Asteroids

Dynamical Perturbations: Myth or Reality?

Ryan Shannon

For Physics 349

7 April 2004

Background

Johannes Titius tried to find a mathematical formula that would describe the locations of the planets

He didn't publish

Johann Bode found the same formula and published

Moral: Publish as much as you can, as soon as possible, then you get to have your one-eyeballed portrait taken (No paintings of Titius)

The Relation

N = 0,1,2,4,8 …

r is in AU (~ 10^{11} m)

$$r = 0.4 + (0.3 \times N)$$

This rule held for all the known planets at the time (Mercury through Uranus)

Doesn't hold for Neptune and Pluto, but they are weird anyways

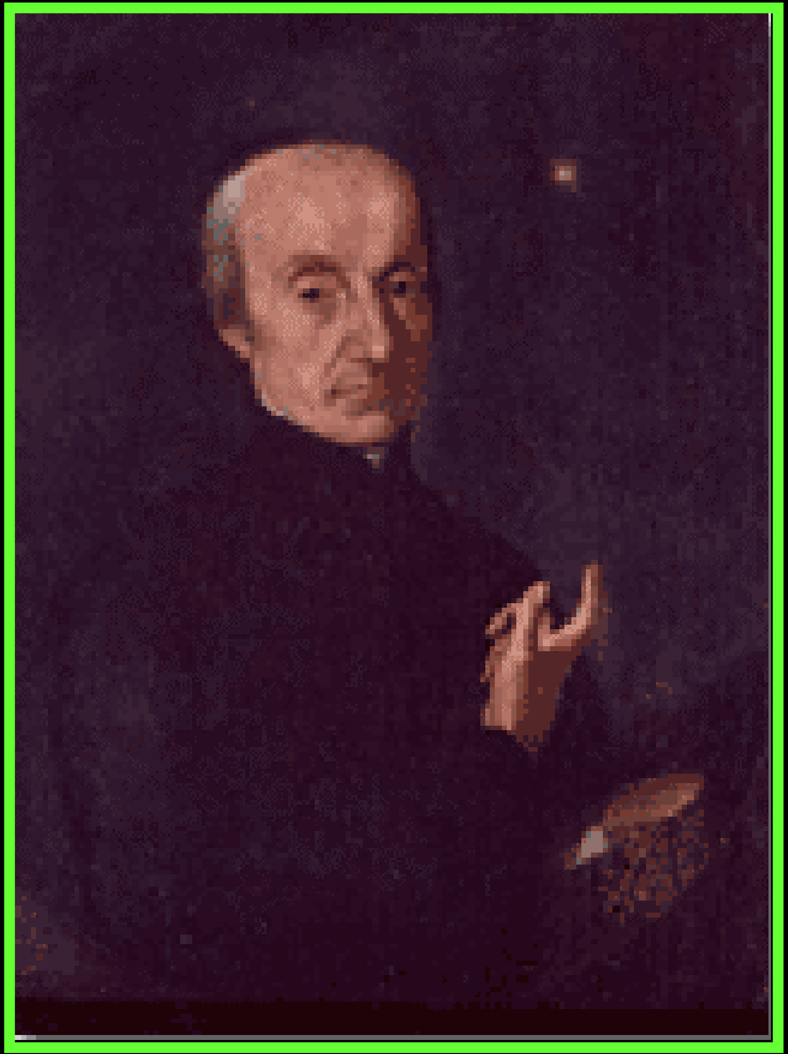
This rule was empirical

The Results

Planet	**N**	**Predicted Distance (Au)**	**Acutal Distance (AU)**
Mercury	0	0.4	0.39
Venus	1	0.7	0.72
Earth	2	1.0	1.0
Mars	4	1.6	1.52
???	8	2.8	2.77
Jupiter	16	5.2	5.20
Saturn	32	10.0	9.54
Uranus	64	19.6	19.19
Neptune	*128*	*38.8*	*30.07*
Pluto	*256*	*77.2*	*39.53*

What the ???

- Something was missing – astronomers went out looking for it (Group of Astronomers got together to look for it)
- However, before they got their act together (do Astronomers every get their act together?), The little Guy from Sicily Found a “minor planet” *Ceres*

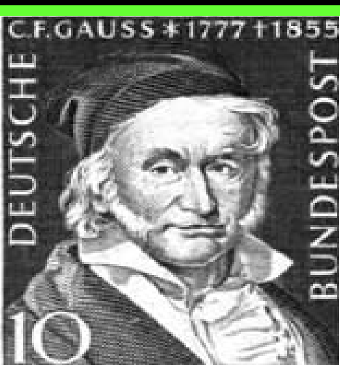


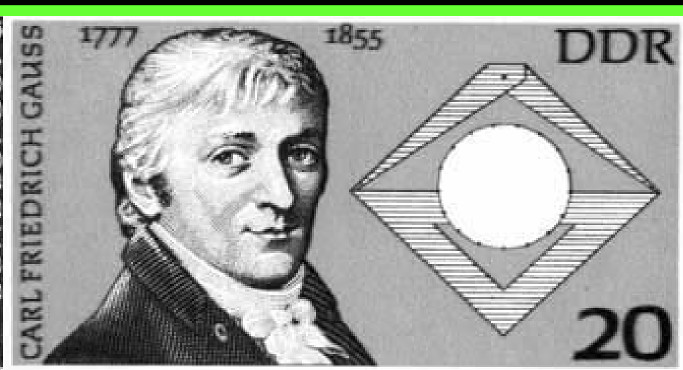
Giuseppe Piazzi, Pondering Life’s Questions

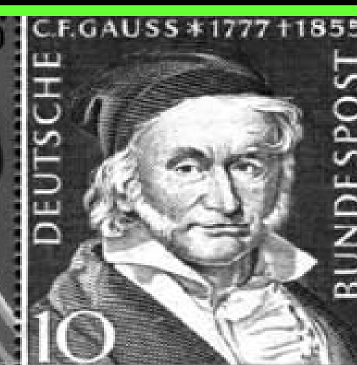
Gauss- Man of Many Talents



The man who made Physics 301 the fun it was also was the first person to figure out how to determine an orbit without having to see much of its motion – Great Triumph in Mathematical Astronomy







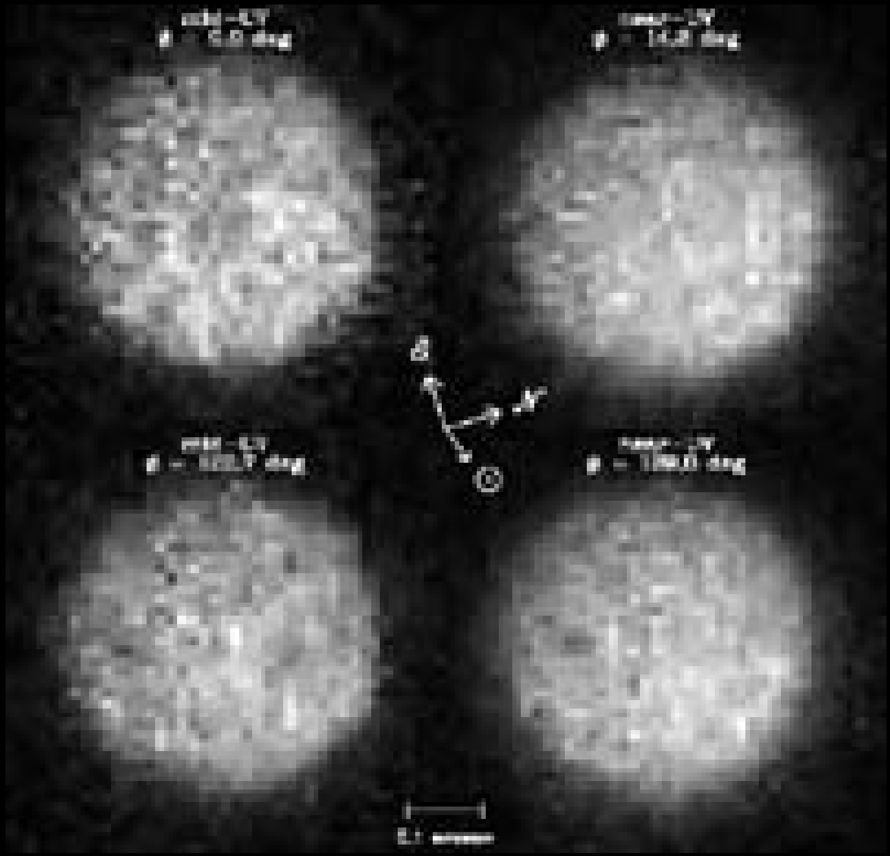
Ceres et. al.

They found Ceres to be in the Region of the Missing Planet

Many more asteroids were soon found by the likes of Olbers and Harding

Looked at stars, looked at chart, tried to figure out what had moved

When Astronomical photography became possible (1890s) asteroids could be found by the hundreds!



Ceres – Mid UV HST Image

Today

- Over 100000 asteroids have been detected.
- Using smallish (1.2 m) telescopes, 200-300 asteroids can be found on a single image.
- So many asteroids, that most of them are being ignored!

Everything You Wanted to Know About Asteroids But Were Afraid To Ask

Total Mass: may be as low as 1/10000 of the Earth's Mass!

Composition (determined by spectroscopy):

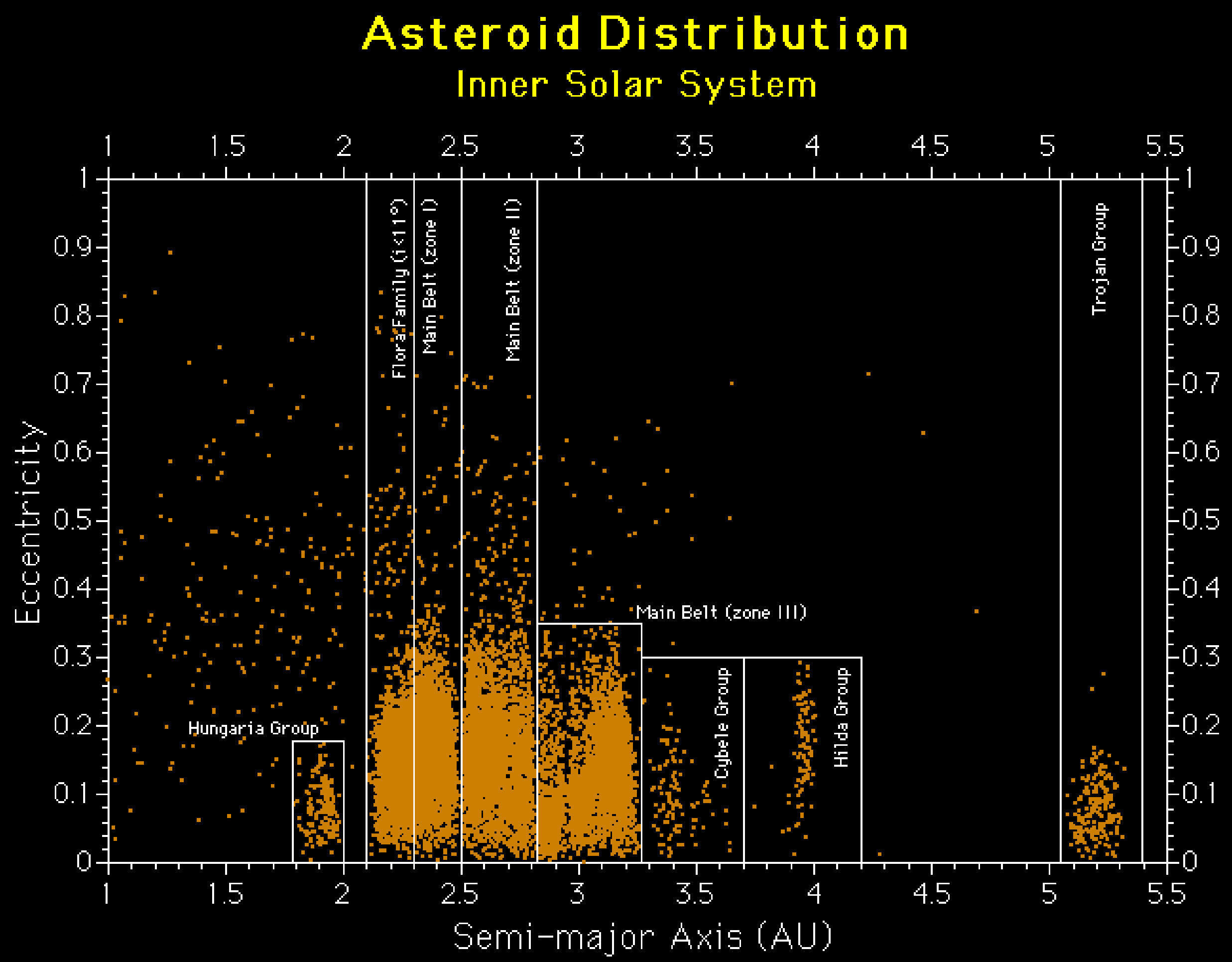
Iron, Magnesium, Silicates, Nickel, Carbon, Water, Ancient Organic Compounds

Shape: Varies, some smooth (like Ceres, some weird)

What Else?

What is the distribution of the Asteroids?

Not uniform. Most asteroids are located in a belt 2.0 – 3.2 AU from the sun. However, the distribution in the belt is not uniform, either.



What I Did:

- Simple model of solar system:
  - Sun, Planet and N Asteroids of the same mass (I could change N)
  - Sun and Planet Interacted with each other (but with the 'stroids)
  - Asteroids Interacted with Sun, Jupiter and each other

$$\vec{F}_p = -\frac{GM_\odot M_p}{r^3_{p,\odot}}\vec{r}_{p,\odot}$$

$$\vec{F}_\odot = -\vec{F}_p$$

$$\vec{F}_i = (\sum_{j=1,j\neq i}^{N} \frac{GM_i M_j}{r^3_{ij}}\vec{r}_{ij}) + \frac{GM_i M_p}{r^3_{i,p}}\vec{r}_{i,p} + \frac{GM_i M_\odot}{r^3_{i,\odot}}\vec{r}_{i,\odot}$$

How I Did It

Computer Simulation (FORTRAN 77)

Separated the equations into (as the other Ryan said) “First Order Canonical Form”

Integrated using a Forward Euler method

justified since gradient of gravitational field is small compared to step size)

More Details

- Motion Constrained to Disk (2D Problem)
- Initial Distribution: Uniform density throughout asteroid belt
- Initial Velocities set randomly
  - Orbital speed 90% - 110% of circular motion
  - Orbital velocity 90% -110% in the direction of circular motion
- Random Number Generated generates uniform distribution

More Details: Part Deux

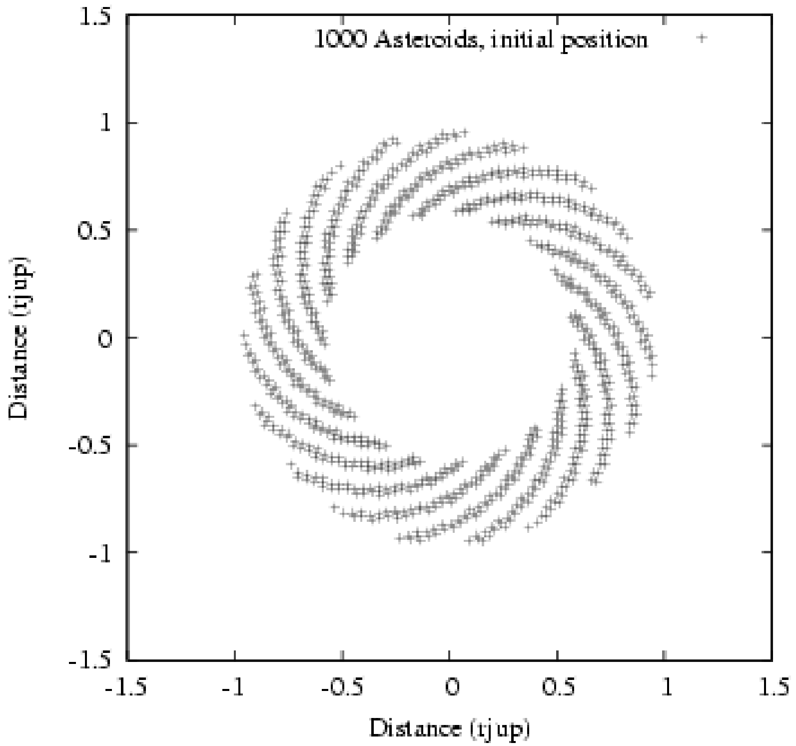
- Two regimes examined:

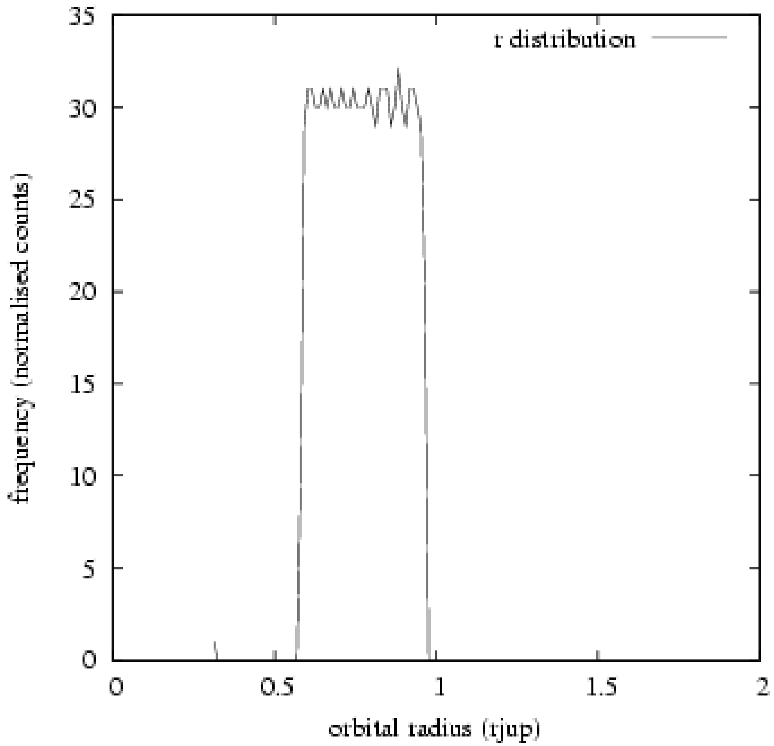
- Asteroids of typical mass (10^13 kg)
  - Total mass not equal to mass of Asteroid Belt
  - This doesn't make much sense for our problem

- Particles represent fluid points (10^18 kg)
  - Total mass equal to mass of asteroid belt
  - Final position represents expected "density" asteroids

- Planet mass and orbit could change. Results discussed here involve only a Jovian Planet at Jovian orbit (ie Jupiter)

Initial Distribution

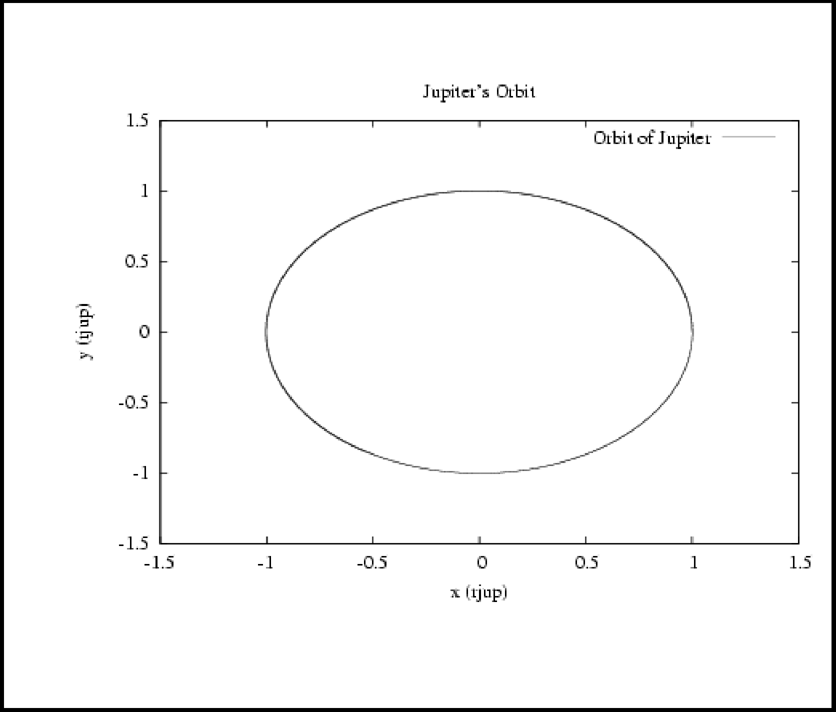




Program Verification

In order to verify the program is working properly, can test simple cases, and show that conserved quantities are conserved.

Simple case: no asteroids, Jupiter should rotate in circular orbit about the Sun.



Tests of Conservation

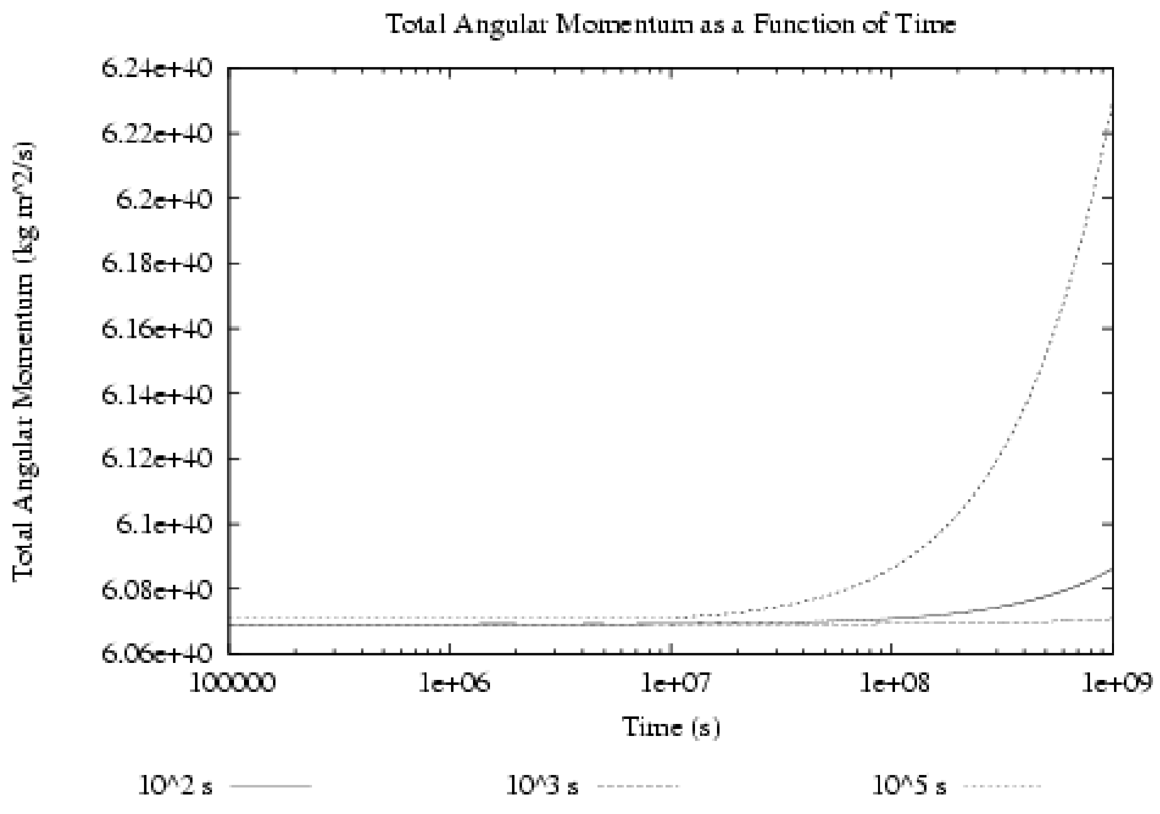
Energy- good, but it is an O(N^2) quantity to compute

$$E = \frac{1}{2}\sum_{i=1}^{N}(m_i v_i^2 - \frac{GM_iM_\odot}{r_{i,\odot}} - \frac{GM_iM_p}{r_{i,p}} - \sum_{j=1,j\neq i}^{N}\frac{GM_iM_j}{r_{ij}})$$

Angular Momentum O(N) quantity – much better!

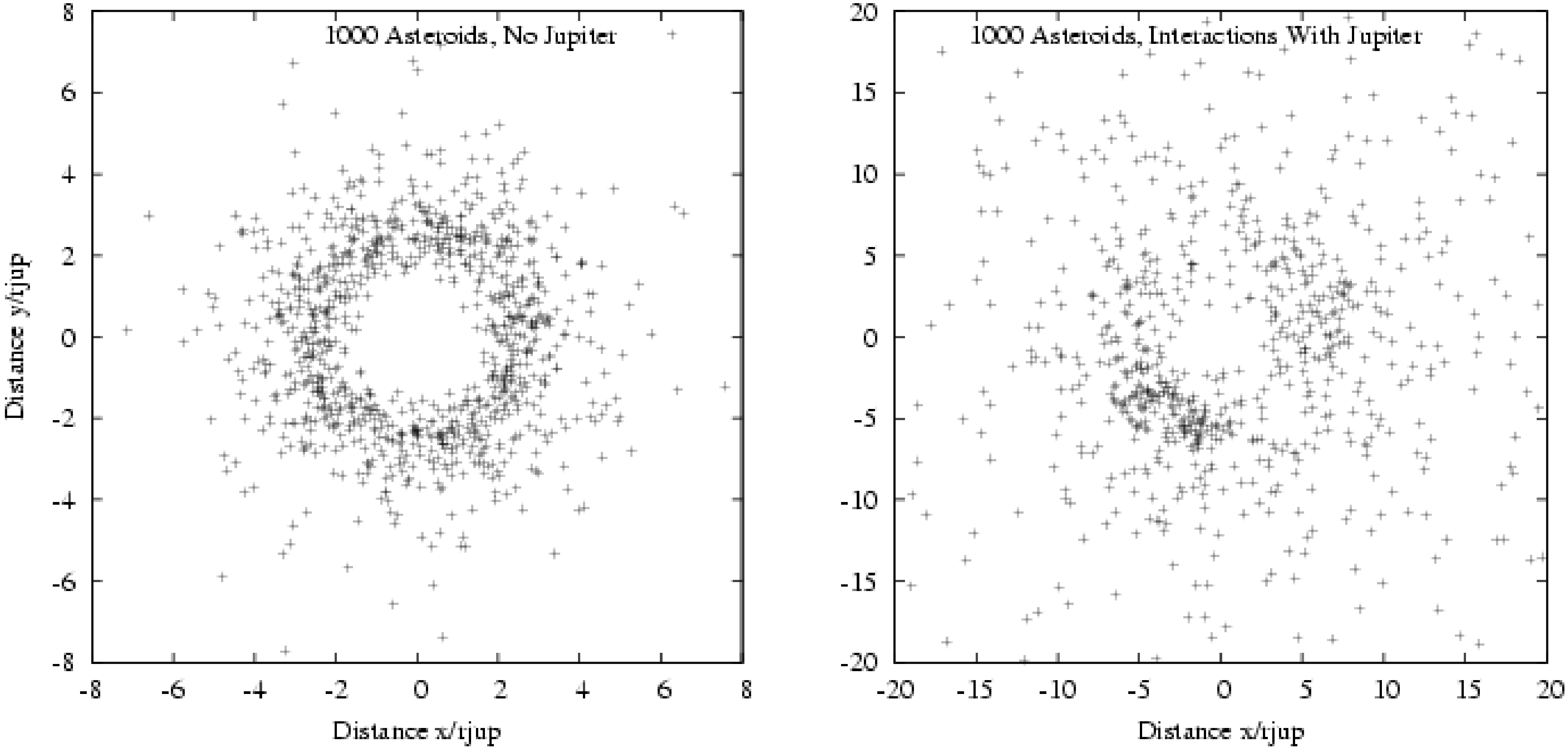
$$\vec{L}_{tot} = M_\odot \vec{r}_\odot \times \vec{p}_\odot + M_p \vec{r}_p \times \vec{p}_p + \sum_{i=1,N} M_i \vec{r}_i \times \vec{p}_i$$

Angular Momentum



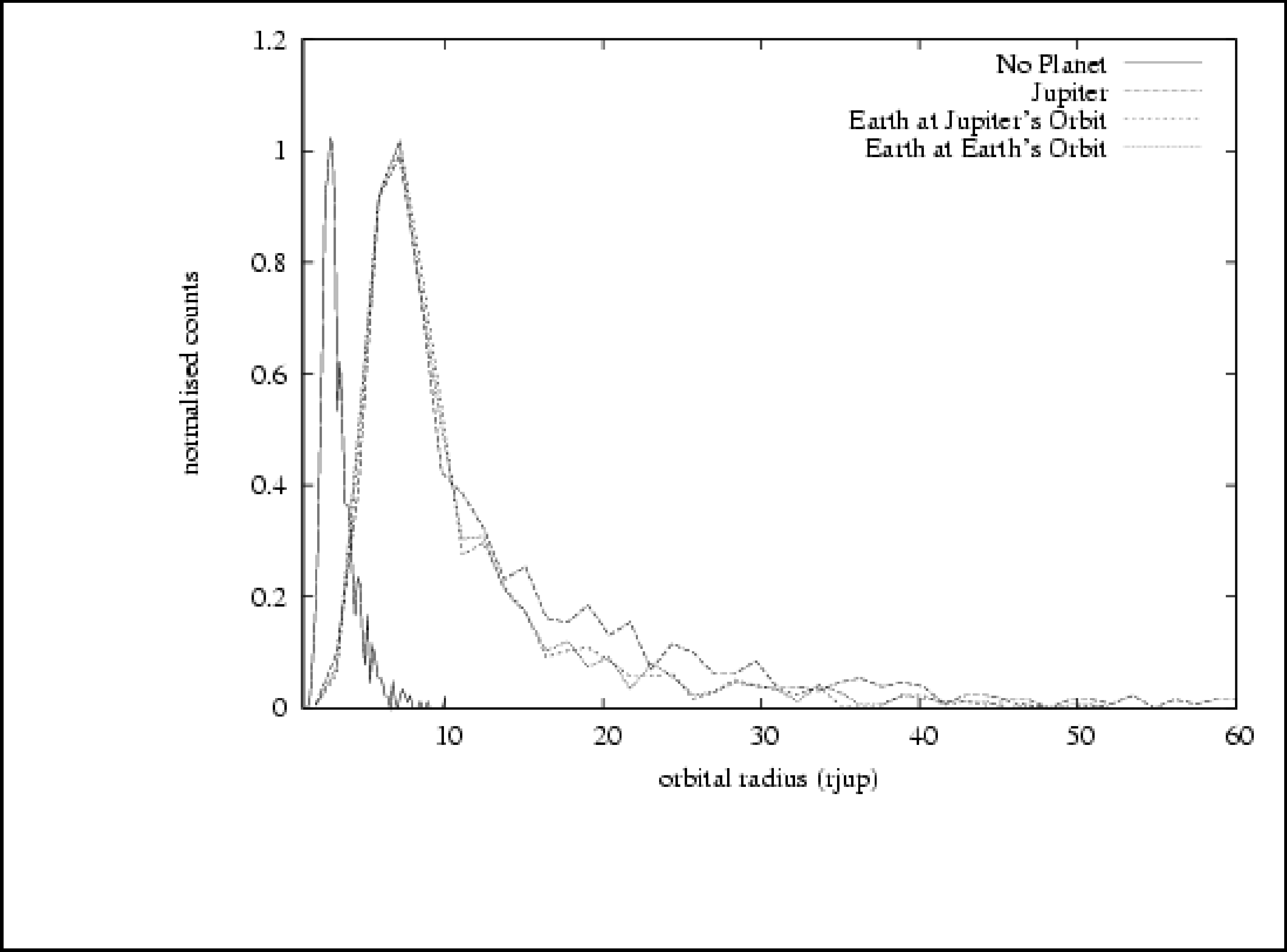
As can be seen, angular momentum tends to increase as time progresses, However, as the step size diminishes, the angular momentum stays more constant

Result 1: Planet Kicks Out Asteroids

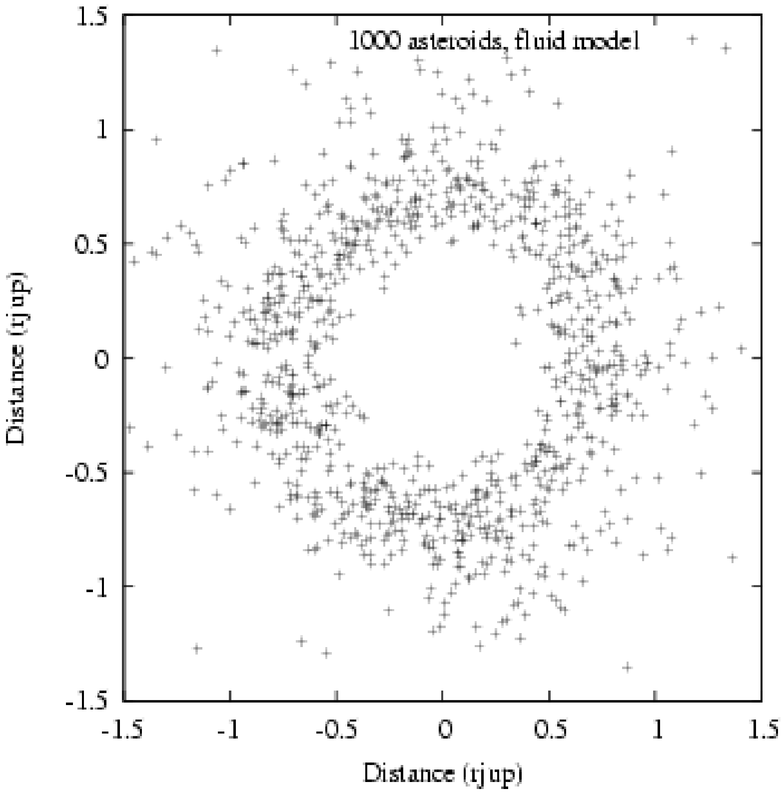


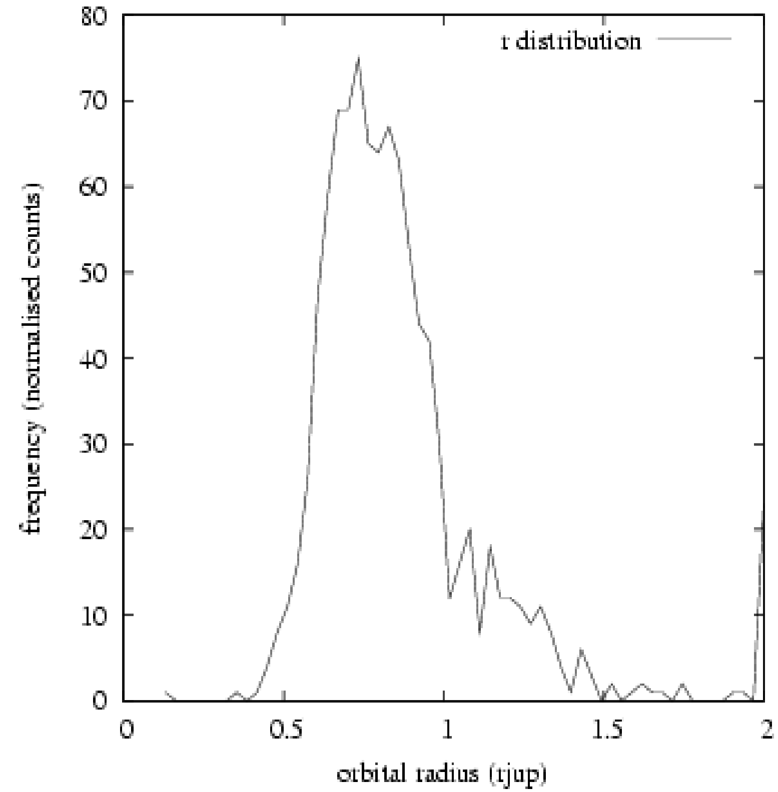
Systems with planets more oblated. This is as expected. The average lifetime of an asteroid in the same orbit as Jupiter is 3000 years

Comparison of Results



Result 2: Distribution of Asteroids





Future Work

More computing needed to refine model (last slide simulation took ~ 24 hours to run for 30 years with a step size of 10^4 seconds)

By reducing step size, the problem of increasing angular momentum will be reduced/removed

By properly testing vs. a null model (no Jupiter), we will be able to see if the structure in the distribution as a result of Jupiter

Requires more computing resources (massively parallel or supercomputer) or a long, long time on a PIII.

Interesting Refinements

- incorporate Other Planets as well
  - Wouldn't be that computationally difficult

- Create a mass distribution of particles

- Treat asteroidal collisions
  - Simplest case: elastic collisions
  - More complicated case:
  - Require a more dynamic language

Conclusion: Asteroids are cool

Note: Atari does not condone this project of presentation