Anyons, fractional charges, and topological order in a weakly interacting system

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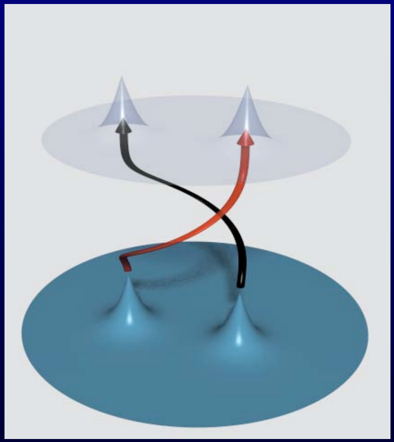
February 16, 2007

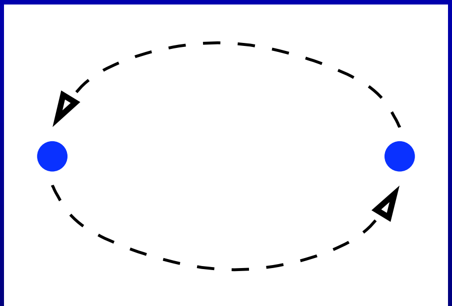


In collaboration with: C. Weeks, G. Rosenberg, B. Seradjeh

Particle statistics

In 3 space dimensions indistinguishable particles can be bosons or fermions,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\Psi(\mathbf{r}_2, \mathbf{r}_1).$$



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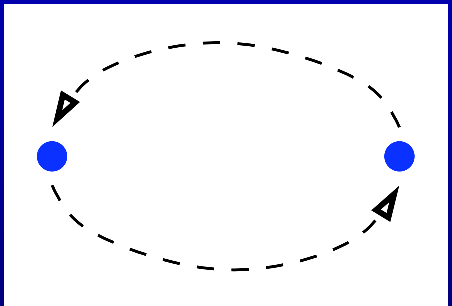

In 2 space dimensions we can have exotic particles called “anyons”, so that

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\theta}\Psi(\mathbf{r}_2, \mathbf{r}_1), \quad \theta \neq 0, \pi.$$

Anyons and quantum computation

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Fault-tolerant quantum computation by anyons

A.Yu. Kitaev*

L.D. Landau Institute for Theoretical Physics, 117940, Kosygina St. 2, Germany

Received 20 May 2002

Abstract

A two-dimensional quantum system with anyonic excitations can be considered as a quantum computer. Unitary transformations can be performed by moving the excitations around each other. Measurements can be performed by joining excitations in pairs and observing the result of fusion. Such computation is fault-tolerant by its physical nature.
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Quantum computation can be performed in a fault-tolerant way by braiding non-abelian anyons.

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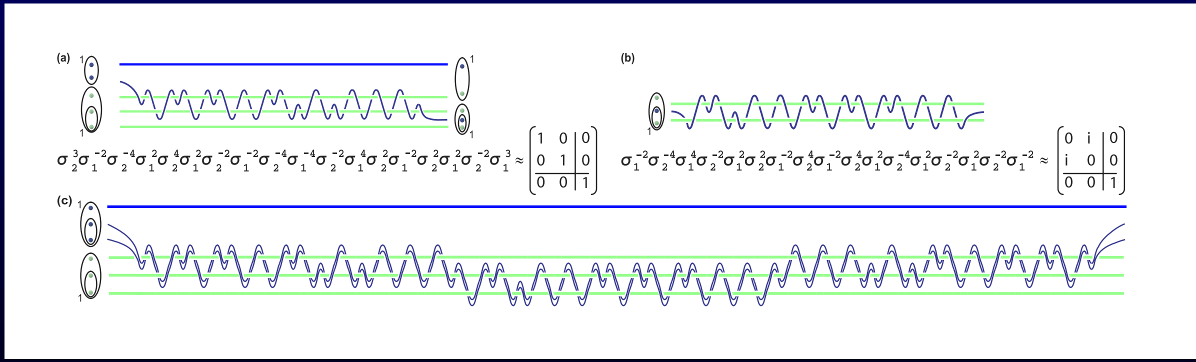

Bonesteel et al., PRL **95**, 140503 (2005).

Anyons in FQH fluids

Abelian anyons are known to occur as excitations of the fractional quantum Hall fluids described by Laughlin wavefunctions

$$\Psi(\{z_i\}) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/4},$$

with m odd integer. These have exchange phase

$$\theta = \frac{\pi}{m}$$

and charge

$$q = \pm\frac{e}{m}.$$

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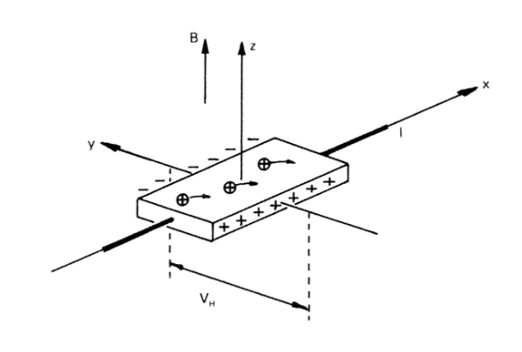

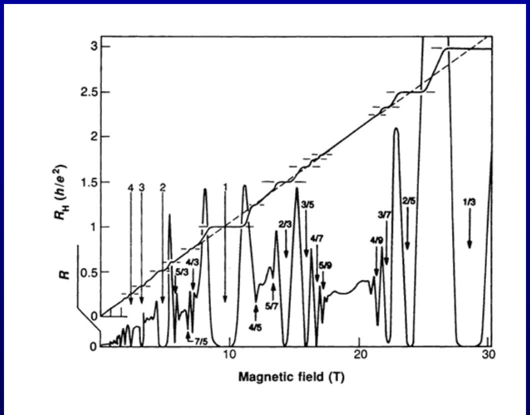


Robert Laughlin (left) receiving the Nobel Prize for physics from Swedish King Carl XVI Gustaf at the ceremonies in Stockholm, Sweden, on December 10, 1998. (AP Photo/Jonas Ekstromer)

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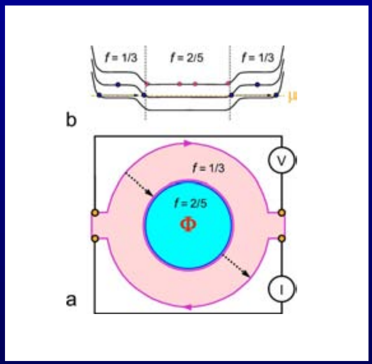

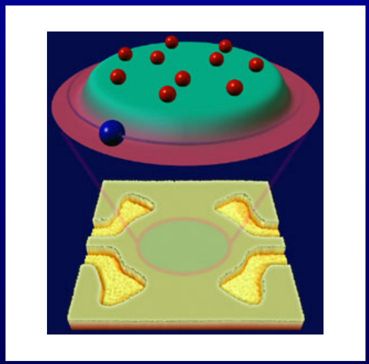

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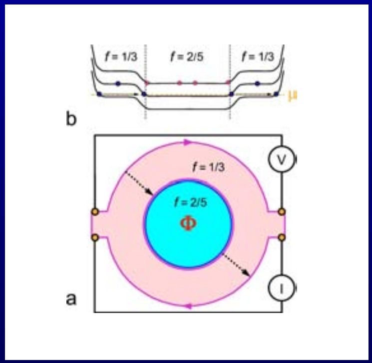

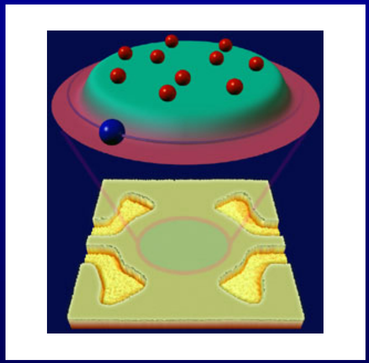

- Non-abelian anyons occur in the so called Moore-Read “Pfaffian” state which may be realized in the $\nu = \frac{5}{2}$ FQH state. Experimentally as yet unconfirmed.

Anyons, fractionalization and strong correlations

Anyons and charge fractionalization typically occur in strongly correlated electron systems.

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Strongly correlated = $\left\{\begin{array}{l}\text{many-body wavefunction } \Psi \textit{ cannot } \text{be} \\ \text{written as a single Slater determinant of} \\ \text{the constituent electron single-particle states.}\end{array}\right.$

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Are fractional statistics and fractional charges inextricably linked to strong correlations?

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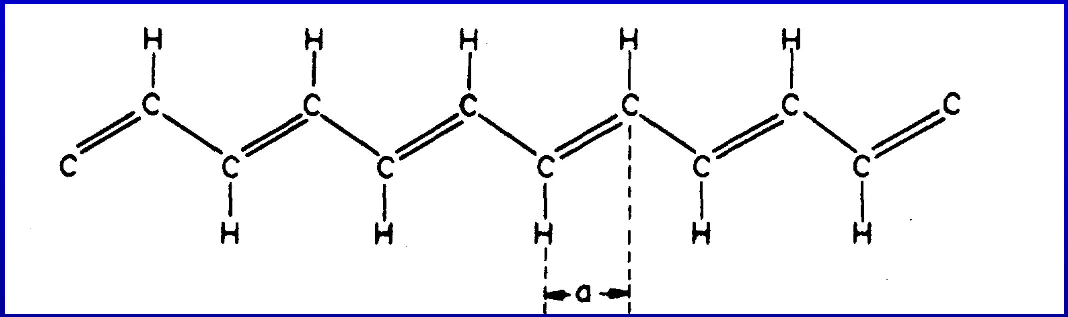
Question:
Are fractional statistics and fractional charges inextricably linked to strong correlations?

Answer:
Not necessarily.

Fractional charges in polyacetylene

[Su, Schrieffer and Heeger, 1979]



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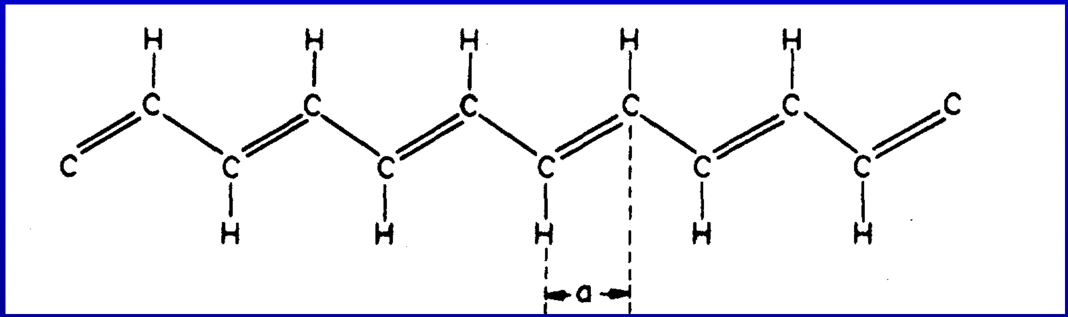

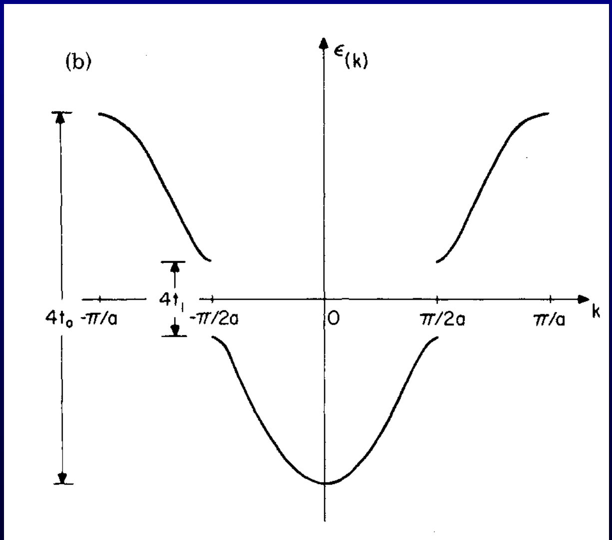

FIG. 1. (a) *Trans* configuration of $(CH)_x$. $a = 1.2$ Å. (b) π-band structure of perfectly dimerized $(CH)_x$.

band structure

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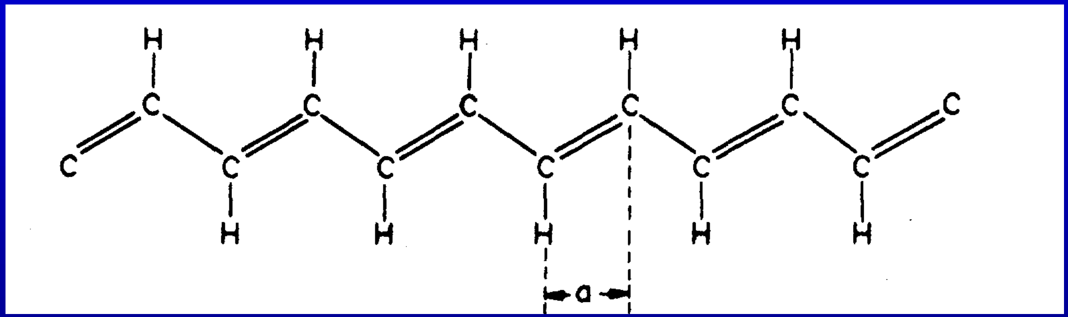

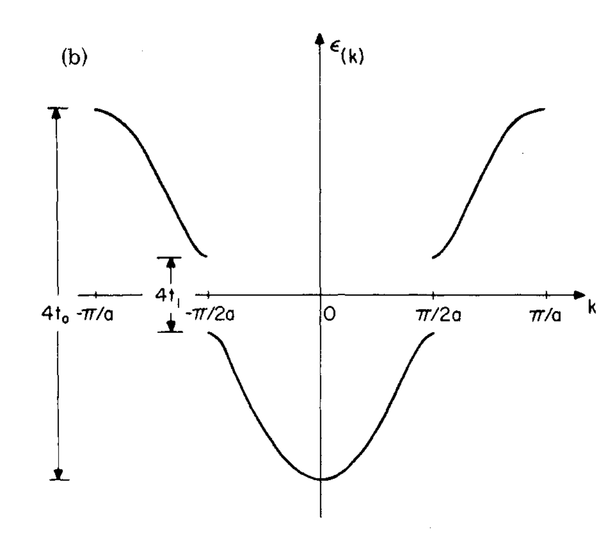

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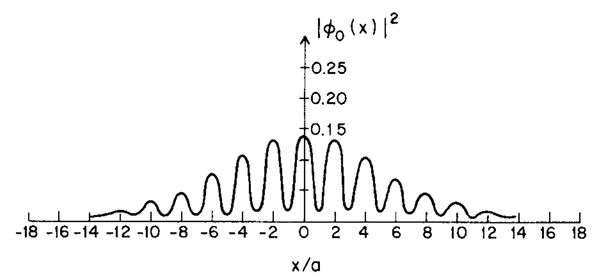

FIG. 3. Probability distribution of the localized electronic state at the center of the gap.

bound state at domain wall

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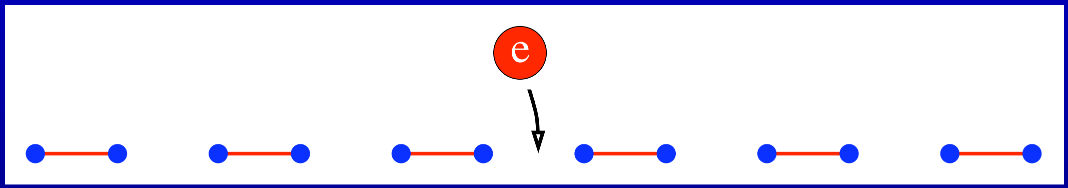

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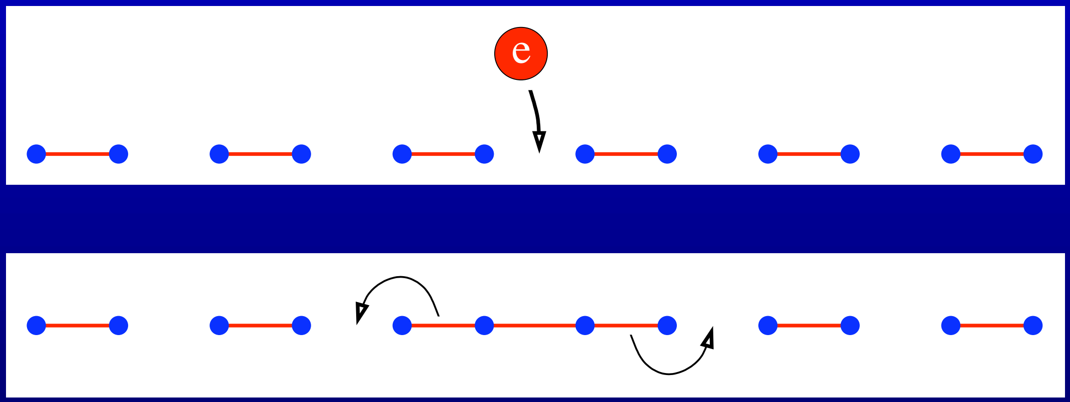

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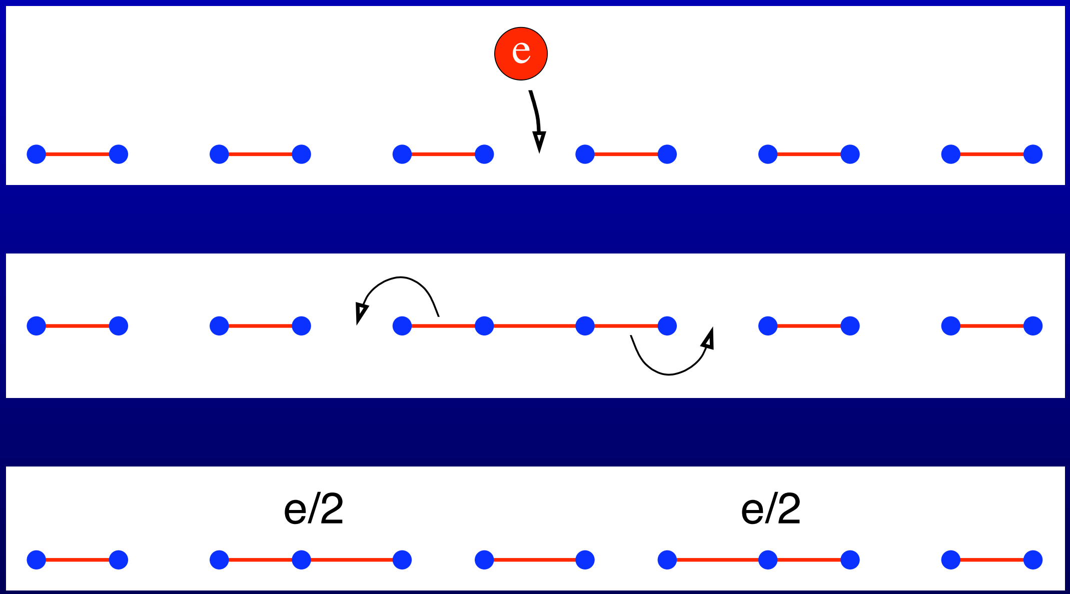

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PHYSICAL REVIEW D VOLUME 13, NUMBER 12 15 JUNE 1976

Solitons with fermion number 1/2*

R. Jackiw and C. Rebbi
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(Received 23 December 1975)

We study the structure of soliton-monopole systems when Fermi fields are present. We show that the existence of a nondegenerate, isolated, zero-energy, *c*-number solution of the Dirac equation implies that the soliton is a degenerate doublet with Fermi number $\pm 1/2$. We find such solutions in the theory of Yang-Mills monopoles and dyons.

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Interactions play no significant role, systems can be regarded as **weakly correlated**.

In $d = 1, 3$ particle statistics is trivial (fermions or bosons):

$\boxed{\text{Need a two-dimensional example!}}$

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A simple toy model based on the Aharonov-Bohm effect.

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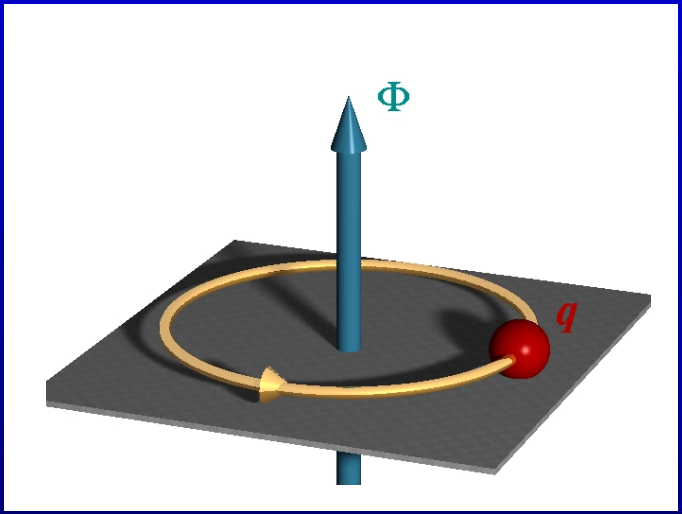

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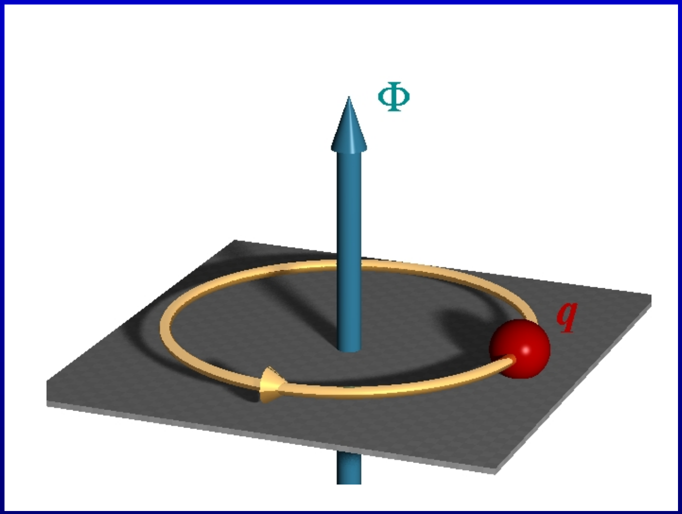

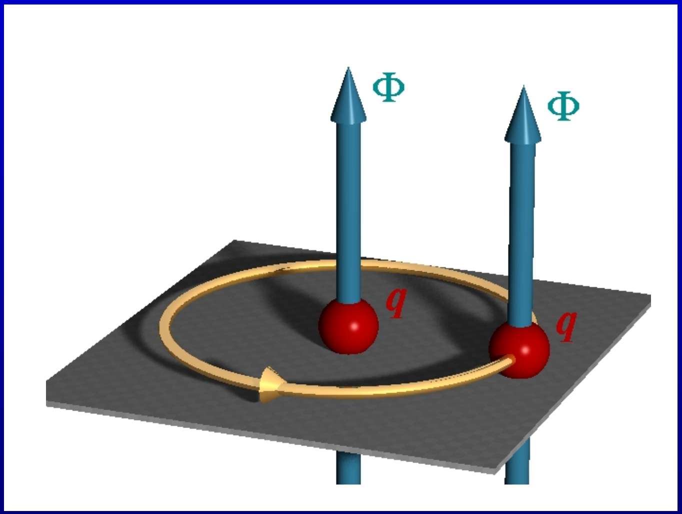

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$$2\pi \left(\frac{q}{e}\right) \left(\frac{\Phi}{\Phi_0}\right)$$

A charge-flux composite (q, Φ) encircling another (q, Φ) acquires

$$4\pi \left(\frac{q}{e}\right) \left(\frac{\Phi}{\Phi_0}\right)$$

Since the encircling operation can be thought of as two exchanges, the (q, Φ) composite particles have exchange phase

$$\theta = 2\pi \left(\frac{q}{e}\right) \left(\frac{\Phi}{\Phi_0}\right).$$

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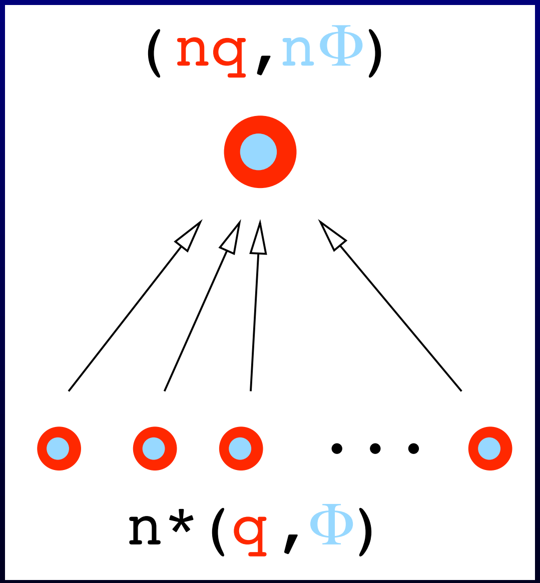
Anyon fusion:

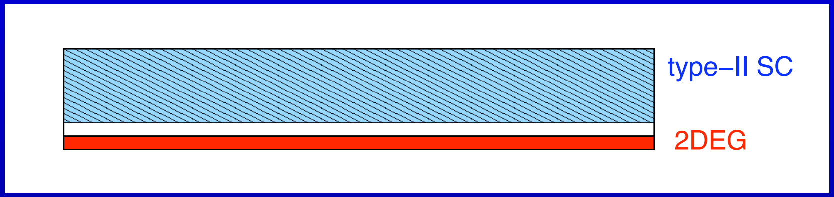


Fusing together n identical particles with statistical phase θ_0 results in a particle with statistical phase

$$\boxed{\theta = n^2\theta_0.}$$

A new device:

Superconductor-semiconductor heterostructure.



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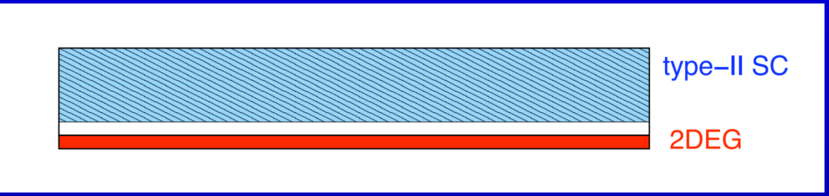

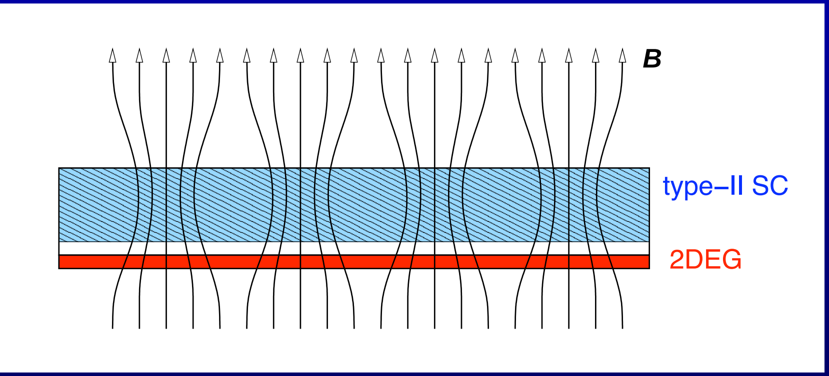

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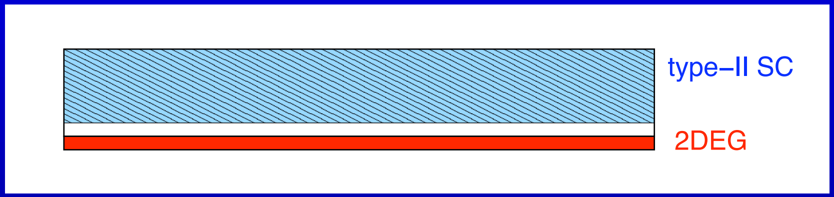
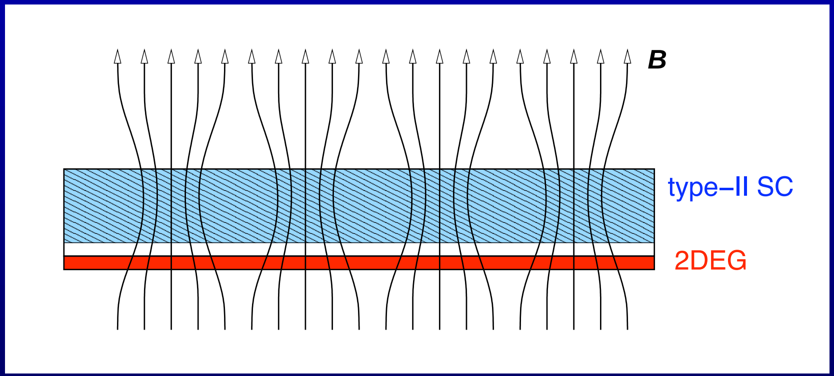
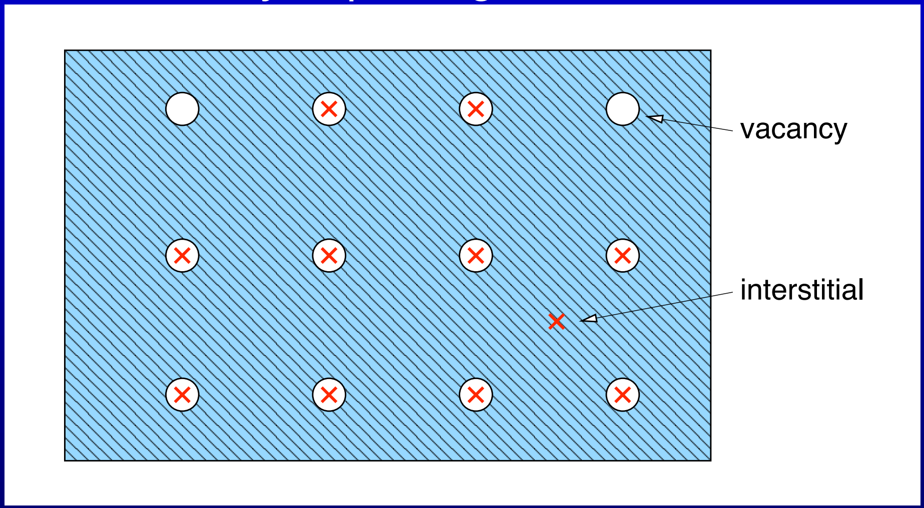




Periodic array of pinning sites for vortices



[top view]

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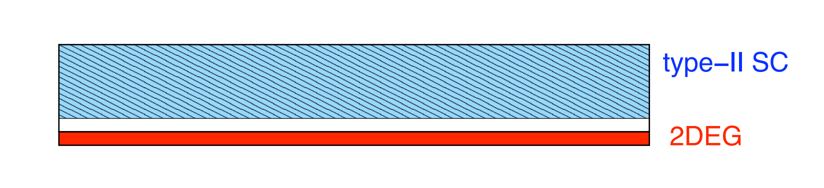

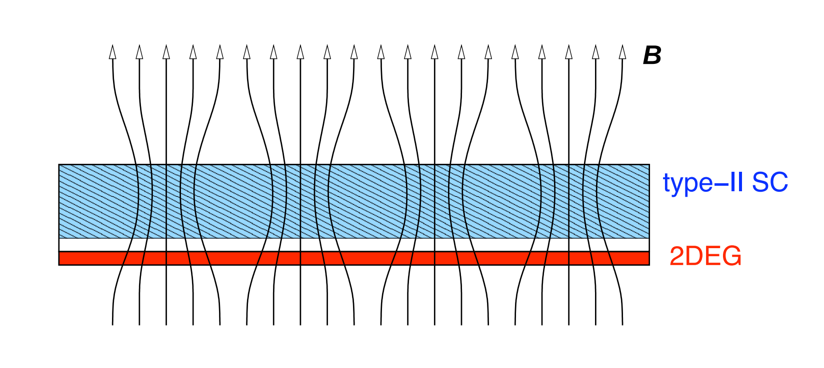

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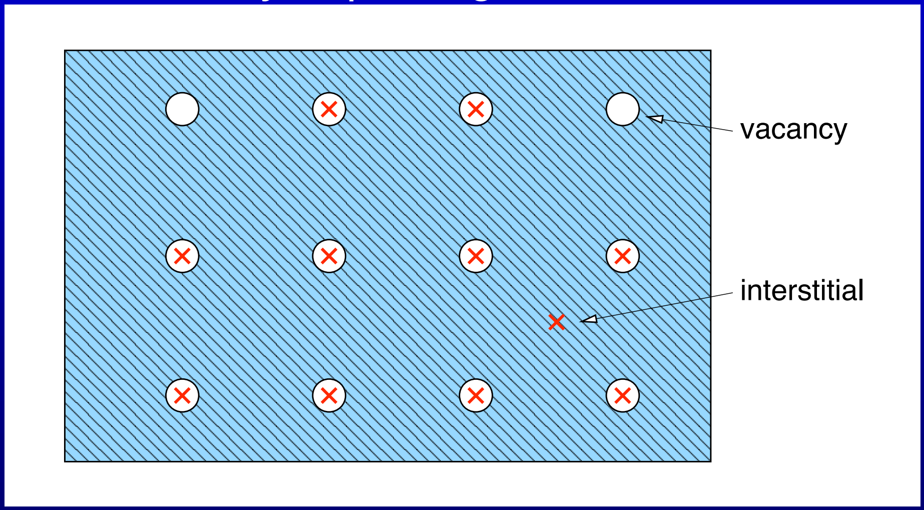
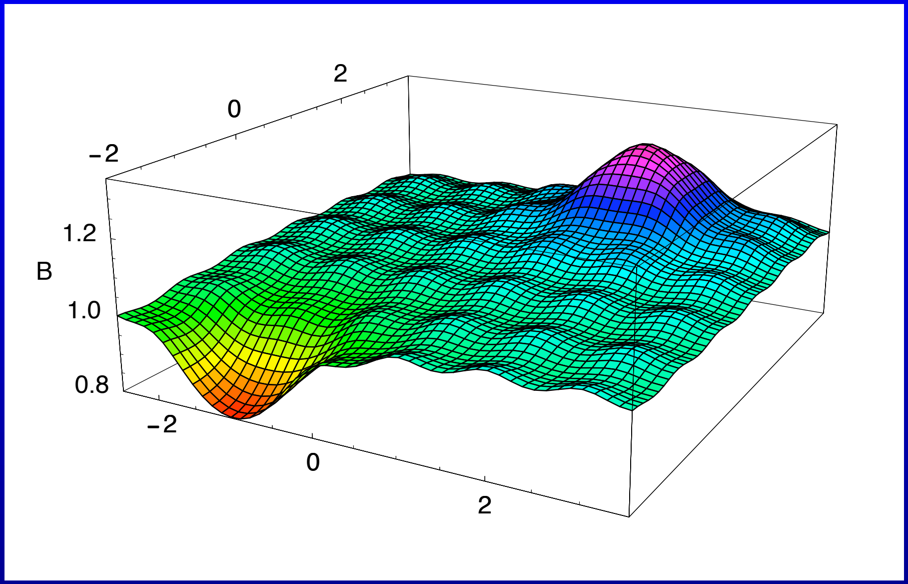


[top view]

- Superconductor quantizes magnetic flux in the units of $\Phi_0/2 = hc/2e$.
- Vortices preferentially occupy the pinning sites.
- Vacancies and interstitials in the vortex lattice produce localized flux surplus or deficit $\pm\Phi_0/2$.



Magnetic field profile with one vacancy and one interstitial, based on a simple London model with penetration depth $\lambda = a$ intervortex spacing.

Three principal claims

At 2DEG filling fraction $\nu = 1$ bound to each defect there is charge $+e/2$ for interstitial and $-e/2$ for vacancy.

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At 2DEG filling fraction $\nu = \frac{5}{2}$ bound to each defect should be a quasiparticle of the Moore-Read pfaffian state with non-Abelian statistics.

Fractional charges: Simple general argument

Consider the effect on 2DEG of *adiabatically* adding or removing vortex in a perfect flux lattice.

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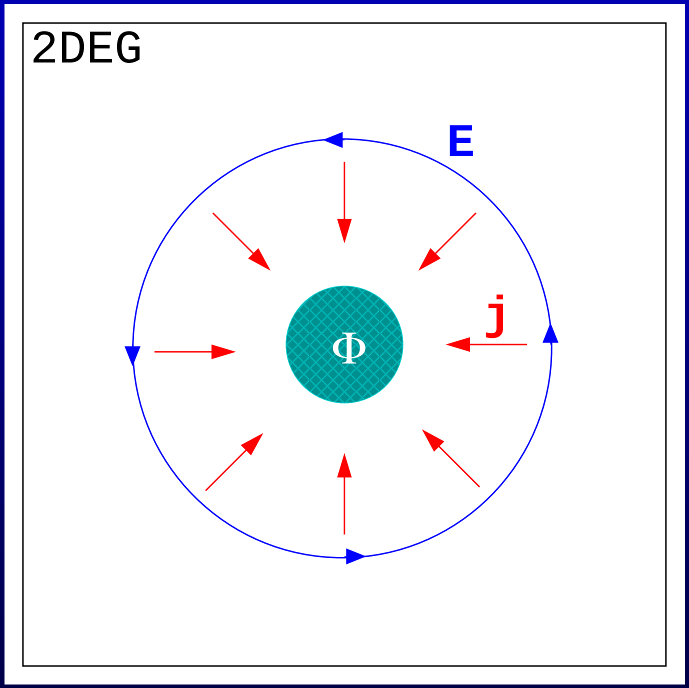

According to the Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},$$

time-dependent flux produces electric field. The field, in turn, gives rise to Hall current

$$\mathbf{j} = \sigma_{xy}(\hat{z} \times \mathbf{E}),$$

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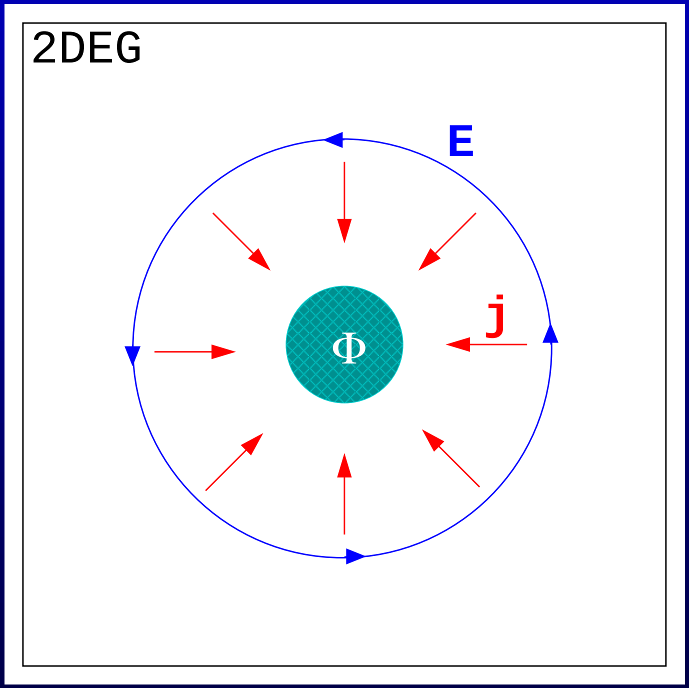

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with $\sigma_{xy} = e^2/h$ at $\nu = 1$. Integrate:

$$\delta Q = \frac{e^2}{hc}\int_{t_1}^{t_2} dt \left(\frac{d\Phi}{dt}\right) = e\left(\frac{\delta\Phi}{\Phi_0}\right)$$

Statistical angle

Naive counting would assign the $(e/2, \Phi_0/2)$ object statistical phase $\theta_0 = 2\pi(\frac{1}{2} \times \frac{1}{2}) = \pi/2$.

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According to the fusion rule $\theta = n^2\theta_0$ with $n = 2$ and $\theta = \pi$ we have

$$\theta_0 = \frac{\pi}{4}.$$

Non-Abelian physics at $\nu = \frac{5}{2}$

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Vacancy or interstitial then should bind a quasiparticle of the pfaffian state. These are known to exhibit non-Abelian exchange statistics.

Practical issues

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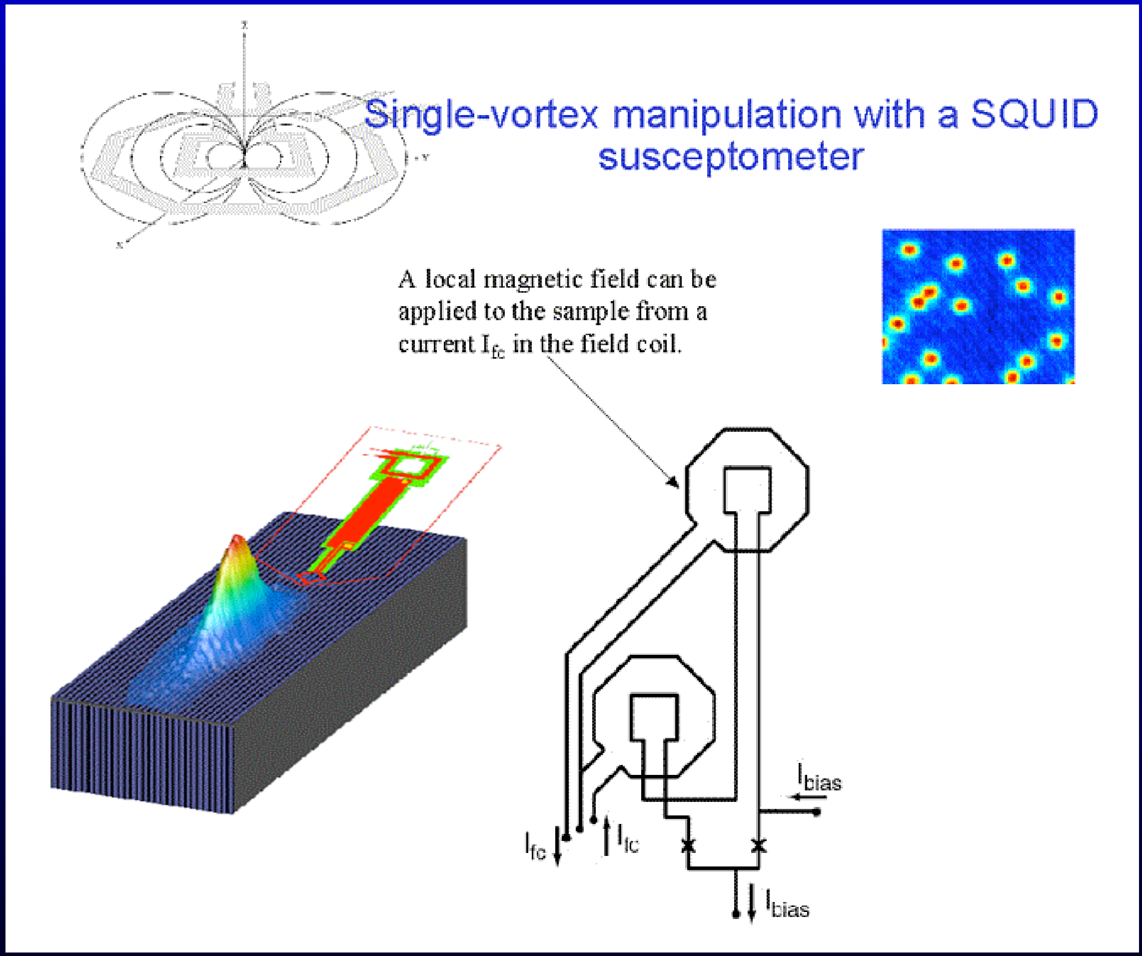

Moler Lab, Stanford

Simple continuum model

Consider 2-dimensional electron Hamiltonian,

$$\mathcal{H} = \frac{1}{2m_e}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2, \qquad \mathbf{A} = \left(\frac{1}{2}B_0 + \frac{\eta\Phi_0}{2\pi r^2}\right)(\mathbf{r} \times \hat{z}),$$

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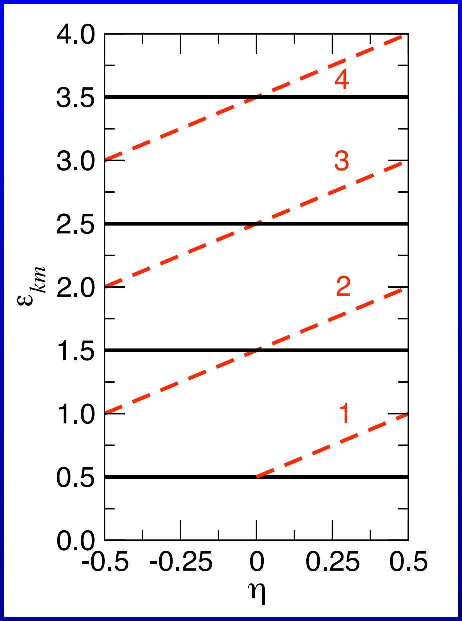
This simple model contains the essence of the physics and is exactly soluble.

The single particle eigenstates $\psi_{km}(\mathbf{r})$ are labeled by the principal quantum number $k = 0, 1, 2, \ldots$ and an integer angular momentum m. The spectrum reads

$$\epsilon_{km} = \frac{1}{2}\hbar\omega_c \left[2k + 1 + |m + \eta| - (m + \eta)\right],$$

where $\omega_c = eB_0/m_e c$ is the cyclotron frequency.



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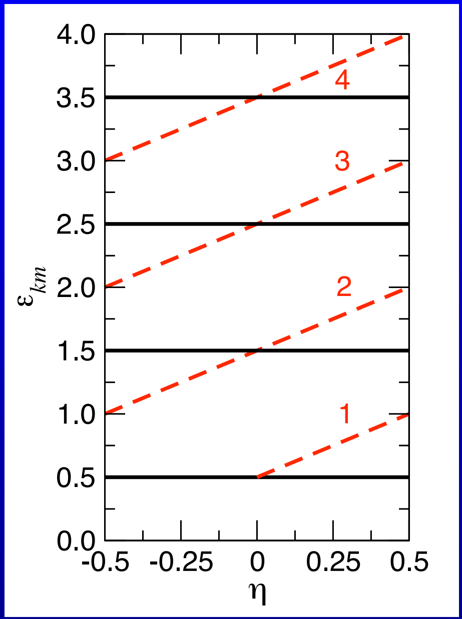

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The eigenstates ψ_{0m} in the lowest Landau level are

$$\psi_{0m}(z) = A_m |z|^{-\eta} z^m e^{-|z|^2/4},$$

where $z = (x + iy)/\ell_B$, and $\ell_B = \sqrt{\hbar c/eB}$ is the magnetic length.

If we fill the lowest Landau level by spin-polarized electrons then the many-body wavefunction can be constructed as a Slater determinant of $\psi_{0m}(z_i)$, where z_i is a complex coordinate of the i-th electron,

$$\Psi(\{z_i\}) = \mathcal{N}_\eta \prod_i |z_i|^{-\eta} \prod_{i<j} (z_i - z_j) e^{-\sum_i |z_i|^2/4}.$$

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The charge density is given as $\rho(\mathbf{r}) = \langle \Psi_0 | \hat{\rho} | \Psi_0 \rangle$. For a droplet composed of N electrons occupying N lowest angular momentum states we obtain

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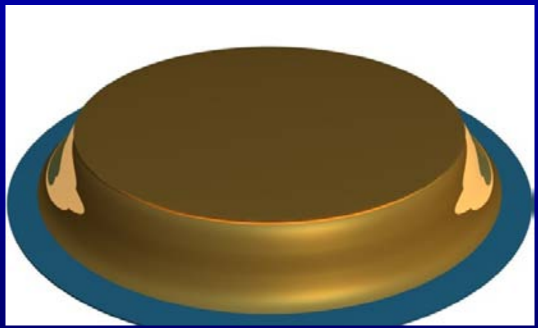

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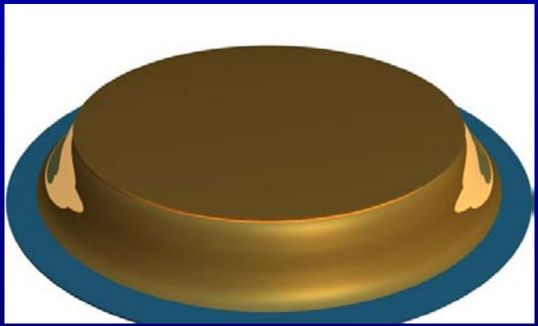

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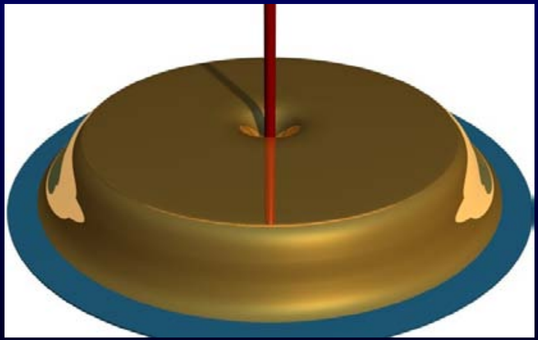
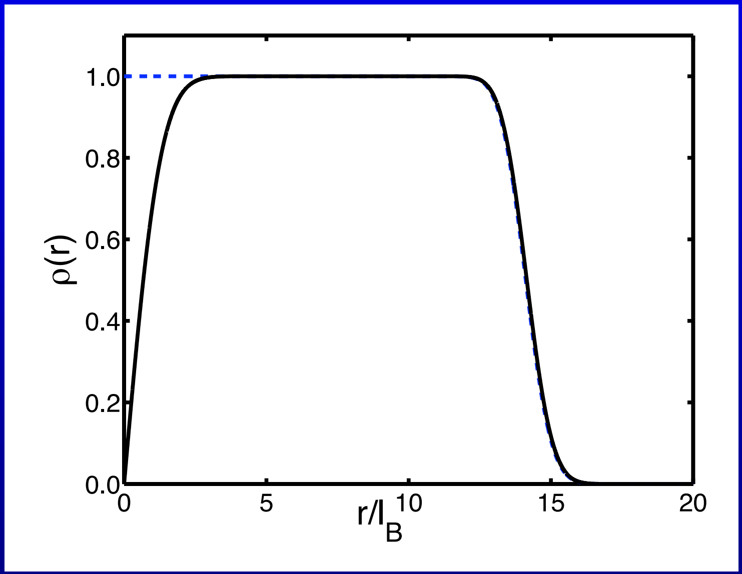


$N = 100$, $\eta = -1/2$

Look at the charge distribution more closely:



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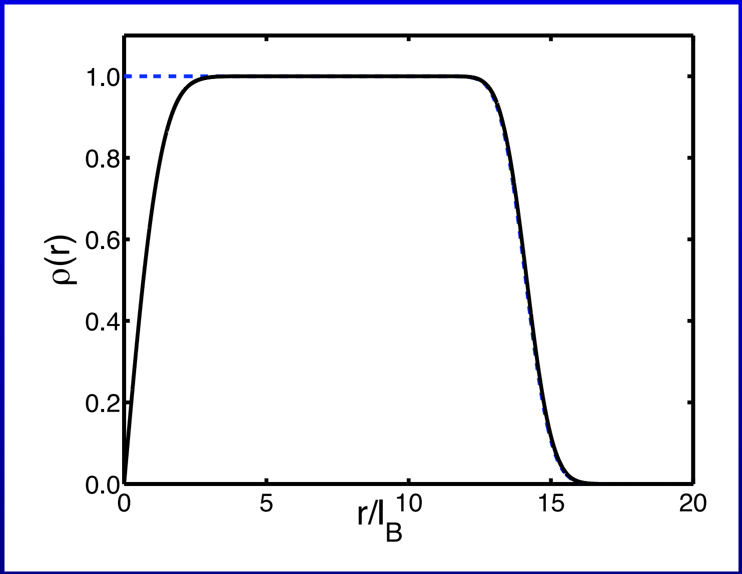

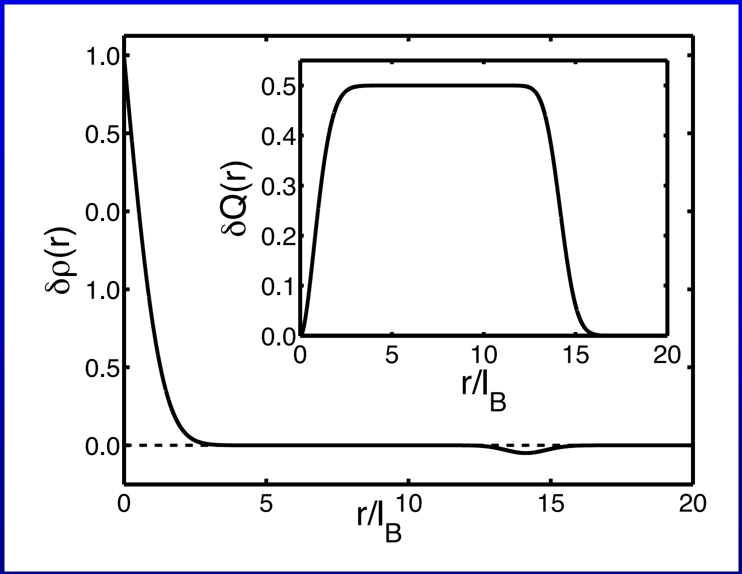

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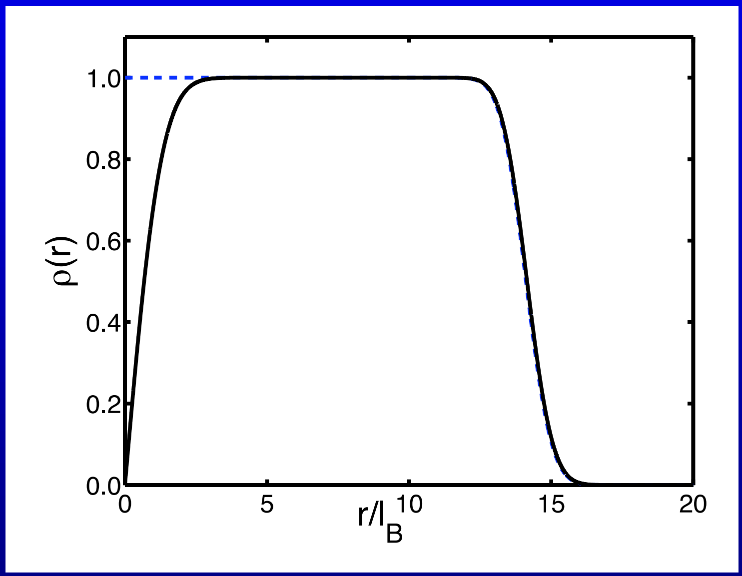

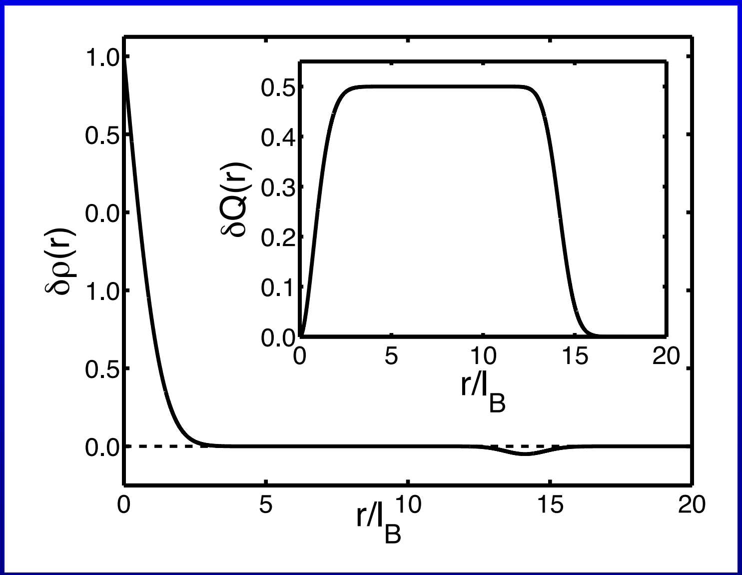

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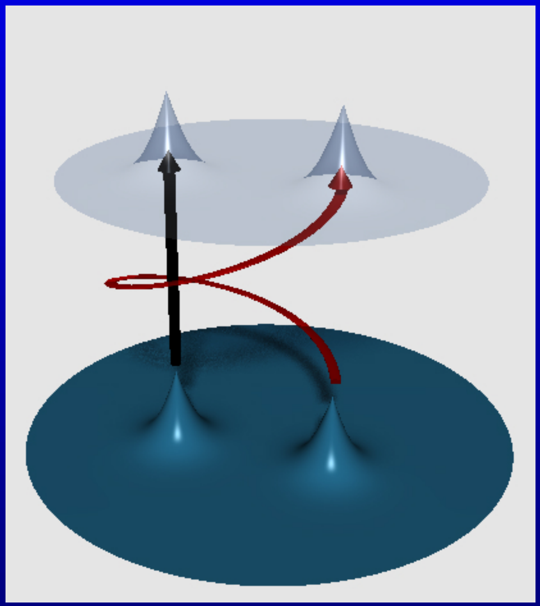
$\boxed{\text{This confirms our first claim that vacancy binds fractional charge } -e/2.}$

Particle statistics

The statistical angle θ can be computed from the present model by evaluating the Berry phase when we adiabatically carry one vacancy around another [Arovas, Schrieffer and Wilczek, 1984]:

$$\gamma(\mathcal{C}) = i \oint_{\mathcal{C}} dw \langle \Psi_w | \frac{\partial}{\partial w} \Psi_w \rangle .$$



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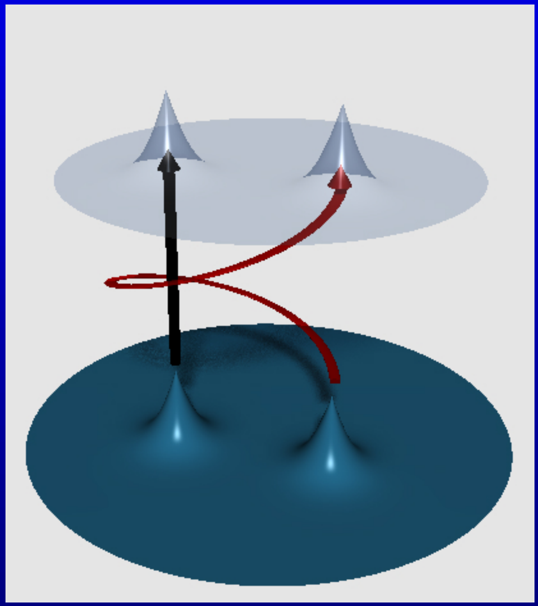

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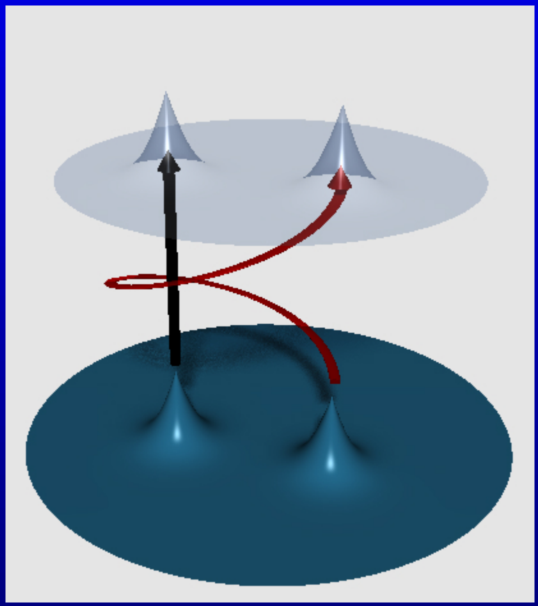

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For two vacancies, located at w_a and w_b the many-body wavefunction reads

$$\Psi_{w_a w_b} = \mathcal{N}_{w_a w_b} \prod_i (z_i - w_a)^{1/2} (z_i - w_b)^{1/2} \prod_{i<j} (z_i - z_j) e^{-\sum_i |z_i|^2/4} . \qquad (1)$$

In the above we have performed a gauge trasformation into the “string gauge” in which all the phase information is explicit in the wavefunction.

We now take $w_a = w$ and $w_b = 0$ and compute the above Berry phase along a closed contour $\mathcal{C}$ that encloses the origin.

This gives

$$\gamma(\mathcal{C}) = -\pi \left(\frac{\Phi}{\Phi_0} - \frac{1}{2} \right) .$$

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- The second represents twice the statistical phase of the vacancy, $\theta = \pi/4$, confirming our earlier heuristic result.

Lattice model

When the effective Zeeman coupling in 2DEG is sufficiently strong, then, in addition to the periodic vector potential, the electrons also feel periodic scalar potential. This effect becomes important in diluted magnetic semiconductors, such as $Ga_{1-x}Mn_xAs$, where the effective gyro-magnetic ration can be of order $\sim$100.

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$$\mathcal{H} = \sum_{ij} (t_{ij} e^{i\theta_{ij}} c_j^\dagger c_i + \mathrm{h.c.}) + \sum_i \mu_i c_i^\dagger c_i,$$

with Peierls phase factors

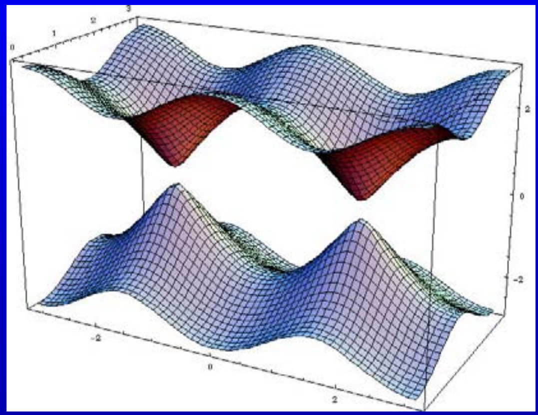
$$\theta_{ij} = \frac{2\pi}{\Phi_0} \int_i^j \mathbf{A} \cdot d\mathbf{l}$$

corresponding to magnetic field of $\frac{1}{2}\Phi_0$ per plaquette.

In *uniform field* and $\mu_i = \mu = \mathrm{const}$ $\mathcal{H}$ is easily diagonalized with the energy spectrum

$$E_{\mathbf{k}} = \mu \pm 2t\sqrt{\cos^2 k_x + \cos^2 k_y + 4\gamma^2 \sin^2 k_x \sin^2 k_y},$$

and $\gamma = t'/t$.



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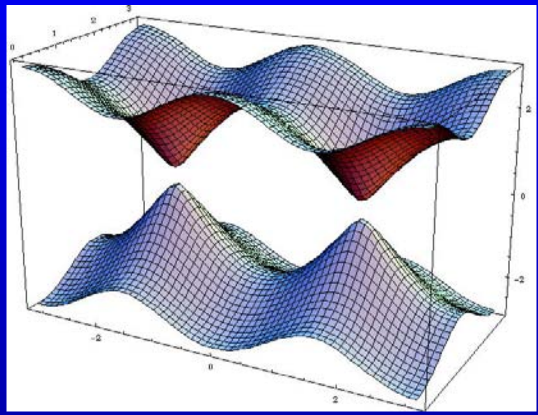

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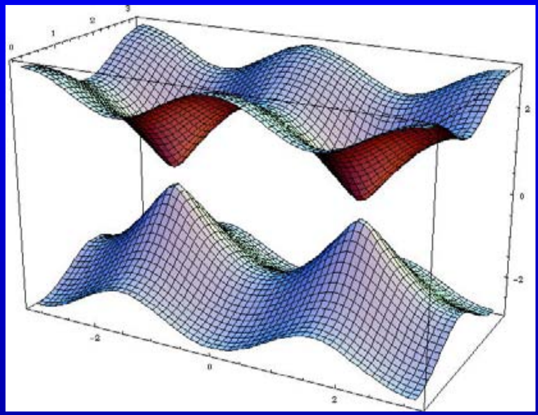

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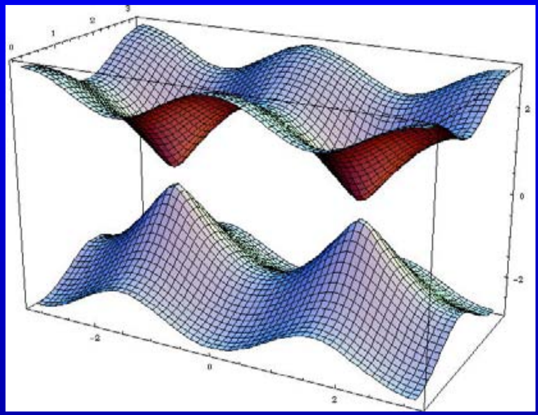

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We have diagonalized the above Hamiltonian numericaly for system sizes up to 50×50 and various configurations of fluxes and μ_i's.

In all cases we find that vacancy/interstitial binds charge $\pm e/2$.

Sample result for a single interstitial and random μ_i with $\Delta\mu_i = 0.05t'$.

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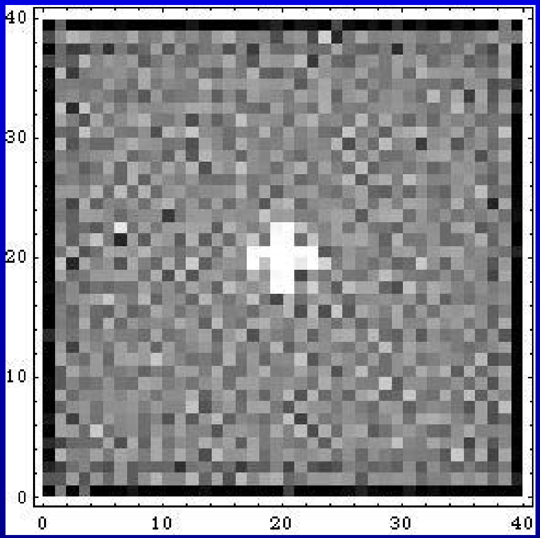

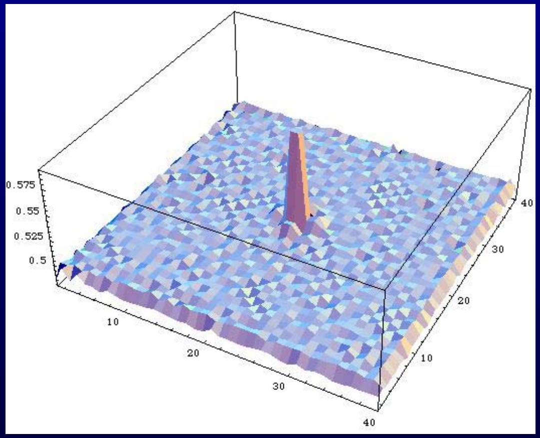

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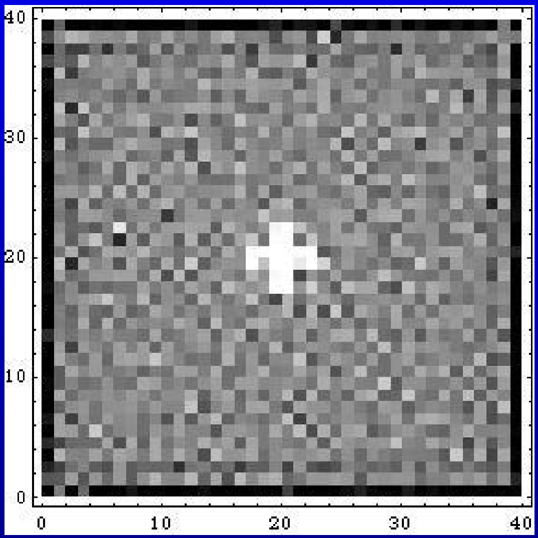

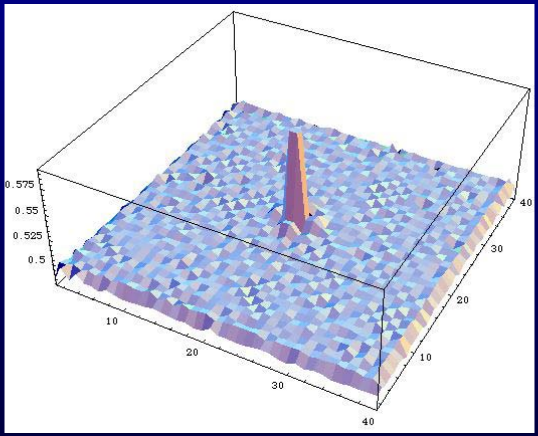

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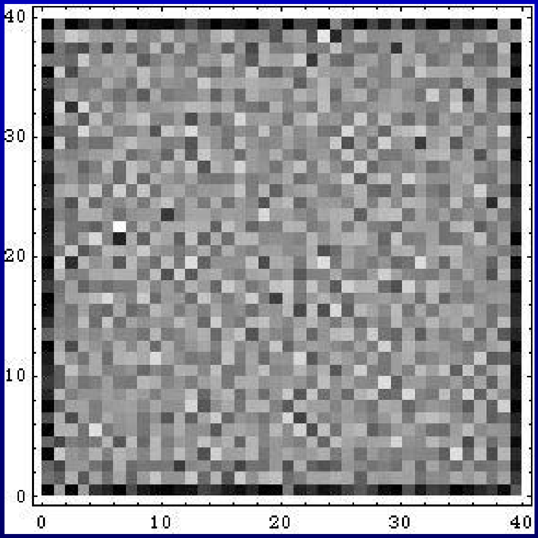

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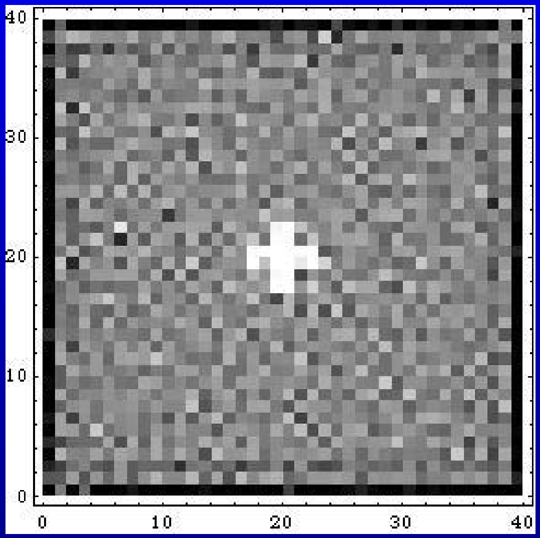

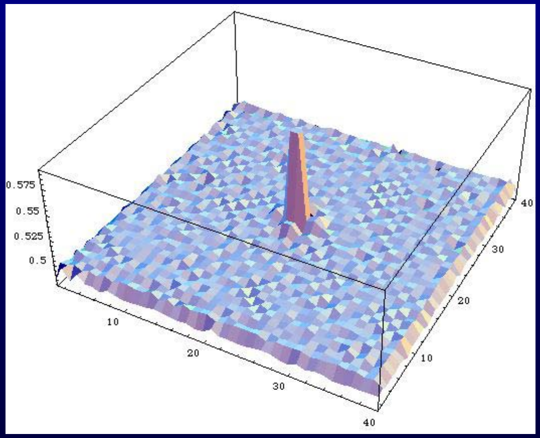

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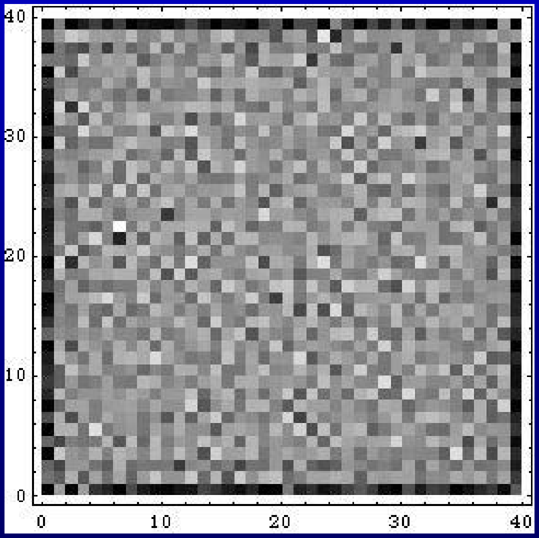

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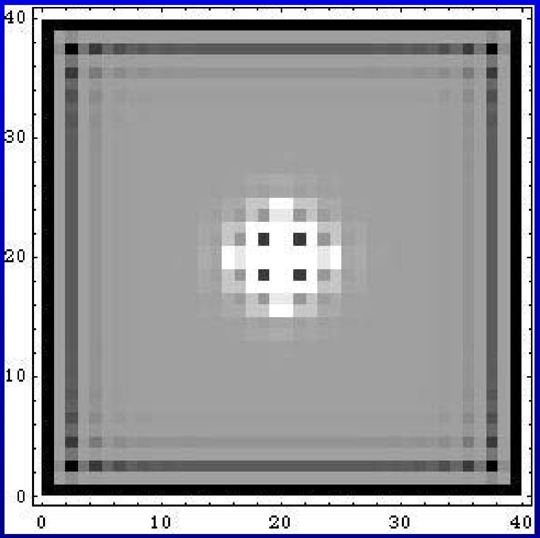

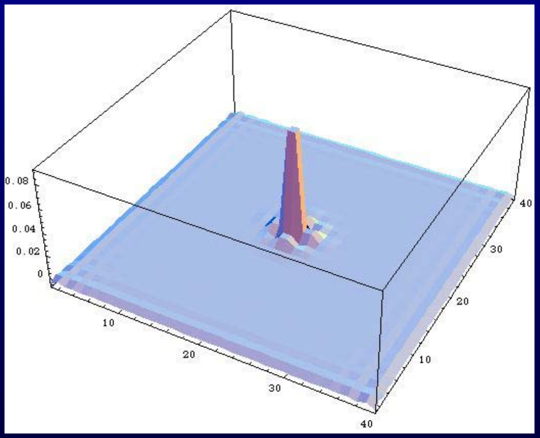

induced charge $\delta\rho(\mathbf{r})$

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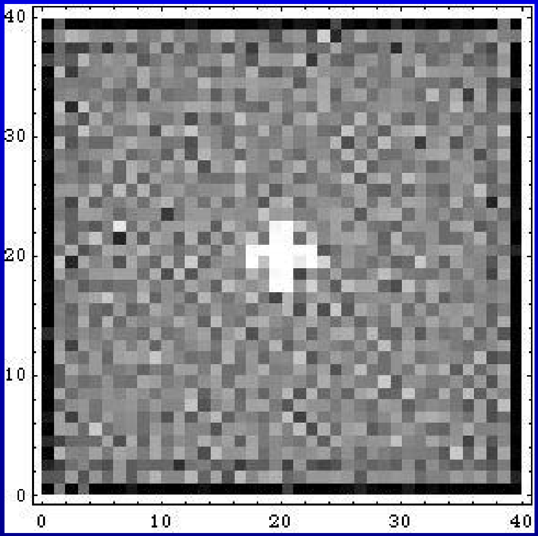

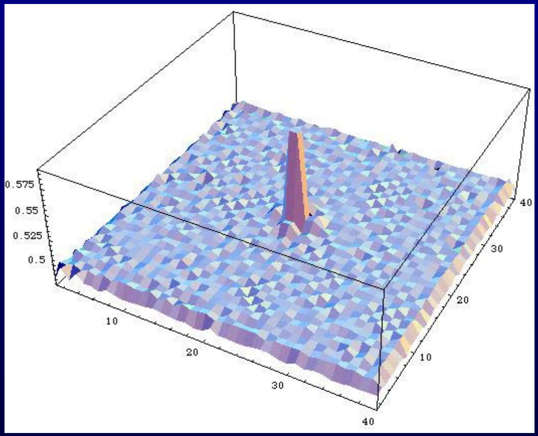

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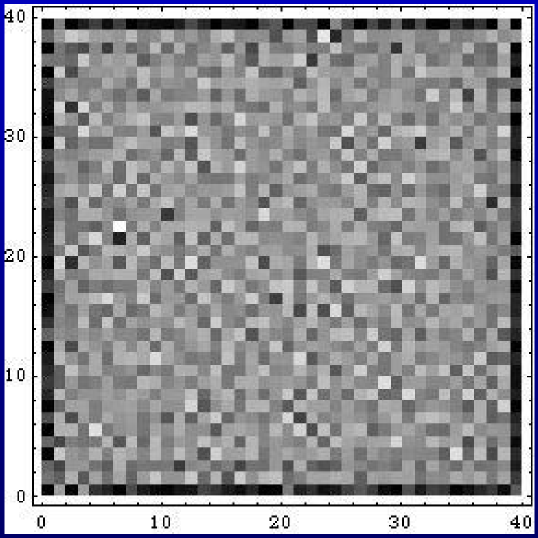

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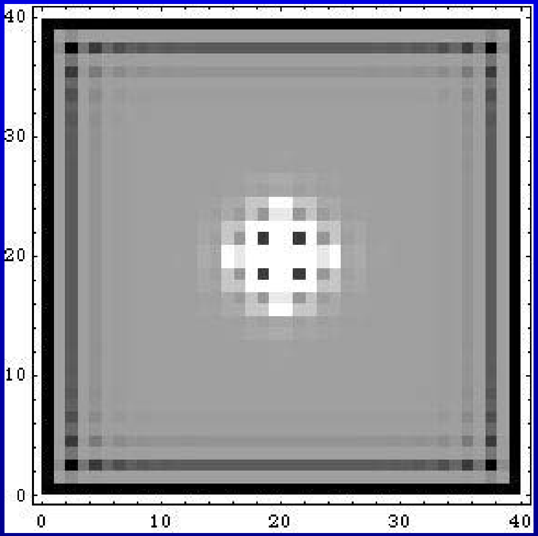

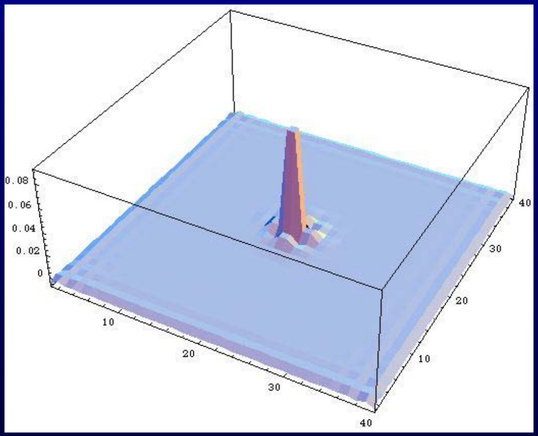

induced charge $\delta\rho(\mathbf{r})$

The induced charge integrates to $e/2$ to within machine accuracy.

Conclusions

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- At $\nu = \frac{5}{2}$ vacancy should bind a Moore-Read "non-Abelion" which may be used to implement topologically protected fault tolerant quantum gates.