PhD(Astro) Qualifying examination - 2014

13:30 - 17:30, August 29/2014

Do not open the exam until instructed to do so but you may read this cover sheet

- A one-page (8.5x11") hand-written double-sided sheet of notes is allowed.
- Scientific calculator allowed and expected.
- Put your name on upper right corner of your exam book.
- All answers should be in the exam books. Start every question on a new page.
- There are 8 questions to choose from. You will only select and provide answers for 5 of them; you may not attempt any portion of the other 3 questions. The five questions you choose are each valued the same, and thus you may wish to time budget about 45 minutes per question.
- On the front of your exam booklet you should clearly indicate which 5 questions you wish graded if you have any writing in your answers on more than 5 questions. If not, the first 5 questions started in your exam booklets will be graded.

Please return this examination with your exam booklet.

(1). Examine Figure 1 which shows the radial relative velocity of high velocity water masers in gas clouds that orbit the center of NGC 4258, a galaxy in Canes Venatici. It is believed that these masers are heavily beamed along the velocity vector direction, and thus only masers travelling radially towards or away from us are visible and thus provide the real circular rotation speed at the given angular separation (in milli-arcseconds, or mas). As shown in the graph, at 5.0 mas, v=980 km/s and at 8.0 mas, v=775 km/s, relative to the galaxies dynamical center (the solid and open symbols correcspond the different senses of rotation).

a) [3 points] Explain what masers are and how they can be used to provide speed relative to the galaxy's center.

b) [5 points] NGC 4258 is about half as far away as the Virgo cluster of galaxies is. To factor of 3 accuracy, estimate the physical scale of region which the masers occupy, and compare it to the scale of the bulge of our own galaxy. If you do not know the distance to the Virgo cluster, use 10^{22} meters.

c) [12 points] Describe/calculate as precisely as possible (clearly explaining your reasoning) the geometry and magnitude of the mass distribution at the center of NGC 4528 implied by these data.

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(2). [20 points] It is beyond doubt that large galaxies harbour central massive black holes. These grow in mass by accreting stars and gas. Smaller black holes are able to tidally disrupt stars before they cross the event horizon, leading to observational signatures from the hot gas. On the other hand, more massive black holes are expected to swallow entire stars whole, which would probably not be detectable. Estimate the maximum mass that a black hole could have and still be able to tidally disrupt a Sun-like star.

(3a) A Hot Interstellar Cloud: [10 points]

X-ray photons are produced in a cloud of radius R at the uniform rate (photons per unit volume per unit time). The cloud is a distance d away. Assume that the cloud is optically thin. A detector at Earth has an angular acceptance beam of half-angle $\Delta\theta$ and an effective area A.

1. If the cloud is fully resolved by the detector, what is the observed intensity of the radiation as a function of position?

2. If the cloud is fully unresolved, what is the average intensity when the source is in the detector?

(3b) The Eddington Luminosity: [10 points]

There is a natural limit to the luminosity a gravitationally bound object can emit. At this limit the inward gravitational force on a piece of material is balanced by the outgoing radiation pressure. Although this limiting luminosity, the Eddington luminosity, can be evaded in various ways, it provides a useful (if not truly firm) estimate of the minimum mass of a particular radiation source. Recall the solar mass is 2×10^{33}g.

Consider ionized hydrogen gas. Each electron-proton pair has a mass more or less equal to the mass of the proton (m_p) and a cross section to radiation equal to the Thompson cross-section (σ_T). The radiation pressure is given by outgoing radiation flux over the speed of light. Equate the outgoing force due to radiation on the pair with the inward force of gravity on the pair.

Solve for the luminosity as a function of mass. What is the Eddington luminosity of the sun?

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(4). The life of a solar mass star. [20 points]

Present as detailed an explanation as possible of the history of a one solar-mass object, beginning from an isolating interstellar cloud and then proceeding for 20 billions years. You should show the evolutionary track on an well-labelled HR diagram, and number sequentially certain points along the evolution track, for which you then develop in writing what is going on in that stage, in terms of time scales, physical state and scale, and energy sources important. Be as quantitative as possible. Judge the level of your answers relative to spending roughly 45 minutes on this question to cover the entire question.

(5). Radiation pressure effects

Consider a dust grain that is suddenly released from an asteroid. Assume that the heliocentric orbital eccentricity of the asteroid is zero and that the grain roughly retains the asteroid's orbit immediately following the grain's release. After being expelled from the asteroid, the grain is now subject to radiation pressure from the star.

(a) [4 pt] Describe in words how radiation pressure affects the grain's orbit.

(b) [4 pt] Derive an approximate relation for β, the ratio of the magnitude of the radiation pressure force over the magnitude of the star-grain gravitational force. Set the radiation pressure to the radiation flux incident on the grain divided by the speed of light. Ignore relativistic effects. Describe all other assumptions in your derivation and define your terms. Write the sum of the two forces (radiation pressure and gravity) using beta.

(c) [2 pt] Describe how the radiation pressure effects on a grain's orbit vary (i) with distance from the host star and (ii) with the grain's diameter.

(d) [12 pt] Derive the value for beta at which radiation pressure will remove the grain from the system.

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(6). Cosmology

a) [3 points] The cosmological Helium mass fraction is observed to be about $Y_{He} \simeq 2/9$. Use this fact to estimate the neutron to proton ratio n_n/n_p at the temperature T_{nuc} when Big Bang Nucleosynthesis (BBN) turns on.

b) [3 points] Deuterium forms in the simple process : $n + p \longleftrightarrow D + \gamma$
The binding energy of deuterium is $B_D = 2.22$ MeV, and it is energetically favourable to form deuterium at temperatures lower than this. Yet, BBN does not begin until much lower temperatures. Why does this occur?

c) [7 points] Write down a Saha-like equation relating the number densities of protons, neutrons, and deuterium. Estimate T_{nuc} from Figure 2 and, assuming all neutrons become temporarily bound in deuterium at T_{nuc}, find an estimate of the baryon-to-photon ratio η_b and the present-day baryon fraction Ω_b.

d) [7 points] The neutron-proton mass difference is $Q = m_n - m_p = 1.293$ MeV. Assume all interactions that interconvert protons and neutrons, other than free neutron decay, shut down at T=1 MeV. Determine the neutron-to-proton ratio at $T = 1$ MeV, and using your knowledge of cosmology and Y_{He}, estimate the lifetime of free neutrons.

(7). Statistical inference

(a) [**10 points**] Explain, using at most one page, the critical difference(s) between Bayesian and frequentist statistical analyses. Discuss pros and cons, using a descriptive example if needed.

(b) [**10 points**] The following two sets of data are photon counts in 10 equal time bins from an astronomical detector. Use the chi-square test to decide if each one obeys Poisson photon statistics as might be expected from random events.

Data set 1 : 21 16 22 14 21 35 27 10 20 17

Data set 2 : 20 16 23 17 18 18 21 24 14 24

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(8). Observational Astronomoy [20 points]

Scientists on the Kepler spacecraft team recently reported the discovery of Kepler-22b, an Earth-like planet in the habitable zone of the star Kepler-22a. The system is estimated to be 600 light-years (LY) from Earth.

a) The host star is type G5. Based on this and astro-seismic analysis, it has an estimated mass of $0.970 \pm 0.060\ M_\odot$, and an estimated radius of $0.979 \pm 0.020\ R_\odot$. What is the apparent visual magnitude of Kepler-22a, assuming it has the same temperature and radius as the Sun? The Sun's absolute visual magnitude is 4.83 and 1 pc = 3.26 LY.

b) Kepler time-series photometry data indicate a brightness drop of 492 ± 10 ppm in Kepler-22a during a transit. Assuming the planet is opaque, estimate the radius of Kepler-22b, in units of the Earth radius. Recall that $R_\odot = 109 R_\oplus$.

c) The photometry data indicate an orbital period of 290 days for Kepler-22b. Assuming a circular orbit, estimate the radius of the orbit, in AU.

d) A Solar System object in thermal equilibrium has a temperature $T = 280\ \text{K}\ (1 - A)^{1/4}/a^{1/2}$, where A is the planet's albedo, and a its semi-major axis, in AU. Assuming this formula applies and that Kepler-22b has an Earth-like albedo (A=0.39), what is the equilibrium surface temperature of Kepler-22b. Explain any approximations you believe are inherent in this T estimate.

e) Assuming the T derived in (d), what is the frequency of peak thermal emission from Kepler-22b? What is the corresponding wavelength, in μm?

f) What is the apparent angular radius of Kepler-22b, as seen from Earth? If one attempted to image Kepler-22b using interferometry in the future, estimate the baseline required if the observations were done at the wavelength of peak emission. For convenience, note $R_\oplus = 6.74 \times 10^{-10}$ LY.)

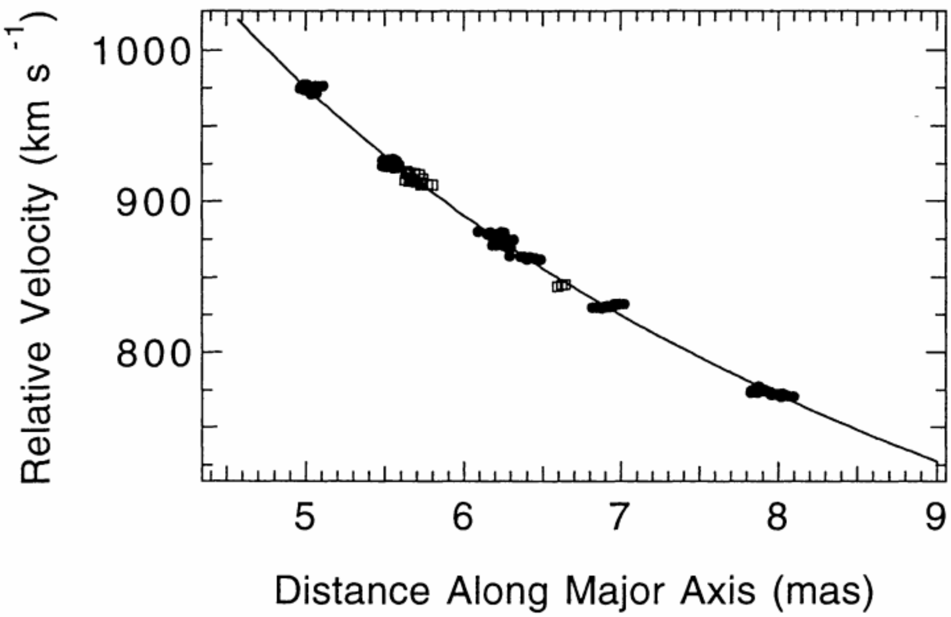

Figure 1. For question 1.