PhD(Astro) Qualifying examination - 2017

September 1, 2017

Do not open the exam until instructed to do so but you may read this cover sheet

- A one-page (8.5x11") hand-written double-sided sheet of notes is allowed.
- Scientific calculator allowed and expected.
- Put your name on upper right corner of your exam book.
- All answers should be in the exam books. Start every question on a new page.

• There are 8 equal-value questions to choose from. **You will only select and provide answers for 5 of them;** you may not attempt any portion of the other 3 questions. The five questions you choose are each valued the same, and thus you may wish to time budget about 45 minutes per question.

• On the front of your exam booklet you should clearly indicate which 5 questions you wish graded if you have any writing in your answers on more than 5 questions. If not, the first 5 questions started in your exam booklets will be graded.

Please return this examination with your exam booklet.

(1). a) [6 points] For a 1.5 solar-mass star ($M = 3.0 \times 10^{30}$ kg) of two solar luminosities, derive (explaining your reasoning) the orbital speed of a 3 $M_\oplus$ super-Earth planet in an a =0.04 au circular orbit around its host star (only 10% accuracy is needed here).

b) [8 points] Given that the 'no-greenhouse' temperature of the Earth is about 270 K, derive the approximate surface temperature if the planet is rapidly spinning. Calculate the wavelength where the thermal re-radiation of the planet peaks.

c) [6 points] If our solar system were to be in the orbital plane of the planet, would the 1 m/s precision of the HARPS radial-velocity echelle spectrograph be sufficient to detect this planet's existence around the star? Explain.

---

(2). [20 points] The vertical component of the gravitational acceleration at a distance z above the plane of a thin stellar disk is given by the cylindrical Poisson equation,

$$|g_z| = 2\pi G\Sigma(z)$$

with

$$\Sigma(z) = \int_{-|z|}^{|z|} \rho(z)dz$$

where ρ is the mass density. From this, show that for an exponential distribution of stars

$$\rho(z) = \rho_0 e^{-|z|/h},$$

the vertical velocity dispersion σ_z^2 is related to the scale height h by

$$\sigma_z^2 = \frac{3}{2}\pi G\Sigma h.$$

where $\Sigma = 2\int_0^\infty \rho(z)dz$ is the surface density.

Hint: First show that for the exponential distribution,

$$g_z(z) = 4\pi G\rho_0 h(1 - e^{-|z|/h})$$

then apply the virial theorem. You may assume this relation if you cannot derive it, at penalty. Rrecall that the potential energy of a star at height z is the (negative of) the work done against gravitaty if you move the star to infinity.

(3) A white dwarf is a degenerate electron gas. The atomic nuclei are mixed in; they balance charge and provide the gravitational mass to hold the star together. Although freshly-formed white dwarfs are some of the hottest stars observed, a permissable approximation in this problem to set T=0.

a) [5 points] Calculate the gravitational potential energy U_{grav} of a uniform density sphere with radius R and mass M.

b) [10 points] Assume that the star contains one proton and one neutron for each electron and that the electrons are non-relativistic. Show that the total kinetic energy

$$U_K = 0.0088 \, \frac{h^2 M^{5/3}}{m_e m_p^{5/3} R^2}$$

where m_e and m_p there the electron and proton mass, respectively. The numerical factor is a combination of factors which you will need to calculate. You will need the Fermi energy

$$E_F = \frac{h^2}{8m_e}\left(\frac{3n_e}{\pi}\right)^{2/3}$$

where n_e is the volume number density of electrons.

c) [5 points] Sketch a plot of the total energy $U_K + U_{grav}$ as a function of radius R. Derive the relation between the equilibrium radius R_{eq} and the white dwarf's total mass M.

---

(4). The life of a solar mass star. [20 points]

Present as detailed an explanation as possible of the history of a one solar-mass object, beginning from an isolating interstellar cloud and then proceeding for 20 billion years. You should show the evolutionary track on an well-labelled HR diagram, and number sequentially certain points along the evolution track, for which you then develop in writing what is going on in that stage, in terms of time scales, physical state and scale, and which energy sources are important at that time. Be as quantitative as possible. Judge the level of your answers relative to spending roughly 45 minutes on this question to cover the entire question.

(5). You have optically observed a minor Solar System body and have a reasonable idea of its distance. You wish to estimate its radius R.

a) [14 points] Show that

$$pR^2 \approx 2.2 \times 10^{16}\ d_{\rm au}^{\,2}\Delta_{\rm au}^{\,2}\ 10^{0.4(m-m_\odot)}\ {\rm km}^2,$$

where p is the object's albedo at the observed wavelengths, $d_{\rm au}$ is the minor body's heliocentric distance in au, and $\Delta_{\rm au}$ is the distance from the Earth to the minor body in au. In the observed band, m is the minor body's apparent magnitude and $m_\odot$ is the Sun's apparent magnitude as observed from Earth. Assume the minor body's phase function is unity. *Hint: It may be helpful to first derive the observed flux for the minor body under the given scenario and then convert the flux to magnitudes.*

b) [2 points] You believe that the object, which was observed at opposition, is at a heliocentric distance of 40 au. For your observing band, $m_\odot = -27$ and $m = 22$. What is the object's size in terms of km $p^{-1/2}$?

c) [4 points] In a few sentences describe how you might break the size-albedo degeneracy.

---

(6). Power-law atmospheres. Assume the following:

- The Rosseland mean opacity is related to the density and temperature of the gas through a power-law relationship,

$$\kappa_R = \kappa_0 \rho^\alpha T^\beta;$$

- The pressure of the gas is given by the ideal gas law;
- The gas is in hydrostatic equilibrium so $p = g\Sigma$ where g is the surface gravity and Σ is the column density; and
- The gas is in radiative equilibrium with the radiation field so the flux is constant with respect to z or Σ.

[20 points] Calculate the temperature of the gas as a function of Σ. Assume that temperature goes to zero as the density goes to zero. For free-free opacity, $\alpha = 1$ and $\beta = -7/2$ and for electron scatting $\alpha = \beta = 0$. What are the relationships in these two cases?

(7). Cosmological He mass fraction

The equilibrium number density of a particle A with mass m_A and g_A internal degrees of freedom in equilibrium at temperature T is

$$n_A = g_A \left(\frac{m_A k_b T}{2\pi\hbar^2}\right)^{3/2} \exp^{-m_A c^2/(k_b T)}$$

We will apply this formula to neutrons (n) and protons (p) being kept in equilibrium by the weak interactions.
a) [4 points] Find an exact expression for the ratio n_n/n_p assuming equilibrium.
b) [3 points] Defining $Q_n = (m_n - m_p)c^2 \simeq 1.29$ MeV, argue that to a very good approximation the neutron to proton fraction in equilibrium is

$$f \equiv \frac{n_n}{n_p} \simeq e^{-Q_n/k_b}$$

c) [7 points] Argue that if the neutron number density fraction is f at the time Helium forms then the maximum possible Helium energy density fraction is

$$Y_{He} \equiv \frac{\rho_{He}}{\rho_b} \simeq \frac{2f}{1+f}$$

where ρ_b is the total density in baryons (neutrons and protons).
d) [6 points] Suppose the weak interations freeze out when T=0.1, 1, or 10 MeV. Find the maximum Helium fraction Y in each case, respectively.

---

(8). A galaxy has $m_{\mathrm{AB}}(550\,\mathrm{nm}) = 22$. It is observed in the V-band ($\lambda = 550\,\mathrm{nm}$ and $\Delta\lambda = 89\,\mathrm{nm}$) by a telescope with a diameter of 1 m and the galaxy completely fills 1 pixel of the CCD. The CCD has a quantum efficiency η of 70% and produces one electron per detected photon. It has a readout noise equivalent to 25 electrons. Assume the sky background and thermal noise in the CCD are negligible.

a) [10 points] How many photons are detected by the CCD in exposure time t? (Hint: $m_{\mathrm{AB}} = 0$ corresponds to a flux density of 3631 Jy and $m_{\mathrm{AB}}(550\,\mathrm{nm}) \simeq m_{\mathrm{V}}$.)

b) [4 points] What is the signal-to-noise ratio (SNR) obtained in time t?

c) [6 points] Sketch a plot of the SNR as a function of exposure time for t ranging from 1 to 100 ms. When does the readout noise significantly affect the SNR?