PHYSICS PH.D. COMPREHENSIVE EXAM 2006

(1) In construction work, a practical means of establishing a vertical reference line is the use of a "plumb line" – a mass hanging in equilibrium from a long vertical cord. However, due to the centrifugal effects of the rotating earth, the plumb line will not hang in a truly vertical direction.

(a) Find an expression for the angular deviation α from vertical for a plumb line at latitude λ ($0 < \lambda < \frac{\pi}{2}$) in the Northern hemisphere. Be sure to indicate the direction of the deviation (N,S,E or W) from the vertical.

(b) Determine the approximate latitude at which the maximum deviation would occur and estimate the magnitude of the deviation at that latitude.

(2) Positronium is a hydrogen-like system consisting of a bound state of an electron and a positron. The lowest energy states are a singlet and triplet sub-state which are almost degenerate. The singlet state is the most stable, lying about $8.2 \times 10^{-4} ev$ below the triplet levels which are themselves degenerate. Field theoretic calculations show that this splitting of the singlet and triplet is due to a spin-spin interaction of the form

$$H_0 = -\frac{A}{\hbar^2}\vec{s}_1 \cdot \vec{s}_2$$

(a) Determine the value of the constant A.

(b) Using the fact that the positron has a charge and magnetic moment that is opposite to that of the electron, calculate the effect of a magnetic field on these levels.

(3) The specific heat capacities of liquid water and ice at atmospheric pressure and for several degrees Celsius below the freezing point are given by the equations

$$c_P(\text{water}) = 4222 - 22.6T$$
$$c_P(\text{ice}) = 2112 + 7.5T$$

in joules per kilogram-degree. Here T is the temperature in Celsius. What is the specific entropy difference between ice and water at -10 Celsius? The latent heat of fusion is 335200J/kg.

(4) A plasma is described by the dielectric function

$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{\omega_P^2}{\omega^2}\right)$$

where ω_P is a constant. In a typical laboratory or astrophysical environment, any attempt to establish a voltage $V(t) = V \cos \omega t$ across the plasma generates a region of vacuum called the "sheath" on either side of the plasma volume as indicated in the one-dimensional sketch below.

(a) Derive expressions for the uniform electric field $E_P(t) = E_p \cos \omega t$ in the plasma and for $E_S(t) = E_S \cos \omega t$ in the sheath. Assume that there is no free charge anywhere. Assume that ω_P is small enough that an electrostatic approximation is always valid.

(b) Plot the fields E_P and E_S on the same graph as a function of frequency. Discuss where the voltage drop occurs and why when (1) $\omega << \omega_P$, (2) $\omega = \omega_P$ and (3) $\omega >> \omega_P$.

(c) Make an LC circuit interpretation of the resonant behavior at $\omega = \omega_P / \sqrt{1 + L/\ell}$.

(5) A photon of wavelength λ is incident upon a stationary electron, collides with the electron and scatters inelastically through an angle θ, achieving the final wavelength λ'. To lowest order in $(\lambda' - \lambda)/\lambda$, and assuming that the electron is non-relativistic, obtain an expression for $\lambda'(\theta)$ in terms of λ and physical constants.

(6) Two neutrons are trapped in a spherical potential $V(r) = 0$ if $r < R$ and $V(r) = +\infty$ if $r > R$. Ignoring the interactions between the neutrons, find the ground state wave function of system (including the spin wave function).

Assume now that the neutrons interact with each other through the potential $V_I = A\vec{s}_1 \cdot \vec{s}_2$, where $\vec{s}_i$ is the Pauli spin matrix for the i-th neutron, and A is a constant. Treating V_I as a perturbation, find the approximate ground state energy.

(7) Denote the energy eigenstates of a one-dimensional simple harmonic oscillator by $|n>$. You are told that, at $t = 0$, there is zero probability that a measurement of the energy will give a value greater than $3\hbar\omega/2$. Moreover, at $t = 0$, the expectation value of p is as large as possible, consistent with the above information. Determine the oscillator's state vector $|t>$ and the expectation value of p for all subsequent times.