Name: ______________ Student #: ______________

**PHYS350: Applications of Classical Mechanics**
**Study Guide**
**Due: April 15, 2019**

# Problem Worksheet B

Name: ______________ Student #: ______________

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*Problem 1*: Inertia Tensors

In each case find the inertia tensor and express it in terms of the total mass $M$ and the length scale of the rotating body.

(a) A uniform density cube with sides of length $a$ about a corner.

(b) A uniform density cube with sides of length $a$ about it's centre.

(c) A uniform density sphere of radius $R$ about it's centre.
[Hint: Use spherical coordinates when performing integrals]

(d) A sphere with of radius $R$ and density and profile $\rho(r) = \rho_R(r/R)^n$. ($n > 0$)

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*Problem 2*: Two Masses and Three Springs

Two masses of mass $m_1$ and $m_2$ at positions $x_1$ and $x_2$ (measured from *equilibrium*) are attached to fixed walls by the springs with spring constants $k_l$ and $k_r$ respectively and to each other by $k_m$.

(a) Draw a picture of this system and write down the Lagrangian for this system using the coordiantes $x_1$ and $x_2$.

(b) What are the generalized mass matrix **M** and spring constant matrix **K** for this system?

(c) For the case $m_1 = m_2 = m$ and $k_l = k_m = k_r = k$ find the two normal frequencies and two normal modes of this system. Qualitatively describe the form of these two oscillations.

(d) If you write down the equations of motion in terms of the normal coordinates $\xi_1$ and $\xi_2$ of this system what form do they take?

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*Problem 3*: Orbits in The Kepler Problem

Two bodies of mass $m_1$ and $m_2$, or equivalently of reduced mass $\mu$ and total mass $M$, interact via a potential of the form $-\gamma/r$.

(a) Write down the Lagrangian for this system in the centre of mass frame using plane polar coordinates and determine the angular and radial equations of motion.

(b) Use the conservation of the magnitude of angular momentum, as expressed in the angular equation, to write an effective 1D radial equation of motion.

(c) Express the radial equation using angle rather than time as the independent variable and change the dependent variable to $u = 1/r$.

(d) Solve to find $u$ and $r$ as a function of angle, and characterize the orbits (bound vs. unbound; circle, ellipse, parabola, hyperbola) as a function of the eccentricity parameter $\epsilon$.

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*Problem* 4: Moments of a Symmetric Body

(a) What are the Principal Moments of a Body? What are the Principal Axes of a Body?

(b) Suppose a body has one axis of cylindrical symmetry. What does this imply about the principal moments of this body? What does this imply about the principal axes of this body?

(c) What are the Euler angles? Describe the 3 rotations needed to define them and the axes that define these 3 rotations.

(d) Write down the angular velocity in terms of the Euler angles.

(e) Write down the Kinetic Energy and Lagrangian of a Symmetric Body

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*Problem 5*: Hamiltonian for a Cart on a Spring

Set up the Hamiltonian $H$ for a cart of mass $m$ on a spring (force constant $k$) whose mass is *not* negligible, using the extension $x$ of the spring as the generalized coordinate.

(a) Find the Total Kinetic Energy of this system? What is the Generalized Momentum?

(b) Write down the Hamiltonian of the system and Hamilton's Equations. What is the frequency of oscillation of the system?

(c) If we interpret this system as a simple harmonic osillator how does the effective mass $m_{eff}$ of the oscillator relate to $m$ and $M$?

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