulty. There seems no chance at all that it could have happened naturally.

However, things were different in the distant past. Both ^{235}U and ^{238}U are radioactive, with half-lives of 0.704×10^9 y and 4.47×10^9 y, respectively. Thus the half-life of the readily fissionable ^{235}U is about 6.4 times shorter than that of ^{238}U. Because ^{235}U decays faster, there must have been more of it, relative to ^{238}U, in the past. Two billion years ago, in fact, this abundance was not 0.72%, as it is now, but 3.8%. This abundance happens to be just about the abundance to which natural uranium is artificially enriched to serve as fuel in modern power reactors.

With this amount of readily fissionable fuel available in the distant past, the presence of a natural reactor (providing certain other conditions are met) is much less surprising. The fuel was there. Two billion years ago, incidentally, the highest order of life forms that had evolved were the blue-green algae.

2. *What is the evidence?* The mere depletion of ^{235}U in an ore deposit is not enough evidence on which to base a claim for the existence of a natural fission reactor. More convincing proof is needed.

If there were a reactor, there must also be fission products; see Fig. 51-2. Of the 30 or so elements whose stable isotopes are produced in this way, some must still remain. Study of their isotopic ratios could provide the convincing evidence we need.

Of the several elements investigated, the case of neodymium is spectacularly convincing. Figure 51-8*a* shows the isotopic abundances of the seven stable neodymium isotopes as they are normally found in nature. Figure 51-8*b* shows these abundances as they appear among the ultimate stable products of the fission of ^{235}U. The clear differences are not surprising, considering their totally different origins. The isotopes shown in Fig. 51-8*a* were formed in supernova explosions that occurred before the formation of our solar system. The isotopes of Fig. 51-8*b* were cooked up in a reactor by totally different processes. Note particularly that ^{142}Nd, the dominant isotope in the natural element, is totally absent from the fission products.

The big question is: What do the neodymium isotopes found in the uranium ore body in Gabon look like? We must expect that, if a natural reactor operated there, isotopes from *both* sources (that is, natural isotopes as well as fission-produced isotopes) might be present. Figure 51-8*c* shows the results after this and other corrections have been made to the raw data. Comparison of Figs. 51-8*b* and 51-8*c* certainly suggests that there was indeed a natural fission reactor at work!

SAMPLE PROBLEM 51-4. The isotopic ratio of ^{235}U to ^{238}U in natural uranium deposits today is 0.0072. What was this ratio 2.0×10^9 y ago? The half-lives of the two isotopes are 0.704×10^9 y and 4.47×10^9 y, respectively.

Solution Consider two samples that, at a time t in the past, contained $N_5(0)$ and $N_8(0)$ atoms of ^{235}U and ^{238}U, respectively. The numbers of atoms remaining at the present time are

$$N_5(t) = N_5(0)e^{-\lambda_5 t} \quad \text{and} \quad N_8(t) = N_8(0)e^{-\lambda_8 t},$$

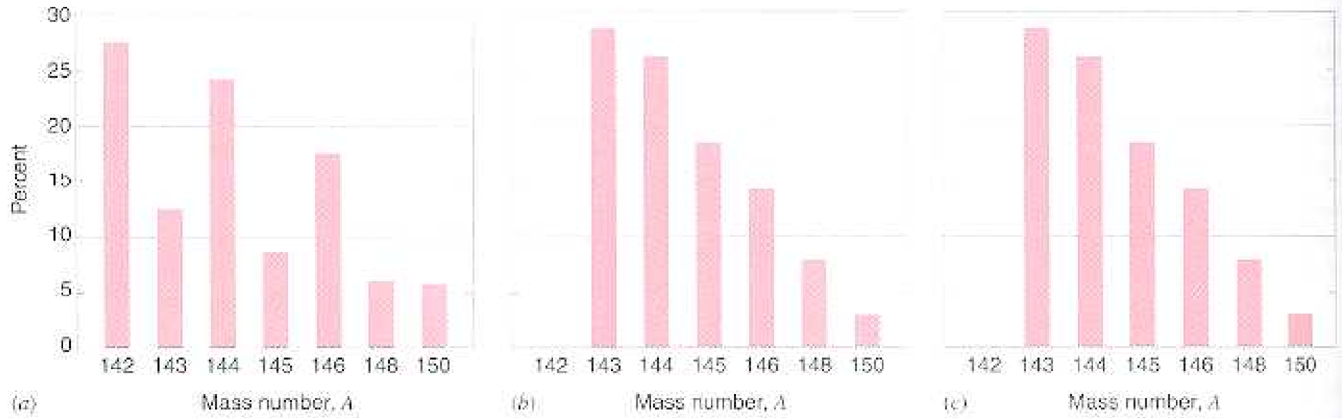

FIGURE 51-8. The distribution by mass number of the isotopes of neodymium as they occur in (*a*) natural terrestrial deposits, (*b*) the spent fuel of a power reactor, and (*c*) the uranium mine in Gabon, West Africa. Note that (*b*) and (*c*) are virtually identical and quite different from (*a*).

respectively, in which λ_5 and λ_8 are the corresponding disintegration constants. Dividing gives

$$\frac{N_5(t)}{N_8(t)} = \frac{N_5(0)}{N_8(0)} e^{-(\lambda_5 - \lambda_8)t}.$$

Expressed in terms of the isotopic ratio $R = N_5/N_8$, this becomes

$$R(0) = R(t)e^{(\lambda_5 - \lambda_8)t}.$$

The disintegration constants are related to the half-lives by Eq. 50-8, or

$$\lambda_5 = \frac{\ln 2}{t_{1/2}(^{235}\mathrm{U})} = \frac{0.693}{7.04 \times 10^8\ \mathrm{y}} = 0.984 \times 10^{-9}\ \mathrm{y}^{-1}$$

and

$$\lambda_8 = \frac{\ln 2}{t_{1/2}(^{238}\mathrm{U})} = \frac{0.693}{4.47 \times 10^9\ \mathrm{y}} = 0.155 \times 10^{-9}\ \mathrm{y}^{-1}.$$

Substituting in the expression for the isotopic ratio gives

$$\begin{aligned} R(0) &= R(t)e^{(\lambda_5 - \lambda_8)t} \\ &= (0.0072)e^{(0.984-0.155)(10^{-9}\ \mathrm{y}^{-1})(2.00\times 10^9\ \mathrm{y})} \\ &= (0.0072)e^{1.66} = 0.0378 \text{ or } 3.78\%. \end{aligned}$$

We see that, two billion years ago, the ratio of ^{235}U to ^{238}U in natural uranium deposits was much higher than it is today. When the Earth was formed (4.5 billion years ago) this ratio was 30%.

51-6 THERMONUCLEAR FUSION: THE BASIC PROCESS

We pointed out in connection with the binding energy curve of Fig. 50-6 that energy can be released if light nuclei are combined to form nuclei of somewhat larger mass number, a process called *nuclear fusion*. However, this process is hindered by the mutual Coulomb repulsion that tends to prevent two such (positively) charged particles from coming within range of each other's attractive nuclear forces and "fusing." This reminds us of the potential barrier that inhibits nuclear fission (see Fig. 51-4) and also of the barrier that inhibits alpha decay (see Fig. 50-9).

In the case of alpha decay, two charged particles—the α particle and the residual nucleus—are initially *inside* their mutual potential barrier. For alpha decay to occur, the α particle must leak through this barrier by the barrier-tunneling process and appear on the *outside*. In nuclear fusion the situation is reversed. Here the two particles must penetrate their mutual barrier from the *outside* if a nuclear interaction is to occur.

The interaction between two deuterons is of particular importance in fusion. Sample Problem 51-5 gives a rough calculation of the potential barrier between two deuterons, which works out to be about 200 keV. The corresponding barrier for two interacting ^{3}He nuclei (charge $= +2e$) is about 1 MeV. For more highly charged particles the barrier, of course, is correspondingly higher.

One way to arrange for light nuclei to penetrate their mutual Coulomb barrier is to use one light particle as a target and to accelerate the other by means of a cyclotron or a similar device. To generate power in a useful way from the fusion process, however, we must have the interaction of matter in bulk, just as in the combustion of coal. The cyclotron technique holds no promise in this direction. The best hope for obtaining fusion in bulk matter in a controlled fashion is to raise the temperature of the material so that the particles have sufficient energy to penetrate the barrier due to their thermal motions alone. This process is called *thermonuclear fusion*.

The average thermal kinetic energy K_{av} of a particle in equilibrium at a temperature T is given, as we have seen in Section 22-4, by

$$K_{av} = \tfrac{3}{2}kT, \qquad (51\text{-}3)$$

where k ($= 8.62 \times 10^{-5}$ eV/K) is the Boltzmann constant. At room temperature ($T \approx 300$ K), $K_{av} = 0.04$ eV, which is, of course, far too small for our purpose.

Even at the center of the Sun, where $T \approx 1.5 \times 10^7$ K, the mean thermal kinetic energy calculated from Eq. 51-3 is only 1.9 keV. This still seems hopelessly small in view of the magnitude of the Coulomb barrier of 200 keV calculated in Sample Problem 51-5. Yet we know that thermonuclear fusion not only occurs in the solar interior but is its central and dominant feature.

The puzzle is solved with the realization that (1) the energy calculated from Eq. 51-3 is a *mean* kinetic energy; particles with energies much greater than this mean value constitute the high-energy "tails" of the Maxwellian speed distribution curves (see Fig. 22-6). Also, (2) the barrier heights that we have quoted represent only the *peaks* of the barriers. Barrier tunneling can occur to a significant extent at energies well below these peaks, as we saw in Section 50-4 in the case of alpha decay.

Figure 51-9 summarizes the situation by a quantitative example. The curve marked $n(K)$ in this figure is a Maxwell distribution of kinetic energies (see Section 22-5) drawn to correspond to the Sun's central temperature,

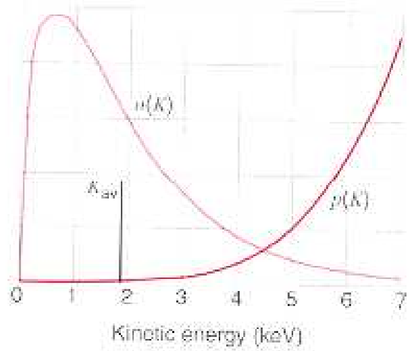

FIGURE 51-9. The curve marked $n(K)$ gives the distribution in energy of protons in the core of the Sun, corresponding to a temperature of 1.5×10^7 K. The vertical line indicates the mean kinetic energy per particle at that temperature. The curve marked $p(K)$ gives the probability of barrier penetration in proton–proton collisions. The two curves are drawn to different arbitrary vertical scales.

1.5×10^7 K. Although the same curve holds no matter what particle is under consideration, we focus our attention on protons, bearing in mind that hydrogen forms about 35% of the mass of the Sun's central core.

For $T = 1.5 \times 10^7$ K, Eq. 51-3 yields $K_{av} = 1.9$ keV, and this value is indicated by a vertical line in Fig. 51-9. Note that there are many particles whose energies exceed this mean value.

The curve marked $p(K)$ in Fig. 51-9 is the probability of barrier penetration for two colliding protons. At $K = 6$ keV, for example, we have $p = 2.4 \times 10^{-5}$. This is the probability that two colliding protons, each with $K = 6$ keV, will succeed in penetrating their mutual Coulomb barrier and coming within range of each other's strong nuclear forces. Put another way, on the average, one of every 42,000 such encounters will succeed.

It turns out that the most probable energy for proton–proton fusion events to occur at the Sun's central temperature is about 6 keV. If the energy is much higher, the barrier is more easily penetrated (that is, p is greater), but there are too few protons in the Maxwellian "tail" (n is smaller). If the energy is much lower, there are plenty of protons but the barrier is now too formidable.

SAMPLE PROBLEM 51-5. The deuteron (^{2}H) has a charge $+e$, and its radius has been measured to be 2.1 fm. Two such particles are fired at each other with the same initial kinetic energy K. What must K be if the particles are brought to rest by their mutual Coulomb repulsion when the two deuterons are just "touching"?

Solution Because the two deuterons are momentarily at rest when they "touch" each other, their kinetic energy has all been transformed into electrostatic potential energy associated with the Coulomb repulsion between them. If we treat them as point charges separated by a distance $2R$, we have

$$2K = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{2R},$$

which yields

$$K = \frac{1}{4\pi\epsilon_0}\frac{e^2}{4R}$$

$$= \frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})^2}{4(2.1 \times 10^{-15}\ \text{m})}$$

$$= 2.7 \times 10^{-14}\ \text{J} = 170\ \text{keV}.$$

This quantity provides a reasonable measure of the height of the Coulomb barrier between two deuterons.

51-7 THERMONUCLEAR FUSION IN STARS

Here we consider in more detail the thermonuclear fusion processes that take place in our Sun and in other stars. In the Sun's deep interior, where its mass is concentrated and where most of the energy production takes place, the (central) temperature is 1.5×10^7 K and the central density is on the order of 10^5 kg/m^3, about 13 times the density of lead. The central temperature is so high that, in spite of the high central pressure (2×10^{11} atm), the Sun remains gaseous throughout.

The present composition of the Sun's core is about 35% hydrogen by mass, about 65% helium, and about 1% other elements. At these temperatures the light elements are essentially totally ionized, so that our picture is one of an assembly of protons, electrons, and α particles in random motion.

The Sun radiates at the rate of 3.9×10^{26} W and has been doing so for as long as the solar system has existed, which is about 4.5×10^9 y. It has been known since the 1930s that thermonuclear fusion processes in the Sun's interior account for its prodigious energy output. Before analyzing this further, however, let us dispose of two other possibilities that had been put forward earlier. Consider first chemical reactions such as simple burning. If the Sun, whose mass is 2.0×10^{30} kg, were made of coal and oxygen in just the right proportions for burning, it would last only about 10^3 y, which of course is far too short (see Exercise 41). The Sun, as we shall see, does not burn coal but hydrogen, and in a nuclear furnace, not an atomic or chemical one.

Another possibility is that, as the core of the Sun cools and the pressure there drops, the Sun will shrink under the action of its own strong gravitational forces. By transferring gravitational potential energy to internal energy (just as we do when we drop a stone onto the Earth's surface), the temperature of the Sun's core will rise so that radiation may continue. Calculation shows, however, that the Sun could radiate from this cause for only about 10^8 y, too short by a factor of 25 (see Problem 7).

The Sun's energy is generated by the thermonuclear "burning" (that is, "fusing") of hydrogen to form helium. Figure 51-10 shows the *proton–proton cycle* by which this is accomplished. Note that each reaction shown is a fusion reaction, in that one of the products (^{2}H, ^{3}He, or ^{4}He) has a higher mass number than any of the reacting particles that form it. The reaction energy Q for each reaction shown in Fig. 51-10 is positive. This characterizes an exothermic reaction, with the net release of energy.

The cycle is initiated by the collision of two protons (^{1}H + ^{1}H) to form a deuteron (^{2}H), with the simultaneous creation of a positron (e^+) and a neutrino (ν). The positron very quickly encounters a free electron (e^-) in the Sun and both particles annihilate, their rest energies appearing as two gamma-ray photons (γ). In Fig. 51-10 we follow the consequences of two such events, as indicated in the top row of the figure. Such events are extremely rare. In fact, only once in about 10^{26} proton–proton collisions is a deuteron formed; in the vast majority of cases the colliding protons simply scatter from each other. It is the slowness of this process that regulates the rate of energy production and keeps the Sun from exploding. In spite of this slowness, there are so very many protons in the huge volume of the Sun's core that deuterium is produced there in this way at the rate of about 10^{12} kg/s!

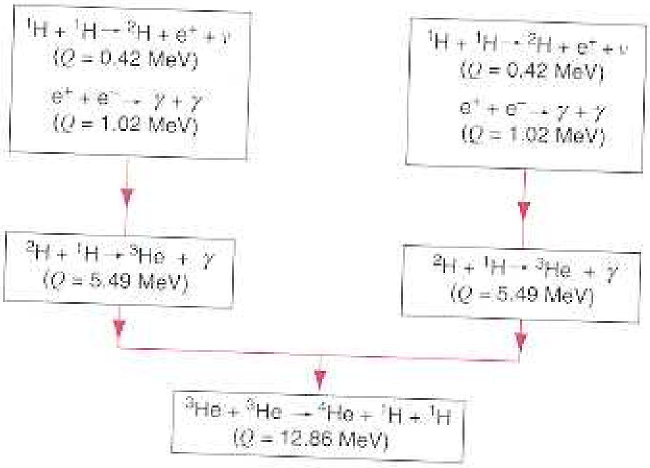

FIGURE 51-10. The proton–proton cycle that primarily accounts for energy production in the Sun.

Once a deuteron has been produced, it quickly (within a few seconds) collides with another proton and forms a ^{3}He nucleus, as the second row of Fig. 51-10 shows. Two such ^{3}He nuclei may then eventually (within about 10^5 y) collide, forming an α particle (^{4}He) and two protons, as the third row of the figure shows. There are other variations of the proton–proton cycle, involving other light elements, but we concentrate on the principal sequence as represented in Fig. 51-10.

Taking an overall view of the proton–proton cycle, we see that it amounts to the combination of four protons and two electrons to form an α particle, two neutrinos, and six gamma rays:

$$4\,^1\mathrm{H} + 2e^- \rightarrow {}^4\mathrm{He} + 2\nu + 6\gamma. \qquad (51\text{-}4)$$

Now, in a formal way, let us add two electrons to each side of Eq. 51-4, yielding

$$4(^1\mathrm{H} + e^-) \rightarrow (^4\mathrm{He} + 2e^-) + 2\nu + 6\gamma. \qquad (51\text{-}5)$$

The quantities in parentheses then represent *atoms* (not bare nuclei) of hydrogen and of helium.

The energy release in the reaction of Eq. 51-5 is, using the atomic masses of hydrogen and helium,

$$\begin{aligned} Q &= (m_i - m_f)c^2 = [4m(^1\mathrm{H}) - m(^4\mathrm{He})]c^2 \\ &= [4(1.007825\ \mathrm{u}) - 4.002603\ \mathrm{u}](931.5\ \mathrm{MeV/u}) \\ &= 26.7\ \mathrm{MeV}. \end{aligned}$$

(Gamma-ray photons are massless, and neutrinos have either zero or negligibly small mass; thus neither particle enters into the calculation of the Q value for the fusion reaction.) This same value of Q follows (as it must) by adding up the Q values for the separate steps of the proton–proton cycle in Fig. 51-10.

Not quite all this energy is available as internal energy inside the Sun. About 0.5 MeV is associated with the two neutrinos that are produced in each cycle. Neutrinos are so penetrating that in essentially all cases they escape from the Sun, carrying this energy with them. Some are intercepted by the Earth, bringing us our only direct information about the Sun's interior.

Subtracting the neutrino energy leaves 26.2 MeV per cycle available within the Sun. As we show in Sample Problem 51-6, this corresponds to a "heat of combustion" for the nuclear burning of hydrogen into helium of 6.3×10^{14} J/kg of hydrogen consumed. By comparison, the heat of combustion of coal is about 3.3×10^7 J/kg, some 20 million times lower, reflecting roughly the general ratio of energies in nuclear and chemical processes.

We may ask how long the Sun can continue to shine at its present rate before all the hydrogen in its core has been converted into helium. Hydrogen burning has been going on for about 4.5×10^9 y, and calculations show that there is enough available hydrogen left for about 5×10^9 y more. At that time major changes will begin to happen. The Sun's core, which by then will be largely helium, will begin to collapse and to heat up while the outer envelope will expand greatly, perhaps so far as to encompass the Earth's orbit. The Sun will become what astronomers call a *red giant*.

If the core temperature heats up to about 10^8 K, energy can be produced by burning helium to make carbon. Helium does not burn readily, the only possible reaction being

$$^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma \qquad (Q = +7.3\ \mathrm{MeV}).$$

Such a three-body collision of three α particles must occur within 10^{-16} s if the reaction is to go. Nevertheless, if the density and temperature of the helium core are high enough, carbon will be manufactured by the burning of helium in this way.

As a star evolves and becomes still hotter, other elements can be formed by other fusion reactions. However, elements beyond $A = 56$ cannot be manufactured by further fusion processes. The elements with $A = 56$ (^{56}Fe, ^{56}Co, ^{56}Ni) lie near the peak of the binding energy curve of Fig. 50-6, and fusion between nuclides beyond this point involves the consumption, and not the production, of energy. The production of the elements in fusion processes is discussed in Chapter 52.

SAMPLE PROBLEM 51-6. At what rate is hydrogen being consumed in the core of the Sun, assuming that all the radiated energy is generated by the proton–proton cycle of Fig. 51-10?

Solution We have seen that 26.2 MeV appears as internal energy in the Sun for every four protons consumed, a rate of 6.6 MeV/proton. We can express this as

$$\frac{(6.6\ \mathrm{MeV/proton})(1.6 \times 10^{-13}\ \mathrm{J/MeV})}{1.67 \times 10^{-27}\ \mathrm{kg/proton}} = 6.3 \times 10^{14}\ \mathrm{J/kg},$$

which tells us that the Sun radiates away 6.3×10^{14} J for every kilogram of protons consumed. The hydrogen consumption rate is then the output power ($= 3.9 \times 10^{26}$ W) divided by this quantity, or

$$\frac{dm}{dt} = \frac{3.9 \times 10^{26}\ \mathrm{W}}{6.3 \times 10^{14}\ \mathrm{J/kg}} = 6.2 \times 10^{11}\ \mathrm{kg/s}.$$

To keep this number in perspective, keep in mind that the Sun's mass is 2.0×10^{30} kg.

51-8 CONTROLLED THERMONUCLEAR FUSION

Thermonuclear reactions have been going on in the universe since its creation in the presumed cosmic "big bang" of some 15 billion years ago. Such reactions have taken place on Earth, however, only since October 1952, when the first fusion (or hydrogen) bomb was exploded. The high temperatures needed to initiate the thermonuclear reaction in this case were provided by a fission bomb used as a trigger.

A sustained and controllable thermonuclear power source—a fusion reactor—is proving much more difficult to achieve. The goal, however, is vigorously pursued because many look to the fusion reactor as the ultimate power source of the future, at least as far as the generation of electricity is concerned.

The proton–proton interaction displayed in Fig. 51-10 is not suitable for use in a terrestrial fusion reactor because the process described in the first row is hopelessly slow. The reaction probability is in fact so small that it cannot be measured in the laboratory. The reaction succeeds under the conditions that prevail in stellar interiors only because of the enormous number of protons available in the high-density stellar cores.

The most attractive reactions for terrestrial use appear to be the deuteron–deuteron (d-d) and the deuteron–triton (d-t) reactions:

$$\text{d-d:}\quad {}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \text{n} \qquad (Q = +3.27\ \text{MeV}), \tag{51-6}$$

$$\text{d-d:}\quad {}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{H} + {}^1\text{H} \qquad (Q = +4.03\ \text{MeV}), \tag{51-7}$$

$$\text{d-t:}\quad {}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + \text{n} \qquad (Q = +17.59\ \text{MeV}). \tag{51-8}$$

Here *triton* indicates ^{3}H, the nucleus of hydrogen with $A = 3$. Note that each of these reactions is indeed a fusion reaction and has a positive Q value. Deuterium, whose isotopic abundance in normal hydrogen is 0.015%, is available in unlimited quantities as a component of seawater. Tritium (atomic ^{3}H) is radioactive and is not normally found in naturally occurring hydrogen.

There are three basic requirements for the successful operation of a thermonuclear reactor.

1. *A high particle density n.* The number of interacting particles (deuterons, say) per unit volume must be great enough to ensure a sufficiently high deuteron–deuteron collision rate. At the high temperatures required, the deuterium gas would be completely ionized into a neutral *plasma* consisting of deuterons and electrons.

2. *A high plasma temperature T.* The plasma must be hot. Otherwise the colliding deuterons will not be energetic enough to penetrate the mutual Coulomb barrier that tends to keep them apart. In fusion research, temperatures are often reported by giving the corresponding value of kT (not $\frac{3}{2}kT$). A plasma temperature of 43 keV, corresponding to 5.0×10^8 K, has been achieved in the laboratory. This is much higher than the Sun's central temperature (1.3 keV, or 1.5×10^7 K).

3. *A long confinement time τ.* A major problem is containing the hot plasma long enough to ensure that its density and temperature remain sufficiently high. It is clear that no actual solid container can withstand the high temperatures necessarily involved, so special techniques, to be described later, must be employed. By use of one such technique, confinement times greater than 1 s have been achieved.

For the successful operation of a thermonuclear reactor, it can be shown that n, T, and τ must be large enough so

$$n\tau T \geq 50 \times 10^{20}\ \text{keV}\cdot\text{s/m}^3, \tag{51-9}$$

a condition sometimes called *Lawson's criterion.* Equation 51-9 tells us, for a high enough plasma temperature, that we have a choice between confining many particles for a relatively short time or confining fewer particles for a somewhat longer time.

Two techniques have been used to attempt to meet Lawson's criterion and so achieve controlled fusion. *Magnetic confinement* uses magnetic fields to confine the plasma while its temperature is increased. In *inertial confinement,* on the other hand, a small amount of fuel is compressed and heated so rapidly that fusion occurs before the fuel can expand and cool. These techniques are discussed in the following two sections.

Magnetic Confinement

Because a plasma consists of charged particles, its motion can be controlled with magnetic fields. For example, charged particles spiral about the direction of a uniform magnetic field. By suitably varying the field strength, it is possible to design a "magnetic mirror" (see Fig. 32-14) from which particles can be reflected. Another design makes use of toroidal geometry, in which the particles spiral around the axis of a toroid inside a "doughnut-shaped" vacuum chamber. The type of fusion reactor based on this principle, which was first developed in Russia, is called a *tokamak,* which comes from the Russian acronym for "toroidal magnetic chamber." Several large machines of this type have been built and tested.

In a tokamak, there are two components to the magnetic field, as illustrated in Fig. 51-11. The *toroidal* field B_t is the one we usually associate with a toroidal winding of wires; Fig. 51-11 shows one small section of an external coil that contributes to the toroidal field. Because the toroidal field decreases with increasing radius, it is necessary to add a second field component to confine the particles. This *poloidal* component B_p of the field adds to the toroidal component to give the total field a helical structure, as illustrated in Fig. 51-11. The poloidal field is produced by a current i' in the plasma itself, which is induced by a set of

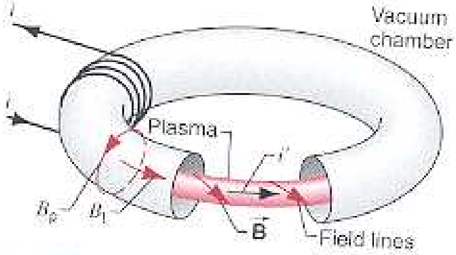

FIGURE 51-11. The toroidal chamber that forms the basis of the tokamak. Note the plasma, the helical magnetic field $\vec{\mathbf{B}}$ that confines it, and the induced current i' that heats it.

windings not illustrated in the figure. This current also serves to heat the plasma. Additional means of heating, such as by firing neutral beams of particles into the plasma, are also necessary to achieve the desired plasma temperature.

Figure 51-12 shows the toroidal vacuum chamber of the Tokamak Fusion Test Reactor at the Princeton Plasma Physics Laboratory. The interior radius of the vacuum chamber is about 2 m, and the major radius of the toroid is 2.5 m. This facility operated until 1998 to develop and test many of the concepts necessary to achieve a workable fusion reactor. It was able to produce 10 MW of fusion power for about 1 second. A larger experimental facility, called the International Thermonuclear Experimental Reactor (ITER) is currently under development as a joint effort by the European Union, Japan, and Russia. This facility, which is planned to be in operation by about the year 2008, is being designed in its initial operation to produce 500 MW of fusion power for times of several minutes.

In designing magnetic confinement devices such as the tokamak, the goal is to increase n, τ, and T. At sufficiently high values of these parameters, Lawson's criterion is satisfied and fusion reactions in the plasma will produce enough energy to equal the energy that must be supplied to

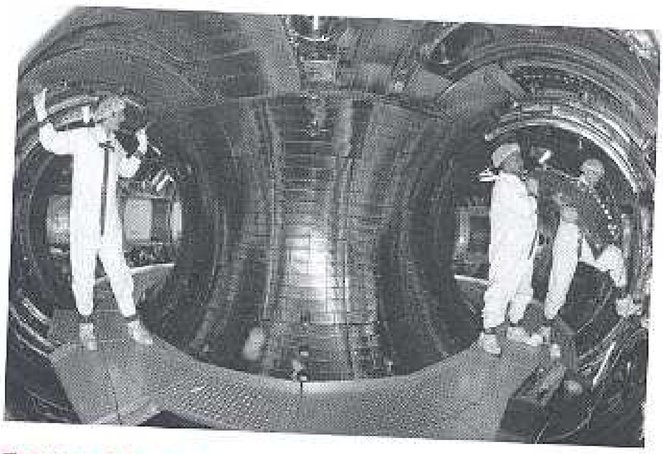

FIGURE 51-12. Workers inside the toroidal chamber of the Tokamak Fusion Test Reactor at Princeton University.

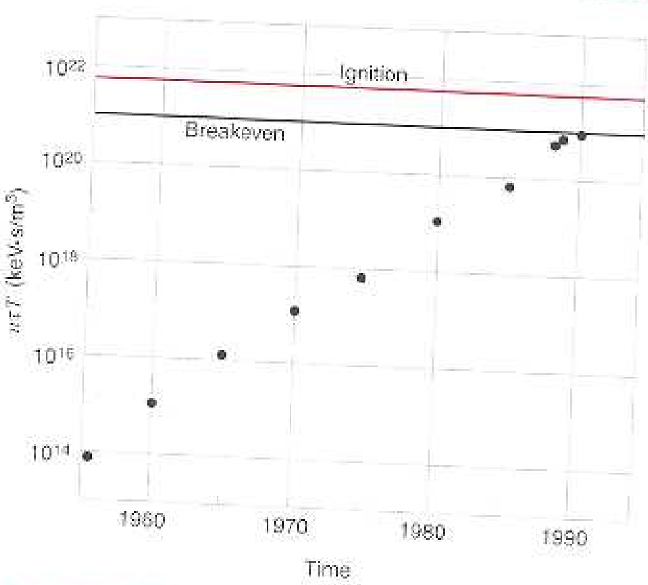

FIGURE 51-13. The approach to breakeven and ignition in controlled fusion reactors, shown as a plot of $n\tau T$ against time.

heat the plasma. This condition is called "breakeven." At still higher values of these parameters, the device will achieve "ignition," where self-sustaining fusion reactions will occur. Figure 51-13 illustrates the steady progress toward these goals that has been made. Despite the near achievement of the breakeven condition, many formidable engineering problems remain to be solved, and the production of electric power from fusion is likely to be many decades away.

SAMPLE PROBLEM 51-7. The Tokamak Fusion Test Reactor at Princeton has achieved a confinement time of 400 ms. (*a*) If the plasma temperature, measured in energy units as kT, is 20 keV, what must be the density of particles in the plasma if Lawson's criterion for ignition is to be satisfied? (*b*) How does this number compare with the particle density of the atoms of an ideal gas at standard conditions? (*c*) Under these same conditions of density and temperature, what confinement time is necessary to achieve the breakeven condition?

Solution (*a*) Using Lawson's criterion (Eq. 51-9), we must have

$$n = \frac{50 \times 10^{20}\ \text{keV} \cdot \text{s/m}^3}{\tau T} = \frac{50 \times 10^{20}\ \text{keV} \cdot \text{s/m}^3}{(400 \times 10^{-3}\ \text{s})(20\ \text{keV})} = 6.3 \times 10^{20}\ \text{m}^{-3}.$$

(*b*) The number density of atoms in an ideal gas at standard conditions is given by $n' = N_A/V_m$, where N_A is the Avogadro constant and V_m ($= 2.24 \times 10^{-2}\ \text{m}^3/\text{mol}$) is the molar volume of an ideal gas at standard conditions, which gives

$$n' = \frac{N_A}{V_m} = \frac{6.02 \times 10^{23}\ \text{mol}^{-1}}{2.24 \times 10^{-2}\ \text{m}^3/\text{mol}} = 2.7 \times 10^{25}\ \text{m}^{-3}.$$

The particle density of the plasma we found in part (*a*) is smaller than that of an ideal gas by a factor of about 4×10^4.

(*c*) Figure 51-13 shows that for breakeven we must have $n\tau T \approx 1 \times 10^{21}$ keV · s/m³, or about $\frac{1}{5}$ of Lawson's criterion for ignition. With the same density and temperature as part (*a*), we must therefore have about $\frac{1}{5}$ of the confinement time, or about 80 ms.

Inertial Confinement

A second technique for confining plasma so that thermonuclear fusion can take place is called *inertial confinement*. In terms of Lawson's criterion (Eq. 51-9), it involves working with extremely high particle densities n for extremely short confinement times τ. These times are arranged to be so short that the fusion episode is over before the particles of the plasma have time to move appreciably from the positions they occupy at the onset of fusion. The interacting particles are confined by their own inertia.

Laser fusion, which relies on the inertial-confinement principle, is being investigated in laboratories throughout the world. At the Lawrence Livermore National Laboratory, for example, in the NOVA laser fusion project (see Fig. 51-14) deuterium–tritium fuel pellets, each smaller than a grain of sand (see Fig. 51-15), are to be "zapped" by 10 synchronized, high-powered laser pulses, symmetrically arranged around the pellet. The laser pulses are designed to deliver in total some 35 kJ of energy to each fuel pellet in less than a nanosecond. This is a delivered power of 4×10^{13} W during the pulse, which is roughly 100 times the total installed electric power-generating capacity of the world!

The laser pulse energy serves to heat the fuel pellet, ionizing it to a plasma and—it is hoped—raising its temperature to around 10^8 K. As the surface layers of the pellet evaporate at these high thermal speeds, the reaction force of the escaping particles compresses the core of the pellet, increasing its density by a factor of perhaps 10^3. If all these things happened, then conditions would be right for thermonuclear fusion to occur in the core of the highly compressed pellet of plasma, the fusion reaction being the d-t reaction given in Eq. 51-8.

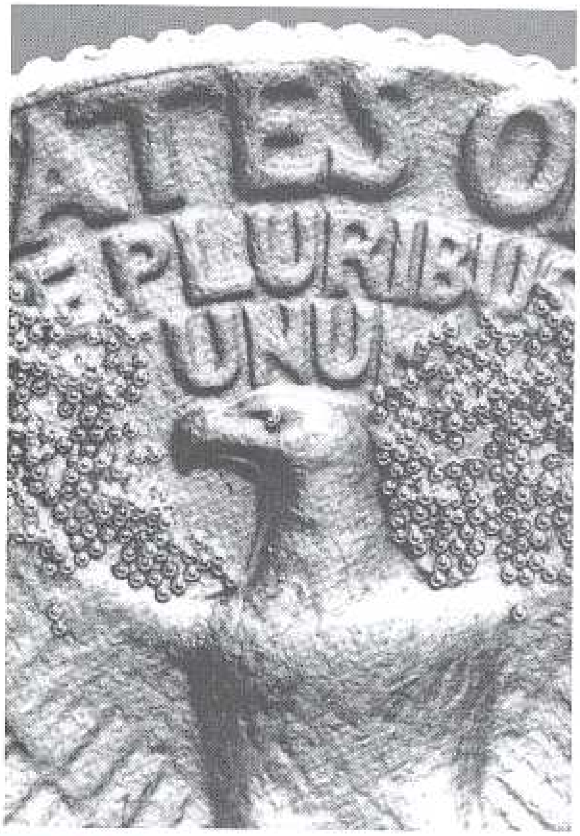

FIGURE 51-15. The tiny spheres, shown resting on a dime, are deuterium–tritium fuel pellets for use in inertial confinement experiments.

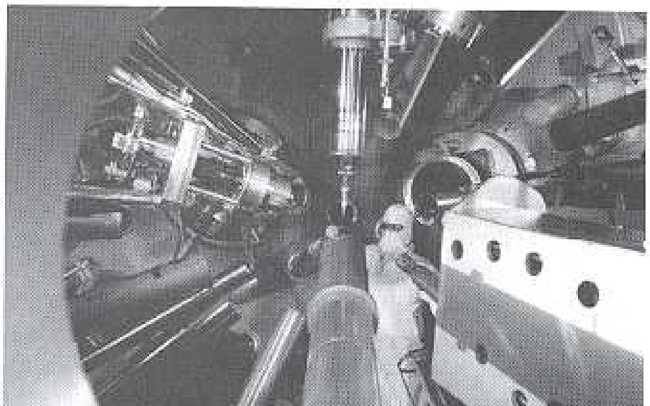

FIGURE 51-14. The target chamber of the NOVA inertial confinement fusion facility at the Lawrence Livermore National Laboratory. The photo shows several of the 10 laser beam tubes.

In an operating thermonuclear reactor of the laser fusion type, it is visualized that fuel pellets would be exploded, like miniature hydrogen bombs, at the rate of perhaps 10–100 per second. The energetic emerging particles of the fusion reaction (^{4}He and n) might be absorbed in a "blanket" consisting of a moving stream of molten lithium, heating it up. Internal energy would then be extracted from the lithium stream at another location and used to generate steam, just as in a fission reactor or a fossil-fuel power plant. Lithium would be a suitable choice for a heat-transfer medium because the energetic neutron would, with high probability, deliver up its energy to the "blanket" by the reaction

$$^6\text{Li} + \text{n} \rightarrow {}^4\text{He} + {}^3\text{H}.$$

The two charged particles would readily be brought to rest in the lithium. The tritium produced in the reaction can be extracted for use as fuel in the reactor.

A new inertial confinement fusion experimental facility is under construction at the Livermore laboratory. The National Ignition Facility, expected to be completed in 2002, will bring 192 laser beams onto a deuterium–tritium target, delivering an energy of 1.8 MJ. Like magnetic confinement, inertial confinement fusion remains a subject of active research and development.

SAMPLE PROBLEM 51-8. Suppose that a fuel pellet in a laser fusion device is made of a liquid deuterium–tritium mixture containing equal numbers of deuterium and tritium atoms. The density d ($= 200\ kg/m^3$) of the pellet is increased by a factor of 10^3 by the action of the laser pulses. (*a*) How many particles per unit volume (either deuterons or tritons) does the pellet contain in its compressed state? (*b*) At a plasma temperature corresponding to $kT = 50$ keV, for how long must this particle density be maintained to satisfy Lawson's criterion?

Solution (*a*) We can write, for the density d' of the compressed pellet,

$$d' = 10^3 d = m_d \frac{n}{2} + m_t \frac{n}{2},$$

in which n is the number of particles per unit volume (either deuterons or tritons) in the compressed pellet, m_d is the mass of a deuterium atom, and m_t is the mass of a tritium atom. These atomic masses are related to the Avogadro constant N_A and to the corresponding molar masses (M_d and M_t) by

$$m_d = M_d/N_A \quad \text{and} \quad m_t = M_t/N_A.$$

Combining these equations and solving for n lead to

$$n = \frac{2d' N_A}{M_d + M_t}$$
$$= \frac{(2)(10^3 \times 200\ kg/m^3)(6.02 \times 10^{23}\ mol^{-1})}{2.0 \times 10^{-3}\ kg/mol + 3.0 \times 10^{-3}\ kg/mol}$$
$$= 4.8 \times 10^{31}\ m^{-3}.$$

(*b*) From Lawson's criterion (Eq. 51-9), we have

$$\tau = \frac{50 \times 10^{20}\ keV \cdot s/m^{-3}}{nT}$$
$$= \frac{50 \times 10^{20}\ keV \cdot s/m^{-3}}{(4.8 \times 10^{31}\ m^{-3})(50\ keV)} = 2 \times 10^{-12}\ s.$$

The pellet must remain compressed for at least this long if breakeven operation is to occur.

A comparison with Sample Problem 51-7 shows that, unlike tokamak operation, laser fusion seeks to operate in the realm of very high particle densities and correspondingly very short confinement times.

MULTIPLE CHOICE

51-1 The Atom and the Nucleus

51-2 Nuclear Fission: The Basic Process

1. Consider the fission reaction $^{235}U + n \rightarrow {}^{99}X + {}^{134}Te + xn$.
   (*a*) What chemical element does X represent?
   (A) Sr (B) Zr (C) Nb
   (D) The answer cannot be determined without more information.
   (*b*) What is the value of x?
   (A) 1 (B) 2 (C) 2.47 (D) 3

2. Why are the fission fragments usually radioactive?
   (A) They come originally from radioactive ^{235}U.
   (B) They have a large neutron excess.
   (C) They have a large binding energy per nucleon.
   (D) They are moving at high speed.

3. In a particular fission process of a uranium atom at rest, fragments with mass numbers A_1 and A_2 are produced. A good estimate of the ratio of kinetic energies K_1/K_2 is
   (A) A_2/A_1. (B) $(A_2/A_1)^2$. (C) A_1/A_2. (D) $(A_1/A_2)^2$.

51-3 Theory of Nuclear Fission

51-4 Nuclear Reactors: The Basic Principles

4. In a nuclear reactor, the function of the moderator is
   (A) to absorb neutrons.
   (B) to keep the reactor from going critical.
   (C) to slow down the neutrons.
   (D) to absorb heat from the core.

51-5 A Natural Reactor

51-6 Thermonuclear Fusion: The Basic Process

5. Extremely high-speed protons are not important in thermonuclear fusion because
   (A) they have more difficulty penetrating the Coulomb barrier.
   (B) they move too fast to collide with another proton.
   (C) there is a relativistic increase in the proton's electric charge.
   (D) there are relatively very few of them.

51-7 Thermonuclear Fusion in Stars

6. On the average, the universe is about 25% helium and 75% hydrogen. The Sun's core is about 65% helium. This excess helium is
   (A) needed for the fusion reactions that produce the Sun's energy.
   (B) only a random fluctuation in the helium concentration of the universe.
   (C) a product of the fusion reactions in the Sun.
   (D) a catalyst that enhances the production of energy by fusion.

51-8 Controlled Thermonuclear Fusion

7. What is the main difficulty associated with the fusion process as a source of electrical power?
   (A) The scarcity of fuel
   (B) The Coulomb barrier
   (C) The radioactivity of the products
   (D) The danger of an explosion

8. Carbon and oxygen are also abundant and cheap elements. Why do we not try to use them instead of hydrogen as fuel for a fusion reactor?
   (A) The energy obtained per nucleon is much smaller.
   (B) The Coulomb barrier is much higher.
   (C) Both (A) and (B).
   (D) Neither (A) nor (B).

9. Compared with the d-d reactions, the energy release in the d-t reaction is so large because
   (A) tritium (^{3}H) is radioactive and therefore packs more energy.
   (B) the reaction product ^{4}He is very tightly bound.
   (C) more nucleons participate in the d-t reaction.
   (D) the Coulomb barrier is lower.

QUESTIONS

1. If it is so much harder to get a nucleon out of a nucleus than to get an electron out of an atom, why try?
2. Can you say, from examining Table 51-1, that one source of energy, or of power, is better than another? If not, what other considerations enter?
3. To which of the processes in Table 51-1 does the relationship $E = \Delta m\, c^2$ apply?
4. Of the two fission fragment tracks shown in Fig. 51-1, which fragment has the larger (*a*) momentum, (*b*) kinetic energy, (*c*) speed, (*d*) mass?
5. In the generalized equation for the fission of ^{235}U by thermal neutrons, $^{235}U + n \rightarrow X + Y + bn$, do you expect the Q of the reaction to depend on the identity of X and Y?
6. Is the fission fragment curve of Fig. 51-2 necessarily symmetrical about its central minimum? Explain your answer.
7. In the chain decays of the primary fission fragments (see Eq. 51-2) why do no β^+ decays occur?
8. The half-life of ^{235}U is 7.0×10^8 y. Discuss the assertion that if it had turned out to be shorter by a factor of 10 or so, there would not be any atomic bombs today.
9. ^{238}U is not fissionable by thermal neutrons. What minimum neutron energy do you think would be necessary to induce fission in this nuclide?
10. The half-life for the decay of ^{235}U by alpha emission is 7×10^8 y; by spontaneous fission, acting alone, it would be 3×10^{17} y. Both are barrier-tunneling processes, as Fig. 50-9 and Fig. 51-4 reveal. Why this enormous difference in barrier-tunneling probability?
11. Compare fission with alpha decay in as many ways as possible. How can a thermal neutron deliver several million electron-volts of excitation energy to a nucleus that absorbs it, as in Fig. 51-3*a*? The neutron has essentially no energy to start with!
12. The binding energy curve of Fig. 50-6 tells us that any nucleus more massive than $A = 56$ can release energy by the fission process. Only very massive nuclides seem to do so, however. Why cannot lead, for example, release energy by the fission process?
13. By bombardment of heavy nuclides in the laboratory it is possible to prepare other heavy nuclides that decay, at least in part, by *spontaneous fission*. That is, after a certain mean life they spontaneously break up into two major fragments. Can you explain this on the basis of the theory of Bohr and Wheeler?
14. Slow neutrons are more effective than fast ones in inducing fission. Can you make that plausible? (*Hint:* Consider how the de Broglie wavelength of a neutron might be related to its capture cross section in ^{235}U.)
15. Compare a nuclear reactor with a coal fire. In what sense does a chain reaction occur in each? What is the energy-releasing mechanism in each case?
16. Not all neutrons produced in a reactor are destined to initiate a fission event. What happens to those that do not?
17. Explain what is meant by the statement that in a reactor core neutron leakage is a surface effect and neutron production is a volume effect.
18. Explain the purpose of the moderator in a nuclear reactor. Is it possible to design a reactor that does not need a moderator? If so, what are some of the advantages and disadvantages of such a reactor?
19. Describe how to operate the control rods of a nuclear reactor (*a*) during initial start-up; (*b*) to reduce the power level; and (*c*) on a long-term basis, as fuel is consumed.
20. A reactor is operating at full power with its multiplication factor k adjusted to unity. If the reactor is now adjusted to operate stably at half power, what value must k now assume?
21. Separation of the two isotopes ^{238}U and ^{235}U from natural uranium requires a physical method, such as diffusion, rather than a chemical method. Explain why.
22. A piece of pure ^{235}U (or ^{239}Pu) will spontaneously explode if it is larger than a certain "critical size." A smaller piece will not explode. Explain.
23. What can you say, if anything, about the value of the multiplication factor k in an atomic (fission) bomb?
24. The Earth's core is thought to be mostly iron because, during the formation of the Earth, heavy elements such as iron would have sunk toward the Earth's center and lighter elements, such as silicon, would have floated upward to form the Earth's crust. However, iron is far from the heaviest element. Why is the Earth's core not made of uranium?
25. From information given in the text, collect and write down the approximate heights of the Coulomb barriers for (*a*) the alpha decay of ^{238}U, (*b*) the fission of ^{235}U by thermal neutrons, and (*c*) the head-on collision of two deuterons.
26. The Sun's energy is assumed to be generated by nuclear reactions such as the proton–proton cycle. What alternative ways of generating solar energy were proposed in the past, and why were they rejected?
27. Elements up to mass number ≈ 56 are created by thermonuclear fusion in the cores of stars. Why are heavier elements not also created by this process?
28. Do you think that the thermonuclear fusion reaction controlled by the two curves plotted in Fig. 51-9 necessarily has its maximum effectiveness for the energy at which the two curves cross each other? Explain your answer.

29. In Fig. 51-9, are you surprised that, as judged by the areas under the curve marked $n(K)$, the number of particles with $K > K_{av}$ is smaller than the number with $K < K_{av}$, where K_{av} is the average thermal energy?

30. The uranium nuclides present in the Earth today were originally built up and spewed into space during the explosion of stars, so-called supernova events. These explosions, which occurred before the formation of our solar system, represent the collapse of stars under their own gravity. Can you then say that the energy derived from fission was once stored in a gravitational field? Does fission energy then, in this limited sense, have something in common with energy derived from hydroelectric sources?

31. Why does it take so long ($\sim 10^6$ y!) for gamma-ray photons generated by nuclear reactions in the Sun's central core to diffuse to the surface? What kinds of interactions do they have with the protons, α particles, and electrons that make up the core?

32. The primordial matter of the early universe is thought to have been largely hydrogen. Where did all the silicon in the Earth come from? All the gold?

33. Do conditions at the core of the Sun satisfy Lawson's criterion for a sustained thermonuclear fusion reaction? Explain.

34. To achieve ignition in a tokamak, why do you need a high plasma temperature? A high density of plasma particles? A long confinement time?

35. Which would generate more radioactive waste products, a fission reactor or a fusion reactor?

36. Does Lawson's criterion hold both for tokamaks and for laser fusion devices?

EXERCISES

51-1 The Atom and the Nucleus

51-2 Nuclear Fission: The Basic Process

1. You wish to produce 1.0 GJ of energy. Calculate and compare (*a*) the amount of coal needed if you obtain the energy by burning coal and (*b*) the amount of natural uranium needed if you obtain the energy by fission in a reactor. Assume that the combustion of 1.0 kg of coal releases 2.9×10^7 J; the fission of 1.0 kg of uranium in a reactor releases 8.2×10^{13} J.

2. In the United States, coal commonly contains about 3 parts per million (3 ppm) of fissionable uranium and thorium. Calculate and compare (*a*) the energy derived from burning 100 kg of coal and (*b*) the energy that could be derived from the fission of the fissionable impurities that remain in its ashes. Assume that the combustion of 1 kg of coal releases 2.9×10^7 J; the fission of 1 kg of uranium or thorium in a reactor releases 8.2×10^{13} J.

3. (*a*) How many atoms are contained in 1.00 kg of pure ^{235}U? (*b*) How much energy, in joules, is produced by the complete fissioning of 1.00 kg of ^{235}U? Assume $Q = 200$ MeV. (*c*) For how many years would this energy light a 100-W lamp?

4. At what rate must ^{235}U nuclei undergo fission by neutrons to generate 2.00 W? Assume that $Q = 200$ MeV.

5. Verify that, as reported in Table 51-1, the fission of 1 kg of ^{235}U could keep a 100-W lamp burning for 3×10^4 y.

6. The fission properties of the plutonium isotope ^{239}Pu are very similar to those of ^{235}U. The average energy released per fission is 180 MeV. How much energy, in joules, is liberated if all the atoms in 1.00 kg of pure ^{239}Pu undergo fission?

7. Very occasionally a ^{235}U nucleus, having absorbed a neutron, breaks up into *three* fragments. If two of these fragments are identified chemically as isotopes of chromium and gallium and if no prompt neutrons are involved, what is at least one possibility for the identity of the fragments? Consult a nuclidic chart or table.

8. Show that, in Sample Problem 51-1, there is no need to take the masses of the electrons emitted during the beta decay of the primary fission fragments explicitly into account.

9. ^{235}U decays by alpha emission with a half-life of 7.04×10^8 y. It also decays (rarely) by spontaneous fission, and if the alpha decay did not occur, its half-life due to this process alone would be 3.50×10^{17} y. (*a*) At what rate do spontaneous fission decays occur in 1.00 g of ^{235}U? (*b*) How many alpha-decay events occur for every spontaneous fission event?

51-3 Theory of Nuclear Fission

10. Fill in the following table, which refers to the generalized fission reaction

$$^{235}U + n \rightarrow X + Y + bn.$$

X	Y	b
^{140}Xe	—	1
^{139}I	—	2
—	^{100}Zr	2
^{141}Cs	^{92}Rb	—

11. Calculate the disintegration energy Q for the spontaneous fission of ^{52}Cr into two equal fragments. The needed masses are ^{52}Cr, 51.94012 u; and ^{26}Mg, 25.982593 u. Discuss your result.

12. Calculate the disintegration energy Q for the fission of ^{98}Mo into two equal parts. The needed masses are ^{98}Mo, 97.905408 u; and ^{49}Sc, 48.950024 u. If Q turns out to be positive, discuss why this process does not occur spontaneously.

13. Calculate the energy released in the fission reaction

$$^{235}U + n \rightarrow {}^{141}Cs + {}^{92}Rb + 3n.$$

Needed atomic masses are

^{235}U	235.043923 u	^{92}Rb	91.919726 u
^{141}Cs	140.920044 u	n	1.008665 u.

14. ^{238}Np has a barrier energy for fission of 4.2 MeV. To remove a neutron from this nuclide requires an energy expenditure of 5.0 MeV. Is ^{237}Np fissionable by thermal neutrons?

15. Consider the fission of ^{238}U by fast neutrons. In one fission event no neutrons were emitted and the final stable end products, after the beta decay of the primary fission fragments, were ^{140}Ce and ^{99}Ru. (*a*) How many beta-decay events were there in the two beta-decay chains, considered together? (*b*) Calculate Q. The relevant atomic masses are

^{235}U	238.050783 u	^{140}Ce	139.905434 u
n	1.008665 u	^{99}Ru	98.905939 u.

16. In a particular fission event of ^{235}U by slow neutrons, it happens that no neutron is emitted and that one of the primary fission fragments is ^{83}Ge. (*a*) What is the other fragment? (*b*) How is the disintegration energy $Q = 170$ MeV split between the two fragments? (*c*) Calculate the initial speed of each fragment.

51-4 Nuclear Reactors: The Basic Principles

17. Many fear that helping additional nations develop nuclear power reactor technology will increase the likelihood of nuclear war because reactors can be used not only to produce energy but, as a by-product through neutron capture with inexpensive ^{238}U, to make ^{239}Pu, which is a "fuel" for nuclear bombs (*breeder* reactors). What simple series of reactions involving neutron capture and beta decay would yield this plutonium isotope?

18. A 190-MW fission reactor consumes half its fuel in 3 years. How much ^{235}U did it contain initially? Assume that all the energy generated arises from the fission of ^{235}U and that this nuclide is consumed only by the fission process. See Sample Problem 51-3.

19. Repeat Exercise 18 taking into account nonfission neutron capture by the ^{235}U. See Sample Problem 51-3.

20. Among the many fission products that may be extracted chemically from the spent fuel of a nuclear power reactor is ^{90}Sr ($t_{1/2} = 29$ y). It is produced in typical large reactors at the rate of about 18 kg/y. By its radioactivity it generates thermal energy at the rate of 2.3 W/g. (*a*) Calculate the effective disintegration energy Q_{eff} associated with the decay of a ^{90}Sr nucleus. (Q_{eff} includes contributions from the decay of the ^{90}Sr daughter products in its decay chain but not from neutrinos, which escape totally from the sample.) (*b*) It is desired to construct a power source generating 150 W (electric) to use in operating electronic equipment in an underwater acoustic beacon. If the source is based on the thermal energy generated by ^{90}Sr and if the efficiency of the thermal–electric conversion process is 5.0%, how much ^{90}Sr is needed? The atomic mass of ^{90}Sr is 89.9 u.

21. The neutron generation time t_{gen} in a reactor is the average time between one fission and the fissions induced by the neutrons emitted in that fission. Suppose that the power output of a reactor at time $t = 0$ is P_0. Show that the power output a time t later is $P(t)$, where

$$P(t) = P_0 k^{t/t_{gen}},$$

where k is the multiplication factor. Note that for constant power output, $k = 1$.

22. The neutron generation time (see Exercise 21) of a particular power reactor is 1.3 ms. It is generating energy at the rate of 1200 MW. To perform certain maintenance checks, the power level must be temporarily reduced to 350 MW. It is desired that the transition to the reduced power level take 2.6 s. To what (constant) value should the multiplication factor be set to effect the transition in the desired time?

23. The neutron generation time t_{gen} (see Exercise 21) in a particular reactor is 1.0 ms. If the reactor is operating at a power level of 500 MW, about how many free neutrons (neutrons that will subsequently induce a fission) are present in the reactor at any moment?

24. A reactor operates at 400 MW with a neutron generation time of 30 ms. If its power increases for 5.0 min with a multiplication factor of 1.0003, find the power output at the end of the 5.0 min. See Exercise 21.

25. The thermal energy generated when radiations from radionuclides are absorbed in matter can be used as the basis for a small power source for use in satellites, remote weather stations, and so on. Such radionuclides are manufactured in abundance in nuclear power reactors and may be separated chemically from the spent fuel. One suitable radionuclide is ^{238}Pu ($t_{1/2} = 87.7$ y), which is an alpha emitter with $Q = 5.59$ MeV. At what rate is thermal energy generated in 1.00 kg of this material?

26. One possible method for revealing the presence of concealed nuclear weapons is to detect the neutrons emitted in the spontaneous fission of ^{240}Pu in the warhead. In an actual trial, a neutron detector of area 2.5 m^2, carried on a helicopter, measured a neutron flux of 4.0 s^{-1} at a distance of 35 m from a missile warhead. Estimate the mass of ^{240}Pu in the warhead. The half-life for spontaneous fission in ^{240}Pu is 1.34×10^{11} y, and 2.5 neutrons, on the average, are emitted in each fission.

51-5 A Natural Reactor

27. How far back in time would natural uranium have been a practical reactor fuel, with a $^{235}U/^{238}U$ ratio of 3.00%? See Sample Problem 51-4.

28. The natural fission reactor discussed in Section 51-5 is estimated to have generated 15 gigawatt-years of energy during its lifetime. (*a*) If the reactor lasted for 200,000 y, at what average power level did it operate? (*b*) How much ^{235}U did it consume during its lifetime?

29. Some uranium samples from the natural reactor site described in Section 51-5 were found to be slightly *enriched* in ^{235}U rather than depleted. Account for this in terms of neutron absorption by the abundant isotope ^{238}U and the subsequent beta and alpha decay of its products.

51-6 Thermonuclear Fusion: The Basic Process

30. For how long could the fusion of 1.00 kg of deuterium by the reaction

$$^2H + ^2H \rightarrow ^3He + n \qquad (Q = +3.27 \text{ MeV})$$

keep a 100-W lamp burning? The atomic mass of deuterium is 2.014 u.

31. Calculate the height of the Coulomb barrier for the head-on collision of two protons. The effective radius of a proton may be taken to be 0.80 fm. See Sample Problem 51-5.

32. The equation of the curve $n(K)$ in Fig. 51-9 is

$$n(K) = \frac{2N}{\sqrt{\pi}} \frac{K^{1/2}}{(kT)^{3/2}} e^{-K/kT},$$

where N is the total density of particles. At the center of the Sun the temperature is 1.5×10^7 K and the mean proton energy K_{av} is 1.9 keV. Find the ratio of the density of protons at 5.0 keV to that at the mean proton energy.

33. Methods other than heating the material have been suggested for overcoming the Coulomb barrier for fusion. For example, one might consider using particle accelerators. If you were to use two of them to accelerate two beams of deuterons directly toward each other so as to collide "head-on," (*a*) what voltage would each require to overcome the Coulomb barrier? (*b*) Would this voltage be difficult to achieve? (*c*) Why do you suppose this method is not presently used?

51-7 Thermonuclear Fusion in Stars

34. We have seen that Q for the overall proton–proton cycle is 26.7 MeV. How can you relate this number to the Q values for the three reactions that make up this cycle, as displayed in Fig. 51-10?

35. Show that the energy released when three alpha particles fuse to form ^{12}C is 7.27 MeV. The atomic mass of ^4He is 4.002603 u and of ^{12}C is 12.000000 u.

36. At the central core of the Sun the density is 1.5×10^5 kg/m^3 and the composition is essentially 35% hydrogen by mass and 65% helium. (*a*) What is the density of protons at the Sun's core? (*b*) What is the ratio of this to the density of particles for an ideal gas at standard conditions of temperature and pressure?

37. Calculate and compare the energy in MeV released by (*a*) the fusion of 1.0 kg of hydrogen deep within the Sun and (*b*) the fission of 1.0 kg of ^{235}U in a fission reactor.

38. The Sun has a mass of 2.0×10^{30} kg and radiates energy at the rate of 3.9×10^{26} W. (*a*) At what rate does the mass of the Sun decrease? (*b*) What fraction of its original mass has the Sun lost in this way since it began to burn hydrogen, about 4.5×10^9 y ago?

39. Let us assume that the core of the Sun has one-eighth the Sun's mass and is compressed within a sphere whose radius is one-fourth of the solar radius. We assume further that the composition of the core is 35% hydrogen by mass and that essentially all of the Sun's energy is generated there. If the Sun continues to burn hydrogen at the rate calculated in Sample Problem 51-6, how long will it be before the hydrogen is entirely consumed? The Sun's mass is 2.0×10^{30} kg.

40. Verify the Q values reported for the reactions in Fig. 51-10. The needed atomic masses are

^1H	1.007825 u	^3He	3.016029 u
^2H	2.014102 u	^4He	4.002603 u
e^-	0.0005486 u.		

(*Hint:* Distinguish carefully between atomic and nuclear masses, and take the positrons properly into account.)

41. Coal burns according to

$$C + O_2 \rightarrow CO_2.$$

The heat of combustion is 3.3×10^7 J/kg of atomic carbon consumed. (*a*) Express this in terms of energy per carbon atom. (*b*) Express it in terms of energy per kilogram of the initial reactants, carbon and oxygen. (*c*) Suppose that the Sun (mass $= 2.0 \times 10^{30}$ kg) were made of carbon and oxygen in combustible proportions and that it continued to radiate energy at its present rate of 3.9×10^{26} W. How long would it last?

42. After converting all its hydrogen to helium, a particular star is 100% helium in composition. It now proceeds to convert the helium to carbon via the triple-alpha process

$$^4He + {^4He} + {^4He} \rightarrow {^{12}C} + \gamma \qquad (Q = 7.27\ \text{MeV}).$$

The mass of the star is 4.6×10^{32} kg, and it generates energy at the rate of 5.3×10^{30} W. How long will it take to convert all the helium to carbon?

51-8 Controlled Thermonuclear Fusion

43. Verify the Q values reported in Eqs. 51-6, 51-7, and 51-8. The needed masses are

^1H	1.007825 u	^3He	3.016029 u
^2H	2.014102 u	^4He	4.002603 u
^3H	3.016049 u	n	1.008665 u.

44. Ordinary water consists of roughly 0.015% by mass of "heavy water," in which one of the two hydrogens is replaced with deuterium, ^2H. How much average fusion power could be obtained if we "burned" all the ^2H in 1 liter of water in 1 day through the reaction $^2H + {^2H} \rightarrow {^3He} + n$ ($Q = 3.27$ MeV)?

45. In the deuteron–triton fusion reaction of Eq. 51-8, how is the reaction energy Q shared between the α particle and the neutron (that is, calculate the kinetic energies K_α and K_n)? Neglect the relatively small kinetic energies of the two combining particles.

46. Figure 51-16 shows an idealized representation of a hydrogen bomb. The fusion fuel is lithium deuteride (LiD). The high temperature, particle density, and neutrons to induce fusion are provided by an atomic (fission) bomb "trigger." The fusion reactions are

$$^6Li + n \rightarrow {^3H} + {^4He}$$

and

$$^2H + {^3H} \rightarrow {^4He} + n \qquad (Q = 17.59\ \text{MeV}),$$

the tritium (^3H) produced in the first reaction fusing with the deuterium (D) in the fuel; see Eq. 51-8. By calculating Q for the first reaction, find the mass of LiD required to produce a fusion yield of 1 megaton of TNT ($= 2.6 \times 10^{28}$ MeV). Needed atomic masses are

^6Li	6.015122 u	^4He	4.002603 u
^3H	3.016049 u	n	1.008665 u.

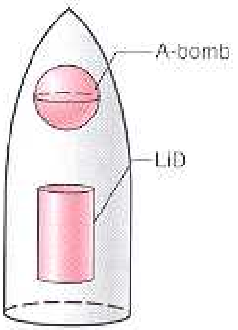

FIGURE 51-16. Exercise 46.

47. Assume that a plasma temperature of 1.3×10^8 K is reached in a laser-fusion device. (*a*) What is the most probable speed of a deuteron at this temperature? (*b*) How far would such a deuteron move in the confinement time calculated in Sample Problem 51-8?

PROBLEMS

1. Assume that immediately after the fission of ^{236}U according to Eq. 51-2, the resulting ^{140}Xe and ^{96}Sr nuclei are just touching at their surfaces. (*a*) Assuming the nuclei to be spherical, calculate the Coulomb potential energy (in MeV) of repulsion between the two fragments. (*Hint:* Use Eq. 50-1 to calculate the radii of the fragments.) (*b*) Compare this energy with the energy released in a typical fission process. In what form will this energy ultimately appear in the laboratory?

2. A ^{236}U nucleus undergoes fission and breaks up into two middle-mass fragments, ^{140}Xe and ^{96}Sr. (*a*) By what percentage does the surface area of the ^{236}U nucleus change during this process? (*b*) By what percentage does its volume change? (*c*) By what percentage does its electrostatic potential energy change? The potential energy of a uniformly charged sphere of radius r and charge Q is given by
$$U = \frac{3}{5}\left(\frac{Q^2}{4\pi\epsilon_0 r}\right).$$

3. In an atomic bomb, energy release is due to the uncontrolled fission of plutonium ^{239}Pu (or ^{235}U). The magnitude of the released energy is specified in terms of the mass of TNT required to produce the same energy release (bomb "rating"). One megaton (10^6 tons) of TNT produces 2.6×10^{28} MeV of energy. (*a*) Calculate the rating, in tons of TNT, of an atomic bomb containing 95 kg of ^{239}Pu, of which 2.5 kg actually undergoes fission. For plutonium, the average Q is 180 MeV. (*b*) Why is the other 92.5 kg of ^{239}Pu needed if it does not fission?

4. (*a*) A neutron with initial kinetic energy K makes a head-on elastic collision with a resting atom of mass m. Show that the fractional energy loss of the neutron is given by
$$\frac{\Delta K}{K} = \frac{4m_n m}{(m + m_n)^2},$$
in which m_n is the neutron mass. (*b*) Find $\Delta K/K$ if the resting atom is hydrogen, deuterium, carbon, or lead. (*c*) If $K =$ 1.00 MeV initially, how many such collisions would it take to reduce the neutron energy to thermal values (0.025 eV) if the material is deuterium, a commonly used moderator? (*Note:* In actual moderators, most collisions are not "head-on.")

5. Calculate the Coulomb barrier height for two ^7Li nuclei fired at each other with the same initial kinetic energy K. See Sample Problem 51-5. (*Hint:* Use Eq. 50-1 to calculate the radii of the nuclei.)

6. In certain stars the *carbon cycle* is more likely than the proton–proton cycle to be effective in generating energy. This cycle is
$$^{12}C + {}^1H \rightarrow {}^{13}N + \gamma, \qquad Q_1 = 1.95 \text{ MeV},$$
$$^{13}N \rightarrow {}^{13}C + e^+ + \nu, \qquad Q_2 = 1.19 \text{ MeV},$$
$$^{13}C + {}^1H \rightarrow {}^{14}N + \gamma, \qquad Q_3 = 7.55 \text{ MeV},$$
$$^{14}N + {}^1H \rightarrow {}^{15}O + \gamma, \qquad Q_4 = 7.30 \text{ MeV},$$
$$^{15}O \rightarrow {}^{15}N + e^+ + \nu, \qquad Q_5 = 1.73 \text{ MeV},$$
$$^{15}N + {}^1H \rightarrow {}^{12}C + {}^4He, \qquad Q_6 = 4.97 \text{ MeV}.$$
(*a*) Show that this cycle of reactions is exactly equivalent in its overall effects to the proton–proton cycle of Fig. 51-10. (*b*) Verify that both cycles, as expected, have the same Q.

7. The gravitational potential energy of a uniform spherical object of mass M and radius R is
$$U = -3GM^2/5R,$$
in which G is the gravitational constant. (*a*) Demonstrate the consistency of this expression with that of Problem 4 in Chapter 50. (*b*) Use this expression to find the maximum energy that could be released by a spherical object, initially of infinite radius, in shrinking to the present size of the Sun. (*c*) Assume that during this shrinking, the Sun radiated energy at its present rate and calculate the age of the Sun based on the hypothesis that the Sun derives its energy from gravitational contraction.

8. (*a*) Calculate the rate at which the Sun is generating neutrinos. Assume that energy production is entirely by the proton–proton cycle. (*b*) At what rate do solar neutrinos impinge on the Earth?

9. Suppose we had a quantity of N deuterons (^2H nuclei). (*a*) Which of the following procedures for fusing these N nuclei releases more energy, and how much more? (A) $N/2$ fusion reactions of the type $^2H + {}^2H \rightarrow {}^3H + {}^1H$, or (B) $N/3$ fusion reactions of the type $^2H + {}^3H \rightarrow {}^4He + n$, using $N/3$ nuclei of ^3H that are first made in $N/3$ reactions of type A. (*b*) List the ultimate product nuclei resulting from the two procedures and the quantity of each.

10. The uncompressed radius of the fuel pellet of Sample Problem 51-8 is 20 μm. Suppose that the compressed fuel pellet "burns" with an efficiency of 10%. That is, only 10% of the deuterons and 10% of the tritons participate in the fusion reaction of Eq. 51-8. (*a*) How much energy is released in each such microexplosion of a pellet? (*b*) To how much TNT is each such pellet equivalent? The heat of combustion of TNT is 4.6 MJ/kg. (*c*) If a fusion reactor is constructed on the basis of 100 microexplosions per second, what power would be generated? (Note that part of this power must be used to operate the lasers.)