A bit about me...

I'm originally from Vancouver.

I'm a theoretical physicsist and cosmologist.

Right now I'm doing research on:

- Dark Matter Physics
- The Matter vs. Antimatter Asymmetry
- Collisions between Universes in a Multiverse
- Cosmological Perturbation Theory
- Cosmic 21-cm Fluctuations (Radio)
- Making a 3D Map of the Universe and Measuring the Expansion Rate
- Dark Energy
- FFT Telescopes CHIME (Radio)
- Strong Gravitational Lensing
- Satellite's and Space Science
- Gravitational Waves

Course Logistics

Physics 350: Applications of Classical Mechanics

Lecture:	M	W	F	3:00 - 3:50	Hennings 201
Tutorial:	M			9:00 - 9:50	Hennings 202

Prof. Kris Sigurdson (krs@phas.ubc.ca)

TA: TBA

Website: www.phas.ubc.ca/~krs/PHYS350

Office Hours: TBA

Textbook: **Required:** "Classical Mechanics" by John R. Taylor

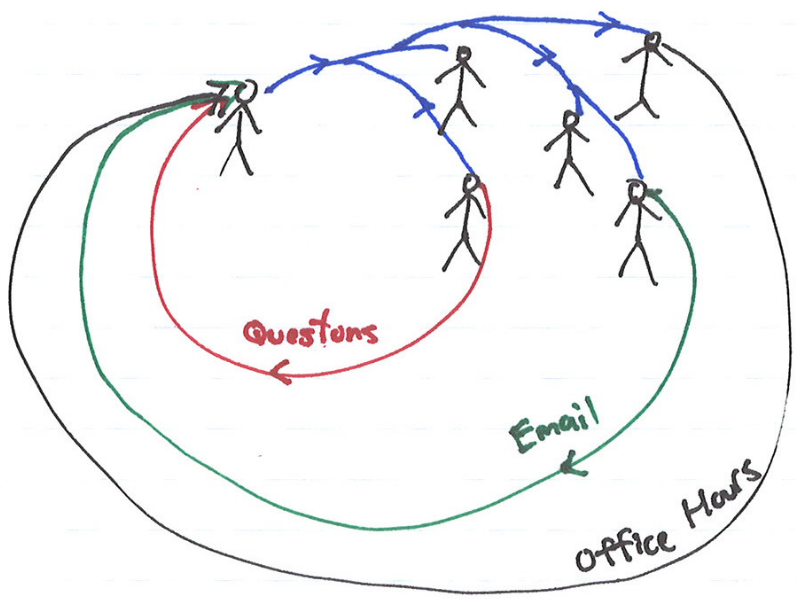
Optional: "Classical Dynamics" by Thornton and Marion

(Advanced) "Classical Mechanics" by Goldstein, Poole, and Safko

Tools: Mathematica, Matlab, Octave, Python

Course Feedback

* Questions Please!
* Office Hours
* Email: krs@phas.ubc.ca



Projects

A Physics project:

* Teams of ~5
* Plan, design, and build a virtual or physical system that demonstrates an advanced concept related to classical mechanics beyond the basic material of the course.
* Research and Development Report
* End of term Presentations!

Start thinking about who you want to work with and what you'd like to do

ASAP

Much more to follow

Course Goals

By the end of PHYS 350 you should be able to:

* Quickly write down differential equations that describe the motion of almost any mechanical system

* Know how to characterize the solutions of these equations with conserved quantities

* Solve, numerically if necessary, for the motion of almost any mechanical system (at least in principle)

* Build a virtual or physical system that demonstrates and communicates an advanced concept or principle of classical mechanics.

* Communicate the basics of Lagrangian Mechanics to a non expert, and an advanced concept to your peers.

Why Know Classical Mechanics?

* Because you want to know how things work.
* Because you want to build something
* Civil, Mechanical, Robotics, Computer
  ↳ "Game" Physics
* An astoundingly good description of the world above the atomic/quantum scale
* You want to know how to approach the same problem from many angles
* Problem solving techniques and adapting to deal with new problems are useful in all sorts of applications outside of physics or engineering
* The obvious generalization of the math and methods we talk about here forms the basis for all known physics.
  "Real physics has a Lagrangian"

What is Classical Mechanics?

Mechanics is the study of how things move.

- * Planets around the Sun
- * Electron in a Magnetic Field
- * Bead on a Wire
- * Dark Matter in the Galaxy
- * Rocket in Space
- * Pulley in a Machine

Classical → Not Quantum Mechanics
Deterministic

→ Not Relativistic Mechanics
$v \ll c$

↓

System of physical principles first described by Galileo (1564-1642) and formulated by Newton (1642-1727) in his three laws of motion.

"Philosophiae Naturalis Principia Mathematica"
published 1687

What is Classical Mechanics?

Later, two alternative formulations were developed.

$\boxed{\text{Lagrange}}$ (1736 - 1813) → Lagrangian Mechanics

$\boxed{\text{Hamilton}}$ (1805 - 1865) → Hamiltonian Mechanics

While we will show these alternative formulations are completely equivalent to Newtonian Mechanics, they often allow for dramatically more straightforward solutions to complex problems.

$$\underbrace{\text{Newtonian, Lagrangian, Hamiltonian}}_{\text{Classical Mechanics}}$$

We will **quickly** remind ourselves of the principles of Newtonian mechanics. The majority of the course will be spent mastering applications of Lagrangian mechanics, and then Hamiltonian mechanics (subject to course/time constraints)

Basic Concepts

Time Space Mass

Force Frames

The Universal Clock

In the (approximate + idealized) world of classical mechanics time is a single universal parameter that all observers agree on up to the answer to "when is $t=0$?"

If a group of observers synchronize their clocks they will forever agree "at what t" any event takes place

NOT TRUE, but a very very good approximation if $v \ll c$.

If all components of a mechanical system move at low velocities then time is universal for all practical purposes.

Basic Concepts

* Vector Basics
Review p 6-8 p33, 34 Taylor

For $\vec{r} = (r_1, r_2, r_3)$ $\vec{s} = (s_1, s_2, s_3)$

$$\vec{r} + \vec{s} = (r_1 + s_1, r_2 + s_2, r_3 + s_3)$$

$$c\vec{r} = (cr_1, cr_2, cr_3)$$

$$\vec{r} \cdot \vec{s} = r_1 s_1 + r_2 s_2 + r_3 s_3 = |\vec{r}||\vec{s}| \cos\theta_{rs}$$

$$|\vec{r}|^2 = \vec{r} \cdot \vec{r} = r_1^2 + r_2^2 + r_3^2$$

$$\vec{r} \times \vec{s} = \begin{Vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ r_1 & r_2 & r_3 \\ s_1 & s_2 & s_3 \end{Vmatrix} = (r_2 s_3 - r_3 s_2)\hat{e}_1 + (r_3 s_1 - r_1 s_3)\hat{e}_2 + (r_1 s_2 - r_2 s_1)\hat{e}_3$$

$$\frac{d}{dt}(\vec{r} + \vec{s}) = \dot{\vec{r}} + \dot{\vec{s}} \qquad \dot{} \equiv \frac{d}{dt}$$

$$\frac{d}{dt}(f\vec{r}) = \frac{df}{dt}\vec{r} + f\frac{d\vec{r}}{dt} = \dot{f}\vec{r} + f\dot{\vec{r}}$$

$$\frac{d}{dt}(\vec{r} \cdot \vec{s}) = \dot{\vec{r}} \cdot \vec{s} + \vec{r} \cdot \dot{\vec{s}}$$

$$\frac{d}{dt}(\vec{r} \times \vec{s}) = \dot{\vec{r}} \times \vec{s} + \vec{r} \times \dot{\vec{s}}$$

Example: $\vec{r} = r_1 \hat{e}_1 + r_2 \hat{e}_2 + r_3 \hat{e}_3$ in general coordinates

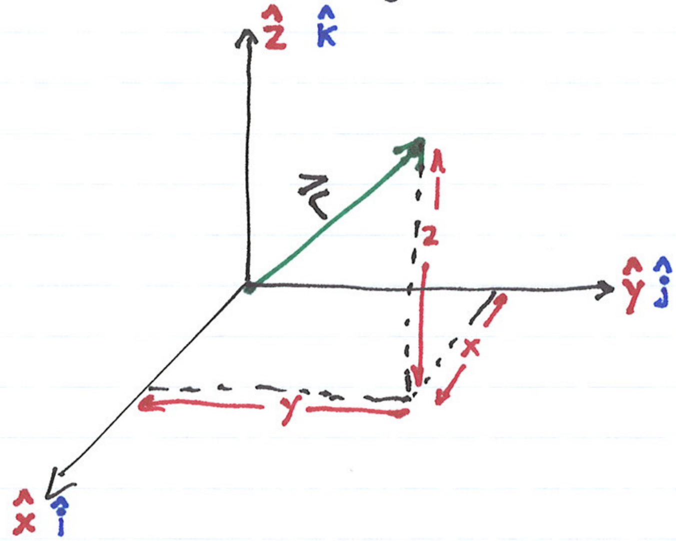
$$\dot{\vec{r}} = \vec{v} = r_1 \dot{\hat{e}}_1 + \dot{r}_1 \hat{e}_1 + r_2 \dot{\hat{e}}_2 + \dot{r}_2 \hat{e}_2 + r_3 \dot{\hat{e}}_3 + \dot{r}_3 \hat{e}_3$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

Basic Concepts

The Stage

Space is the stage where things take place.



Notation:

$$\vec{r} = \begin{cases} x\hat{x} + y\hat{y} + z\hat{z} = (x, y, z) \\ x\hat{i} + y\hat{j} + z\hat{k} \\ x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3 \\ r_1\hat{e}_1 + r_2\hat{e}_2 + r_3\hat{e}_3 = (r_1, r_2, r_3) \end{cases}$$

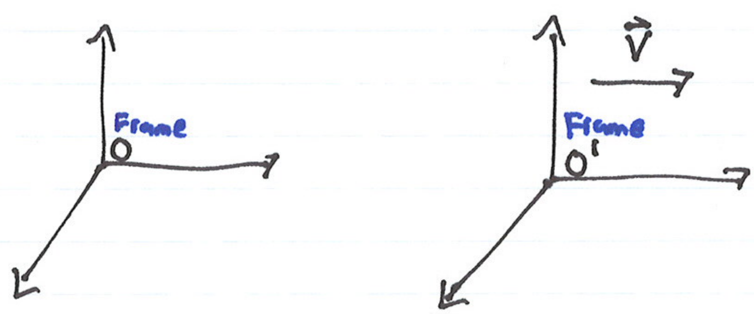
Be familiar with different conventions for cartesian coordinates.

Basic Concepts

Mass is the "inertia" or resistance to acceleration. Scalar quantaty.

Force is a vector quantity that characterizes a "push or pull" in simple terms.

Frame is a choice of spatial axes and their state of motion.



Newtonian Formalism: Ch 1-4

Types of Forces

Conservation of Momentum
Angular Momentum
Energy

Potential   Kinetic

Single Particle Systems

Many Particle Systems

Constraints + Coordinates

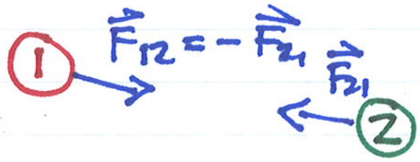
Newton's Laws of Motion

For a point particle (or each particle in an extended body)

① Newton's First Law: (Law of Inertia)
In the absence of forces a particle moves with a constant velocity $\vec{v}$

② Newton's Second Law: ($\vec{F} = m\vec{a} = m\dot{\vec{v}} = \dot{\vec{p}} = m\ddot{\vec{r}}$)
For any particle of mass m, the net force $\vec{F}$ on it is always equal to mass times the instantaneous accelleration.

③ Newton's Third Law: (Action Reaction; $\vec{F}_{21} = -\vec{F}_{12}$)
If object 1 exerts a force $\vec{F}_{21}$ on object 2 then object 2 always exerts a reaction force $\vec{F}_{12}$ on object 1 given by $\vec{F}_{21} = -\vec{F}_{12}$



An inertial frame is a frame where ① holds.
A rotating frame is NOT an inertial frame.
An accelerating frame is NOT an inertial frame.

Extended to each particle in a system of N particles these laws complet[e]ly characterize the motion of mechanical systems!!

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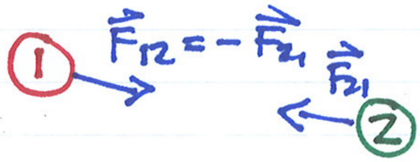

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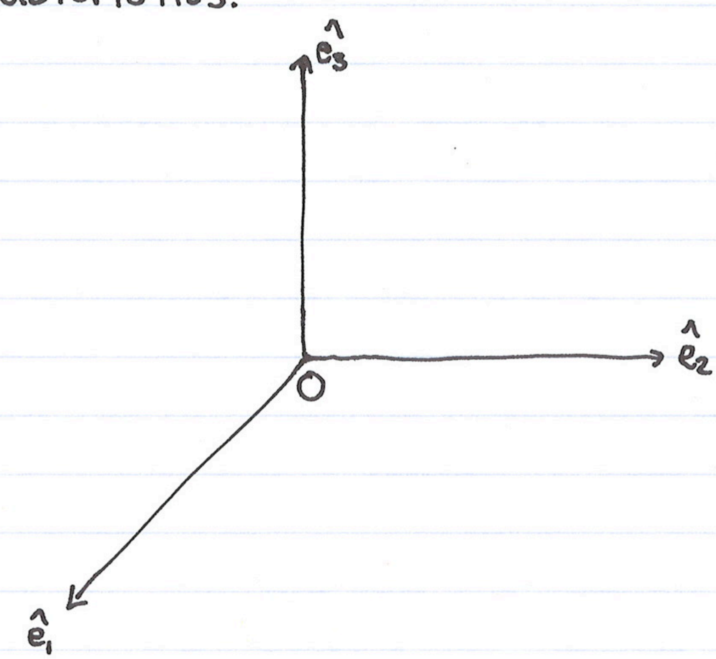
Frames

Choice of spatial origin $(\vec{r} = 0)$

Choice of temporal origin $(t = 0)$

Choice of spatial directions $\hat{e}_1, \hat{e}_2, \hat{e}_3$

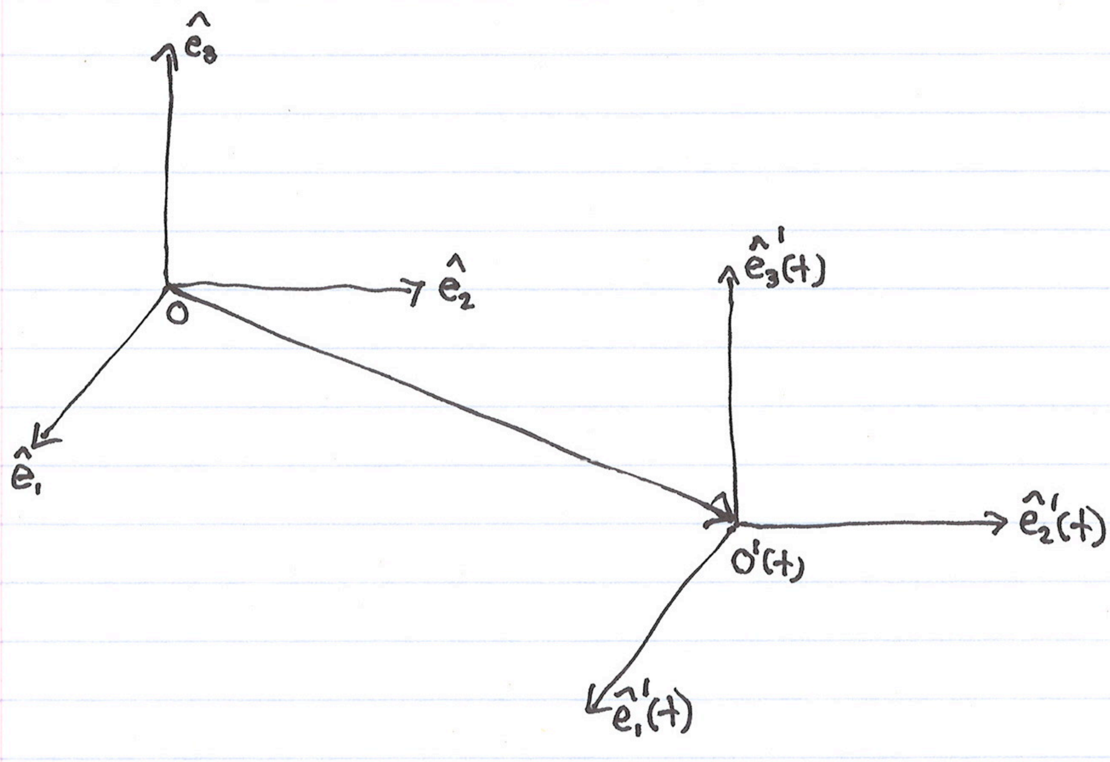
A frame S consists of specifying these characteristics.



Frames cont.

For instance: S could be a frame at rest with respect to the Earth's surface

For two frames S and S' we can specify their relative relationship by giving the coordinates of the origin and directions of one frame in terms of the other



Coordinates

Once we have chosen a frame to work in we still have the choice of the coordinate system we use.

Coordinate Independence:

"Physics doesn't care about coordinates"

Once we have specified an inertial frame, Newton's 2nd Law is valid no matter which coordinate system we choose to write it in.

The Point: $\vec{F} = m\vec{a} = m\ddot{\vec{r}}$

Vector equation true in all coordinate systems.

However, we can choose coordinates convenient to our problem!

Frames cont.

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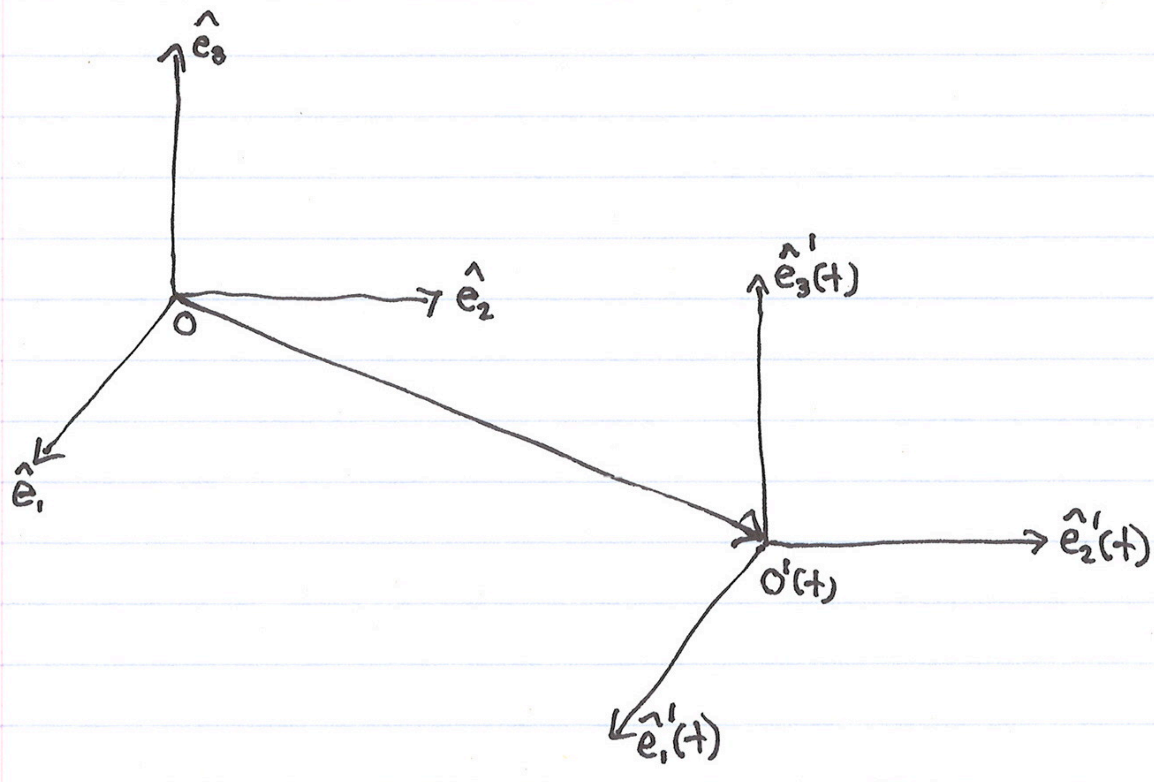

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Cartesian Coordinates

Position Vector $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

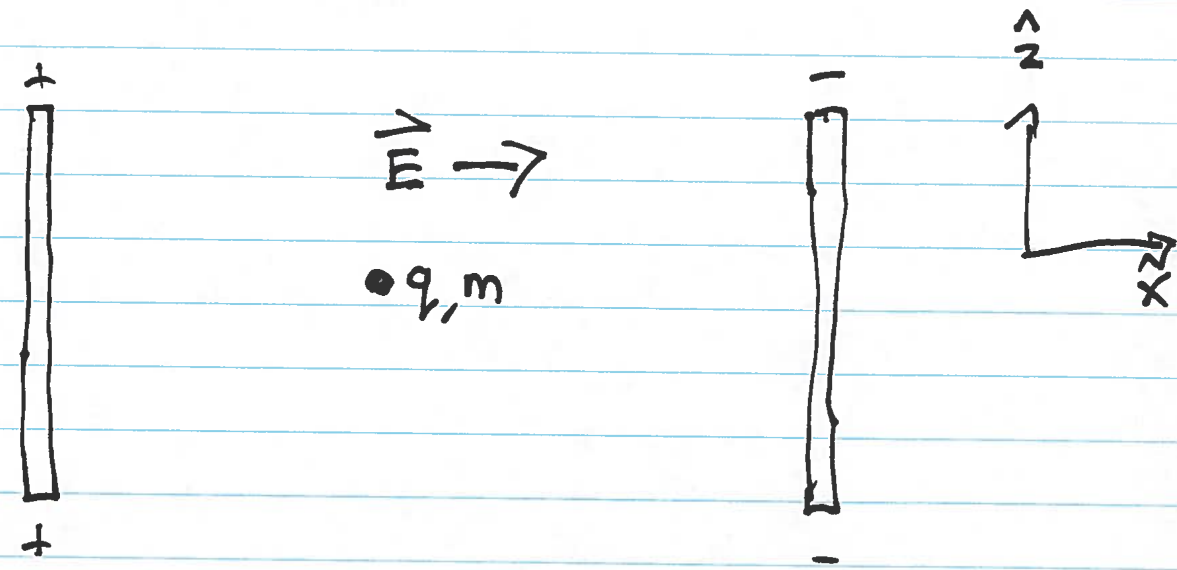
Acceleration Vector $\ddot{\vec{r}} = \ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z}$

Force Vector $\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$

$$\vec{F} = m\ddot{\vec{r}} \quad \Longleftrightarrow \quad \begin{aligned} F_x &= m\ddot{x} \\ F_y &= m\ddot{y} \\ F_z &= m\ddot{z} \end{aligned}$$

In cartesian coordinates the three-dimensional vector equation breaks up into three one-dimensional equations which are identical (up to what we decide to write fore "x", "y", or "z")

Example: Charge between two conducting plates on Earth's surface.



Electric Field $\quad \vec{E} = E\hat{x} \quad \vec{F}_E = q\vec{E}$

Gravitational Field $\quad \vec{g} = g(-\hat{z}) \quad \vec{F}_g = m\vec{g}$

$$\vec{F} = m\ddot{\vec{r}}$$

$$\vec{r} = x(t)\hat{x} + z(t)\hat{z}$$

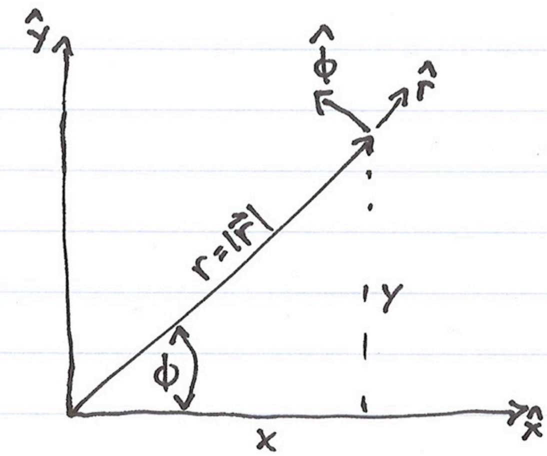
z $\qquad\qquad$ x

$$-mg = m\ddot{z} \qquad\qquad qE = m\ddot{x}$$

$$z = z_0 + v_{z0}t - \frac{1}{2}gt^2$$
$$x = x_0 + v_{x0}t + \frac{1}{2}\frac{qE}{m}t^2$$

Plane Polar Coordinates

constrain $z=0$ for time being

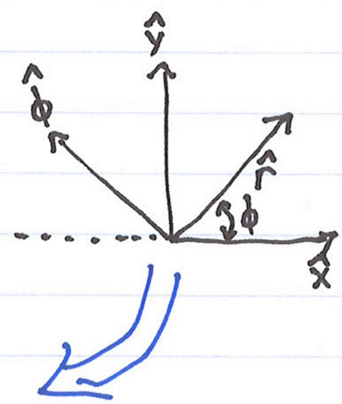


$x = r\cos\phi$ $\qquad$ $r = \sqrt{x^2+y^2}$

$y = r\sin\phi$ $\qquad$ $\phi = \arctan(y/x)$

Position Vector: $\vec{r} = r\hat{r}$

$$\Rightarrow \quad \dot{\vec{r}} = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$



What is $\hat{r}$? What is $\hat{\phi}$?

$$\hat{r} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Plane Polar Coordinates cont.

$$\hat{r} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$

$$\frac{d\hat{r}}{dt} = -\sin\phi\,\dot{\phi}\hat{x} + \cos\phi\,\dot{\phi}\hat{y} = \dot{\phi}\,\hat{\phi}$$

similarly $\displaystyle \frac{d\hat{\phi}}{dt} = -\cos\phi\dot{\phi}\,\hat{x} - \sin\phi\,\dot{\phi}\hat{y} = -\dot{\phi}\,\hat{r}$

so $\longrightarrow$ $\displaystyle \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\phi}\,\hat{\phi}$

$$\downarrow \qquad\qquad \downarrow$$

$$V_r = \dot{r} \qquad V_\phi = r\dot{\phi}$$

Similarly

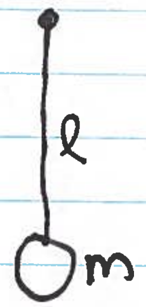
Acceleration Vector $\ddot{\vec{r}} = (\ddot{r} - r\dot{\phi}^2)\hat{r} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\,\hat{\phi}$

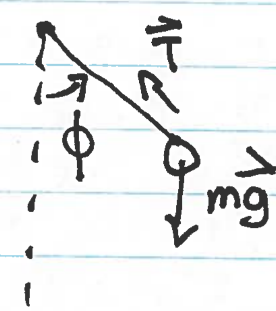
Force Vector $\vec{F} = F_r\,\hat{r} + F_\phi\,\hat{\phi}$

$$\vec{F} = m\ddot{\vec{r}} \quad \Longleftrightarrow \quad \begin{aligned} F_r &= m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi &= m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \end{aligned}$$

Simplifies for $r = \text{const}$ or $\phi = \text{const}$

Example: Ball on a String
(a.k.a. The Simple Pendulum)





$$F_r = -T + mg\cos\phi$$

$$F_\phi = -mg\sin\phi$$

radial $\quad r = \ell \quad \dot{r} = 0 \quad \ddot{r} = 0$

$$F_r = m(\cancel{\ddot{r}}^{\,0} - r\dot{\phi}^2) = -m\ell\dot{\phi}^2$$

$$v_\phi = \ell\dot{\phi}$$

$$F_r = -m\frac{v_\phi^2}{\ell}$$

ϕ equations $\quad F_\phi = m(r\ddot{\phi} + 2\cancel{\dot{r}}^{\,0}\dot{\phi})$

$$-mg\sin\phi = m\ell\ddot{\phi}$$

$$\ddot{\phi} = -\frac{g}{l}\sin\phi$$

$$\frac{d^2\phi}{dt^2} = -\frac{g}{l}\sin\phi$$

$$\phi \ll \frac{\pi}{2} \qquad \sin\phi \simeq \phi$$

$$\ddot{\phi} = -\frac{g}{l}\phi \qquad \omega^2 = \frac{g}{l}$$

$$\ddot{\phi} = -\omega^2\phi$$

$$\phi(t) = \begin{cases} e^{\pm i\omega t} \\ \cos(\omega t) \\ \sin(\omega t) \end{cases}$$

$$\phi(t) = A\cos(\omega t) + B\sin(\omega t) \qquad A = \phi_0$$

$$\omega B = \frac{V_{\phi_0}}{l}$$

$$\phi(0) = \phi_0 \qquad l\dot{\phi}(0) = V_{\phi_0}$$

$$\phi(t) = \phi_0\cos(\omega t) + \frac{V_{\phi_0}}{\omega l}\sin(\omega t)$$