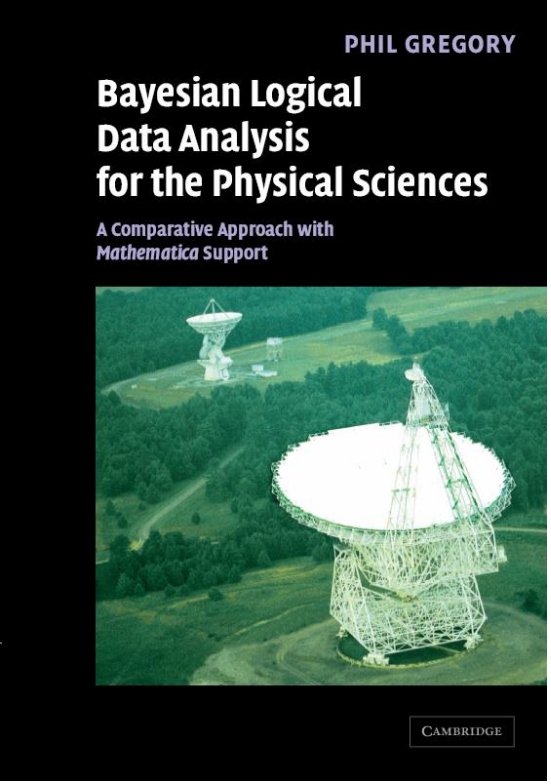


Bayesian Planet Searches for the 10 cm/s Radial Velocity Era

Phil Gregory
University of British Columbia
Vancouver, Canada

Aug. 4, 2015

IAU Honolulu
Focus Meeting 8
On Statistics and Exoplanets

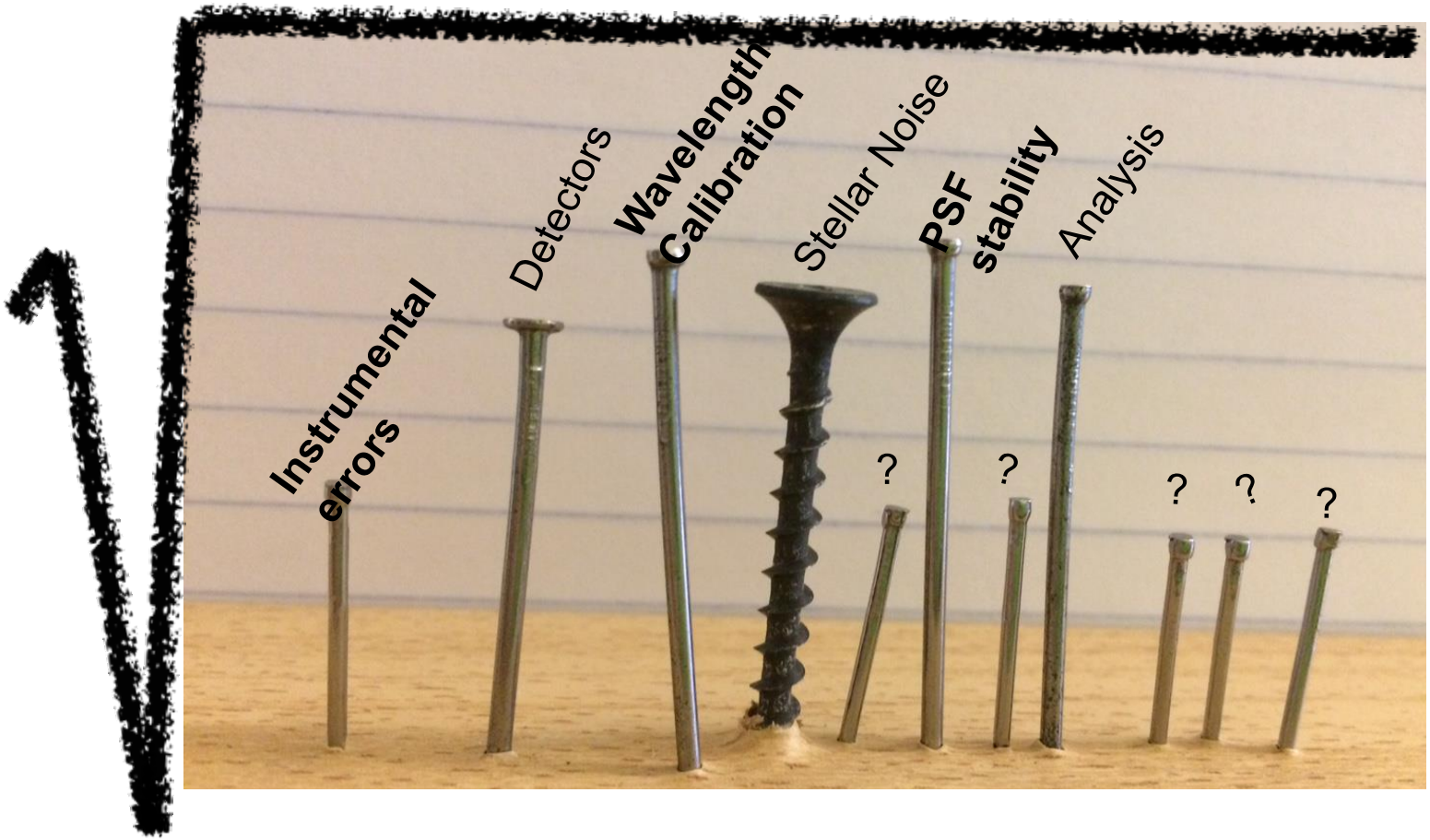
Bayesian planet searches for the 10 cm/s radial velocity era

Intrinsic stellar variability has become the main limiting factor for planet searches in both transit and radial velocity (RV) data. New spectrographs are under development like ESPRESSO and EXPRES that aim to improve RV precision by a factor of approximately 10 over the current best spectrographs, HARPS and HARPS-N. This will greatly exacerbate the challenge of distinguishing planetary signals from stellar activity induced RV signals.

At the same time good progress has been made in simulating stellar activity signals. At the Porto 2014 meeting, "Towards Other Earths II," Xavier Dumusque challenged the community to a large scale blind test using the simulated RV data at the 1 m/s level of precision, to understand the limitations of present solutions to deal with stellar signals and to select the best approach.

My talk will focus on some of the statistical lesson learned from this challenge with an emphasis on Bayesian methodology.

This is how Debra Fischer portrayed the problem at the recent "Extreme Precision Radial Velocity" meeting at Yale (2015)



We have worked hard over the past 2 decades to improve RV precision. Now seem to be at a point where the largest terms in the error budget are similar magnitude. As we push down, we may encounter new surprises.

Need to use the right tool

Debra Fischer

If we eliminate all other error sources except stellar noise, we won't see significant precision gains. We'll be… well… screwed.

Debra Fischer

A key challenge for statistical analysis is to separate planetary signals from stellar activity induced signals.

Debra Fischer

Stellar activity

Time Scale	Vel. noise	Type of activity	Partial solutions
~ 10 years	1 – 20 m/s	Magnetic cycle	correlation
10 – 50 d	few m/s	Active regions spots and plages	a) correlation b) FF’ analysis + Gaussian process
15 min – 2 d	few m/s	Granulations	ave. 3x10 min/night reduce to ~ 0.5 m/s
~ 1 hr	< 1 m/s	Flares	
< 15 min	few m/s	Oscillations	ave. for 15 min reduce to ~ 0.2 m/s

The Kepler Fitting Challenge

(https://rv-challenge.wikispaces.com)
(Google rv challenge wikispace)

Idea

Porto
September 14

Simulation

Stellar signals
Planets

Challenge

Can we find
the planets ?

Xavier CfA Dumusque

The Dataset

(https://rv-challenge.wikispaces.com)
(Google rv challenge wikispace)

DATA

15 data sets

OBSERVABLES

RV, BIS SPAN, FWHM, Log(R'hk)

Analysis

- 24 groups where interested (~55 persons)
- Results from 8 groups
  - 5 groups analyzed the 15 RV curves
  - 2 groups only looked at the first 5 systems
  - 1 group analyzed only 2 systems

Planet detection

Bona-fide planet

Probable planet and correct

Probable planet but mistake

Bona-fide planet but wrong K or P

Bona-fide but mistake

Probable planet and wrong K or P

Planet not detected with K > 1 m/s

Xavier Dumusque CfA

Podium

3rd 

Phil Gregory

2nd 

INAF Torino
Mario Damasso, Aldo Bonomo, Paolo Giacobbe, Raphaëlle Haywood, Matteo Pinamonti, Alessandro Sozzetti

1st 

Mikko Tuomi &
Guillem Anglada Escude

Take Away Message

Best Techniques

Moving Average

Gaussian Process

Apodized Keplerian

Red Noise models

Bayesian Framework

Take Away Message

Simulated DATA

Are not bad, similarity with real datasets

K > 1 m/s

90% of signal recovered

K < 1 m/s

10% of signal recovered

Developed a new approach for the RV challenge based on Apodized Keplerian Models

The Apodized Kepler (AK) model approach

Phil Gregory (July 2015)
University of British Columbia

The Kepler radial velocity parameter K is multiplied by an apodization term of the form

$$\exp\left[-\frac{(t_i - t_a)^2}{2\tau^2}\right]$$

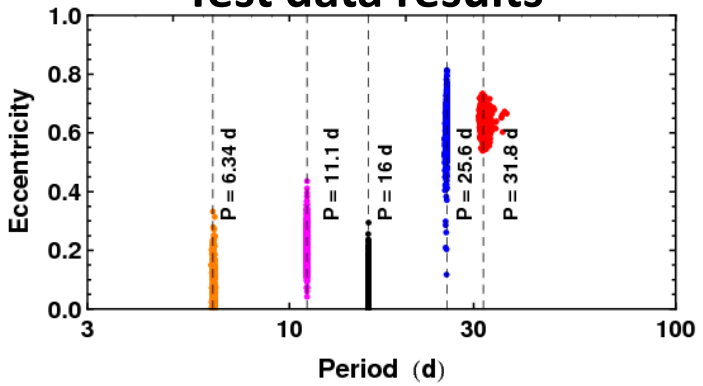
Since a true planetary signal spans the duration of the data the apodization time, τ, will be large while a stellar activity induced signal will generally have a small τ value.

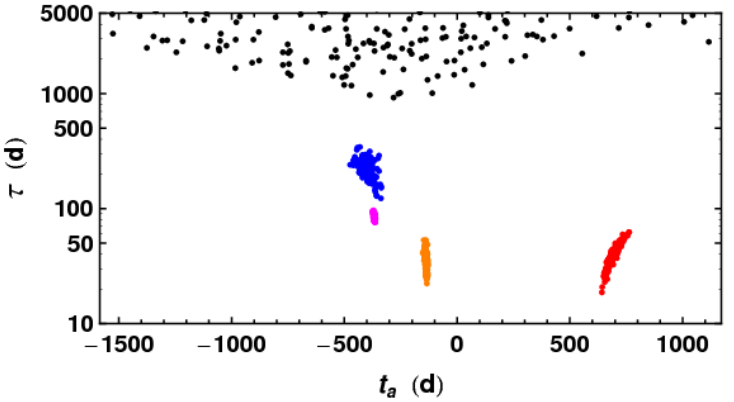
Each model also included a correlation term between RV and the stellar activity diagnostic log(*R'hk*) and an extra Gaussian noise term.

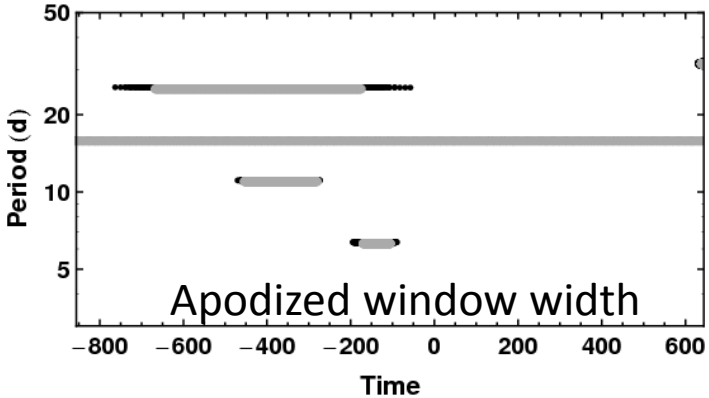
The model parameters were explored using my fusion MCMC code and a differential version of the Generalized Lomb-Scargle algorithm.

The figure shows plots of MCMC parameter estimates for a 5 signal model fit to the test data, known to have one planet with a period of 16 d.

Test data results







Radial velocity model for *m* signals (planets + stellar activity) plus ln(R'hk) linear regression term

$$v(t_i) = V + \sum_{j=1}^{m} \left[K_j \exp[-\frac{(t_i - t_{ja})^2}{2\tau_j^2}] \times (\cos\{\theta_j(t_i + \chi_j P_j) + \omega_j\} + e_j \cos\omega_j) \right] + \beta \times \ln[R'hk](t_i)$$

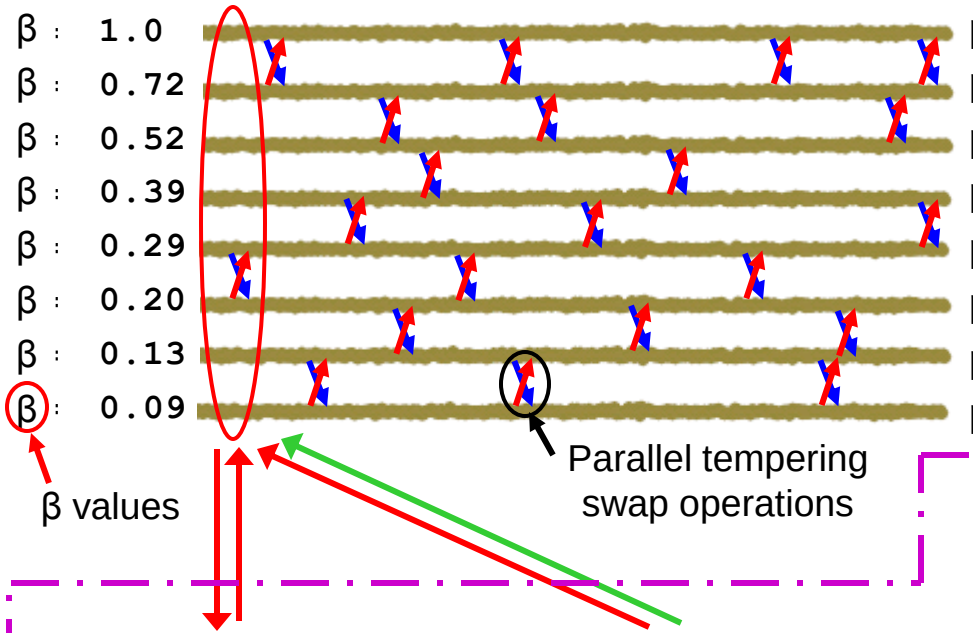
m **= the number of apodized Kepler (AK) signals in model.**
Linear regression term β is just another fit parameter in the MCMC.

The AK models were explored using an automated fusion MCMC algorithm (FMCMC), a general purpose tool for nonlinear model fitting and regression analysis (Gregory 2013). The AK models combined with the FMCMC algorithm constitute a multi-signal AK periodogram.

Current analysis assumes multiple independent Keplerian orbits which breaks down for near resonant orbits.

Fusion MCMC with Automatic proposal scheme

8 parallel tempering Metropolis chains



Output at each iteration

parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike
parameters,	logprior +	loglike,	logprior + loglike

'I' proposals
Independent Gaussian proposal scheme employed 50% of the time

'C' proposals
Proposal distribution with built in param. correlations used 50% of the time

Peak parameter set:
If (logprior + loglike) > previous best by a threshold then update and reset burn-in

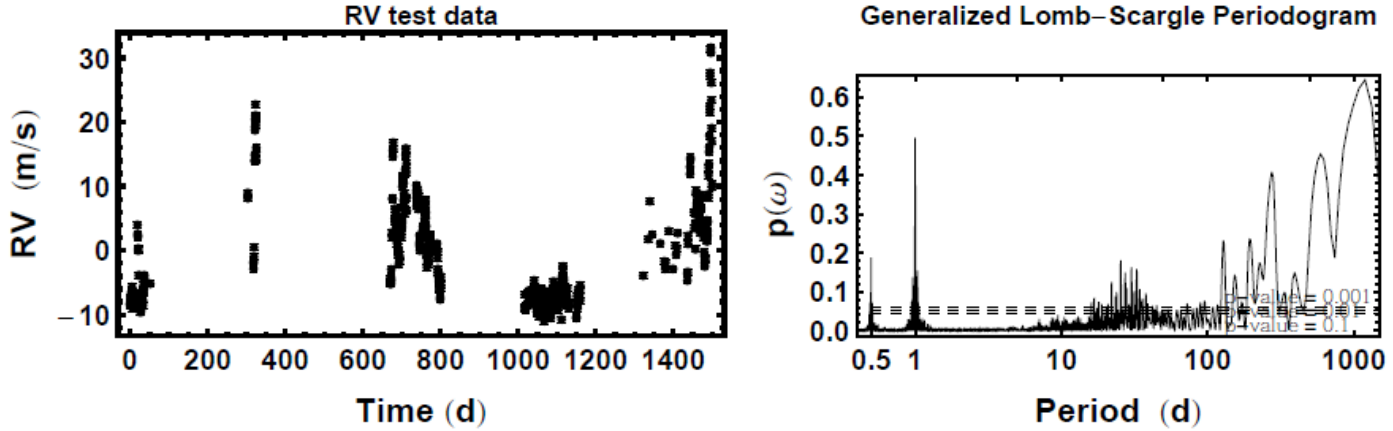
Monitor for parameters with peak probability

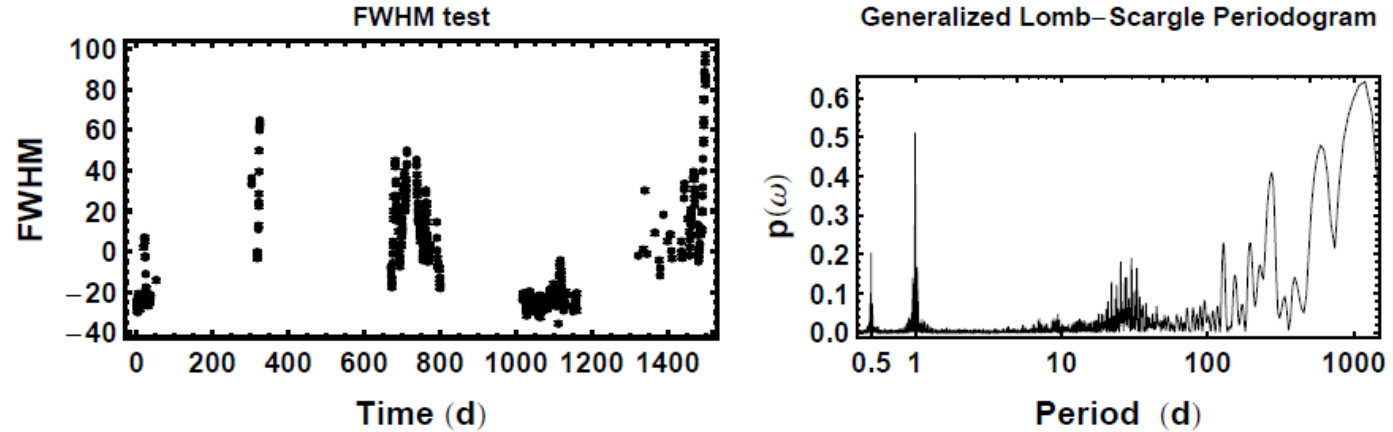
Genetic algorithm

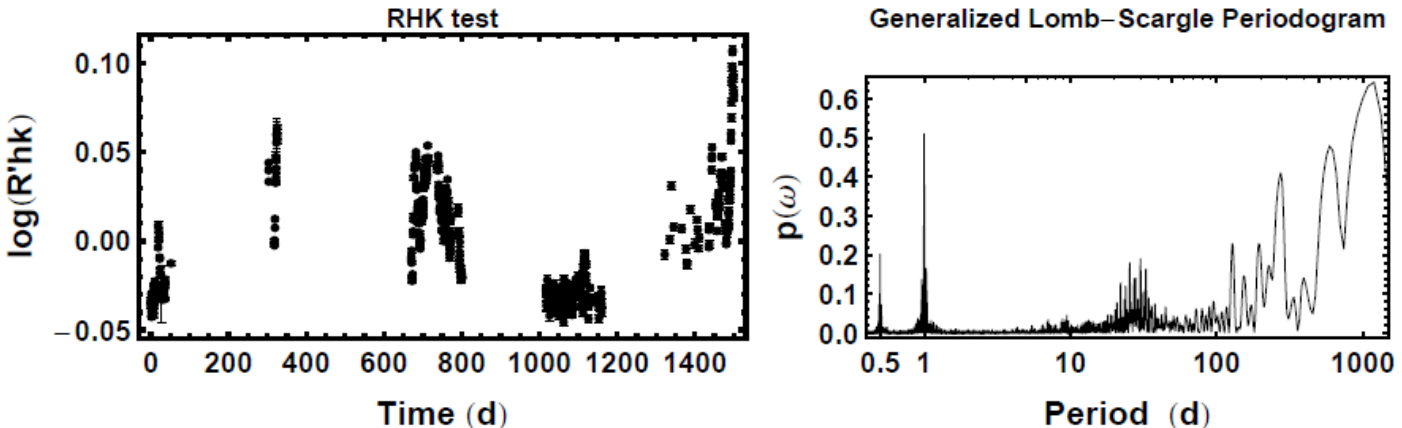
Every 40th iteration perform gene swapping operation to breed a more probable parameter set.

MCMC adaptive control system

Raw RV and the FWHM and ln(R'hk) diagnostics for Test data set





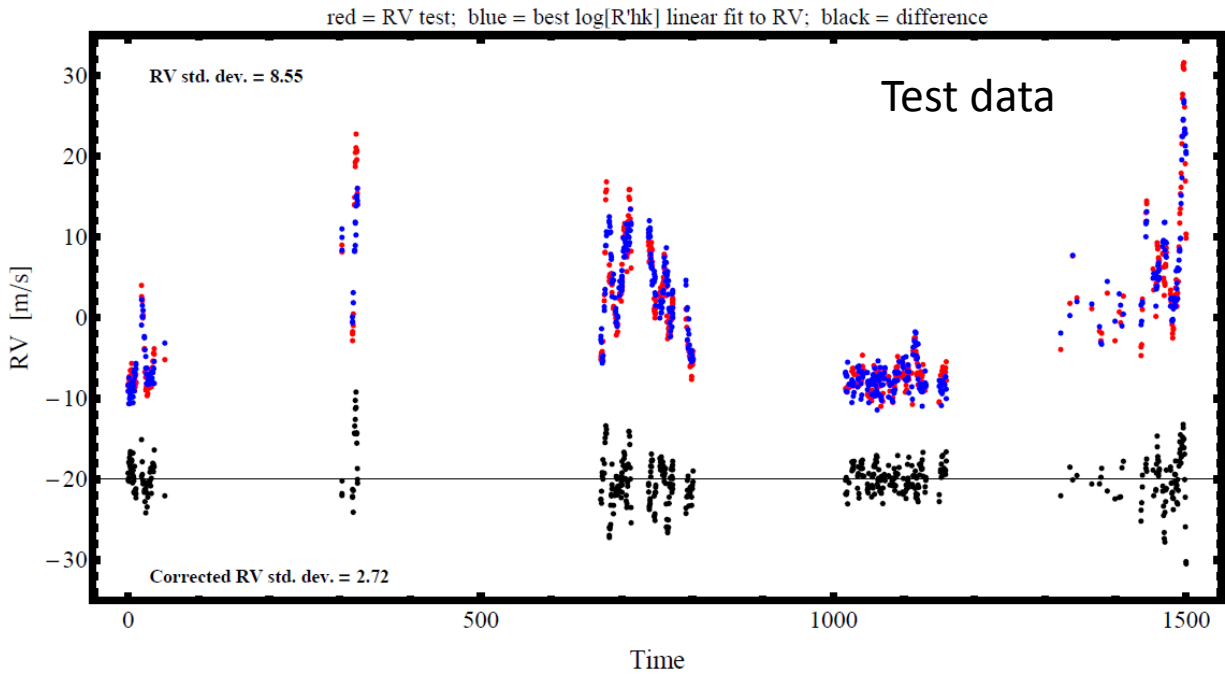


Top panel

Red points shows the raw RV test data,

Blue points show the best log(R'hk) linear regression fit to the RV data, and

Black points = the difference. (call this RV (rhk corrected))

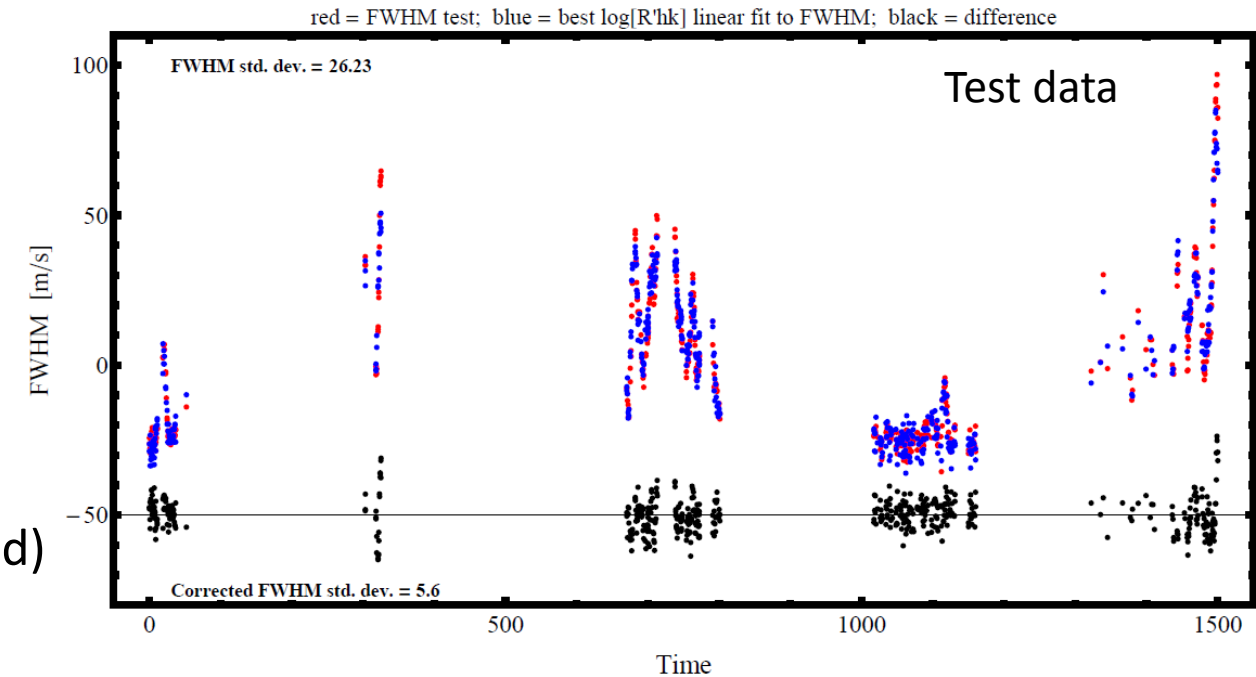


Bottom panel

Red points shows the raw FWHM test data,

Blue points show the best log(R'hk) linear regression fit to the FWHM data, and

Black points = the difference. (call this FWHM (rhk corrected) which is used as a control.)

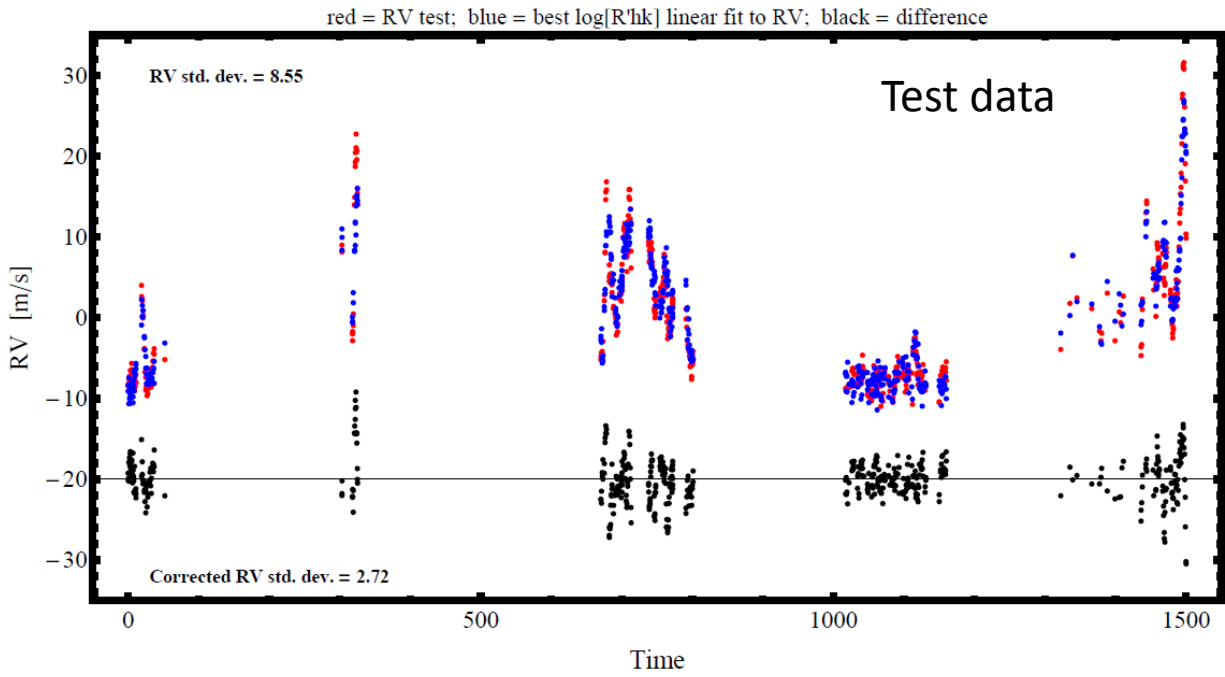


Top panel

Red points shows the raw RV test data,

Blue points show the best log(R'hk) linear fit to the RV data, and

Black points = the difference. (call this RV (rhk corrected))

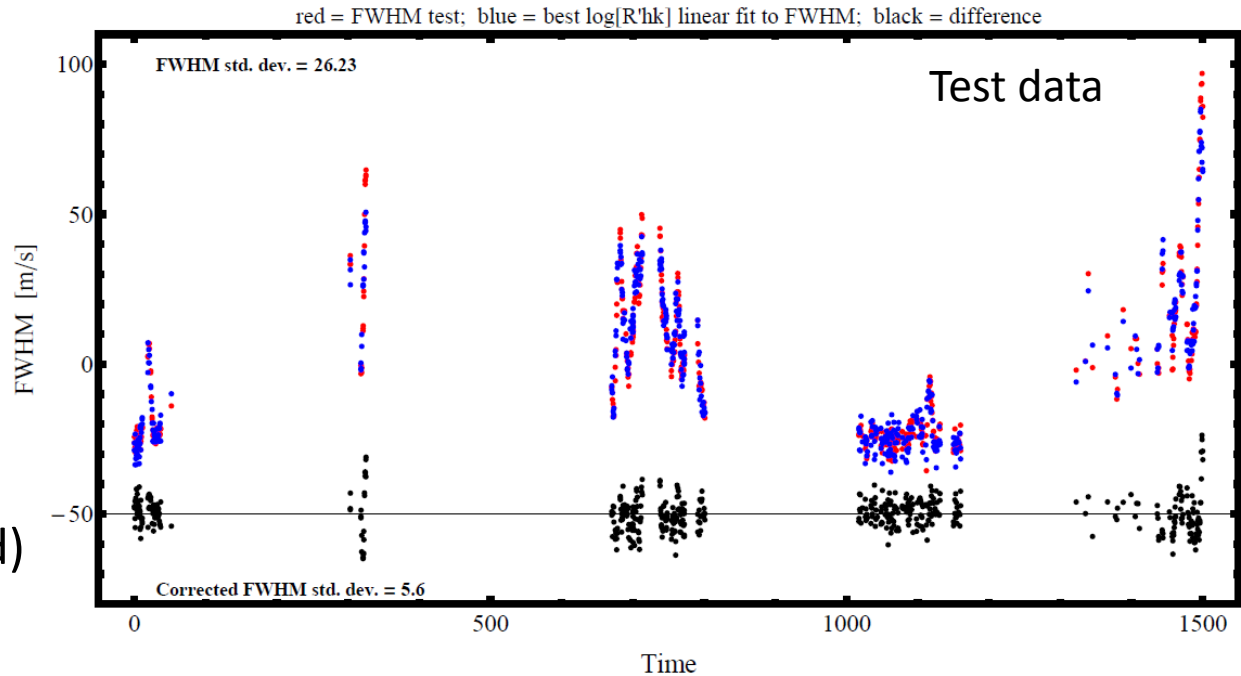


Bottom panel

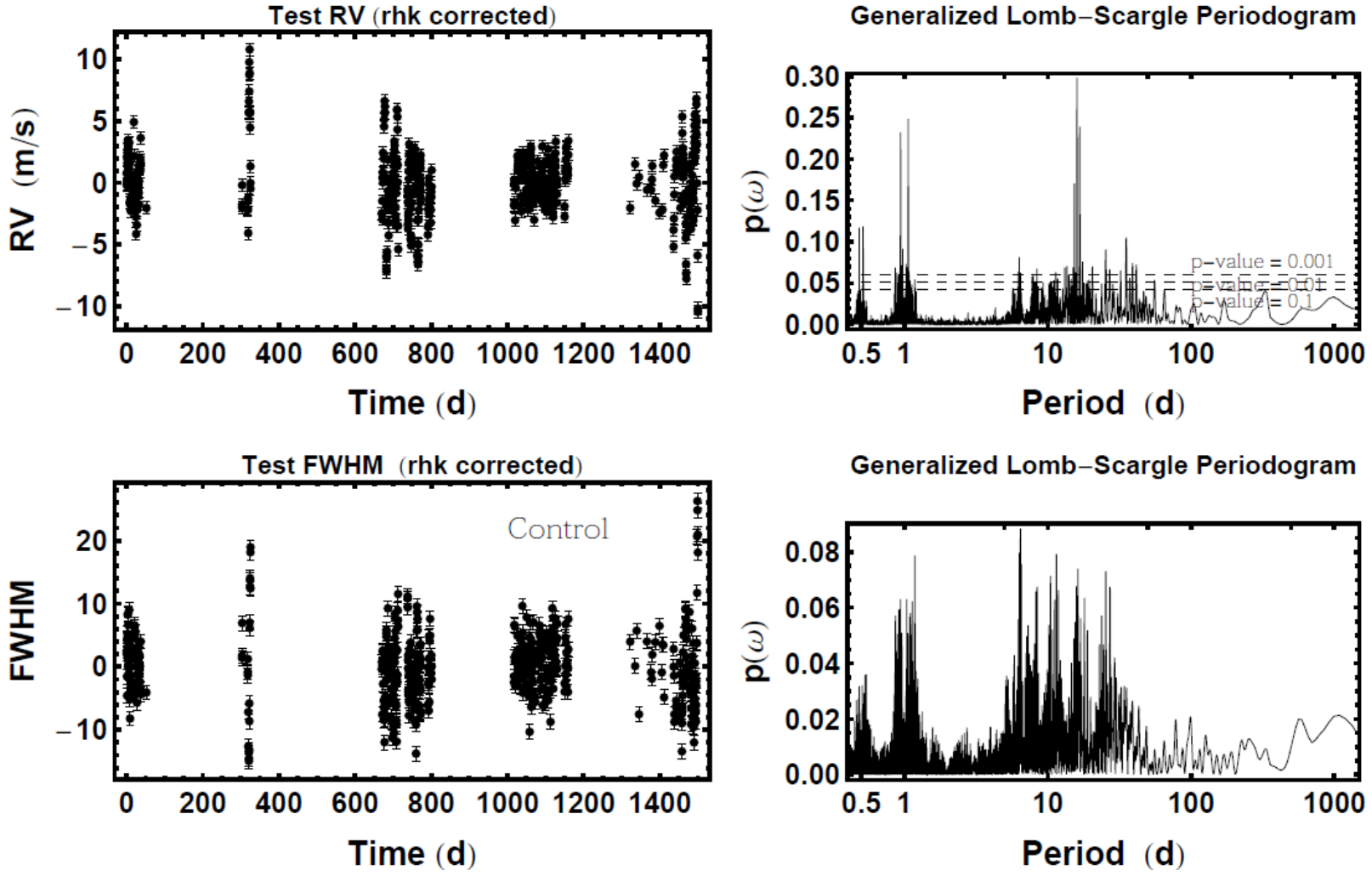
Red points shows the raw FWHM test data,

Blue points show the best log(R'hk) linear fit to the FWHM data, and

Black points = the difference. (Call this FWHM (rhk corrected) which is used as a control.)



Generalized Lomb-Scargle (GLS) periodogram of RV and FWHM (both rhk corrected).



The GLS periodogram measures the relative χ^2-reduction, $p(\omega)$, as a function of frequency ω and is normalised to unity by χ^2_0 (the χ^2 for the weighted mean of the data).

New: a Bayesian version of GLS now available (Mortier et al., arXiv:1412.0467.pdf)

GLS Spectral difference

of significant spectral regions

Black = RV (rhk corr.)

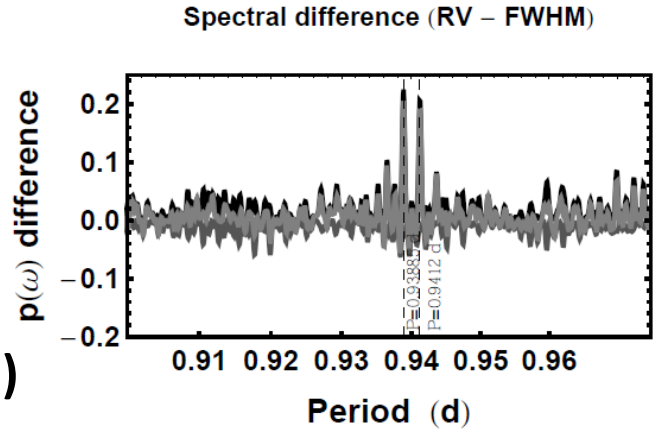
Gray = - FWHM (rhk corr.)

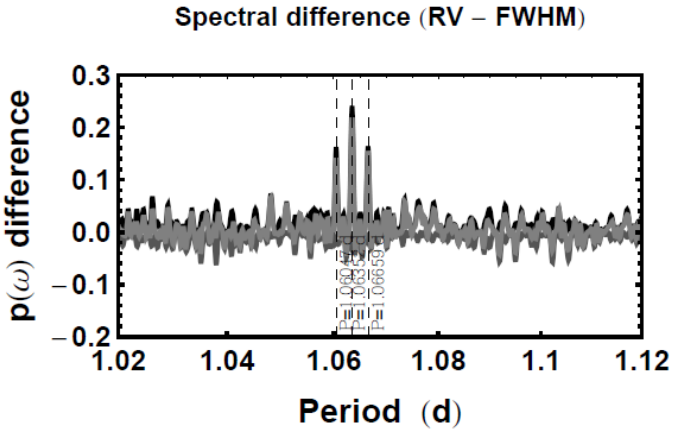
Light Gray = Black + Gray

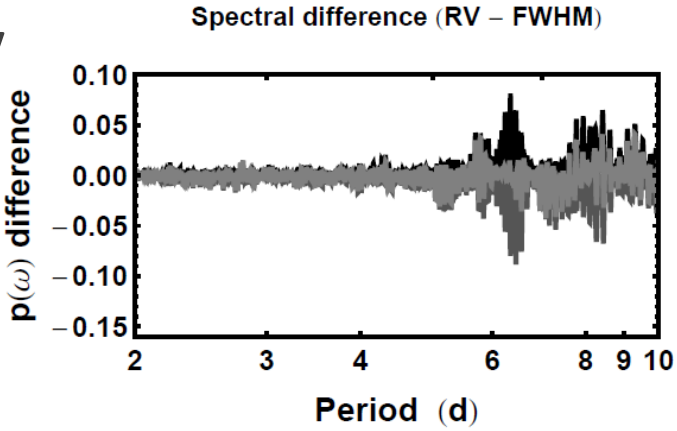
Signals in common to both indicate stellar activity. Gray trace acts as a control.

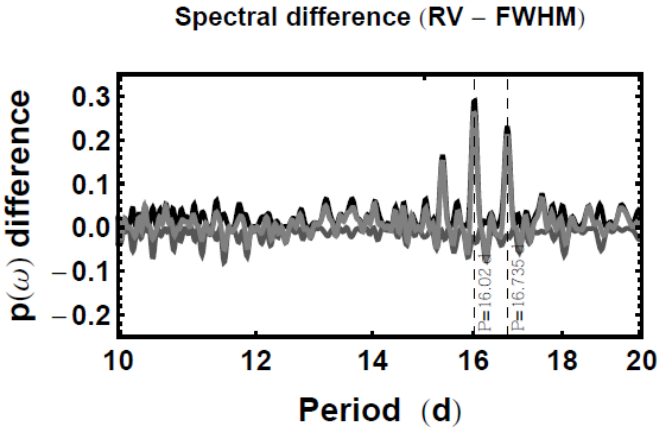
Dominant 16 d signal clearly visible. The next big peak on either side is a 1 yr alias.

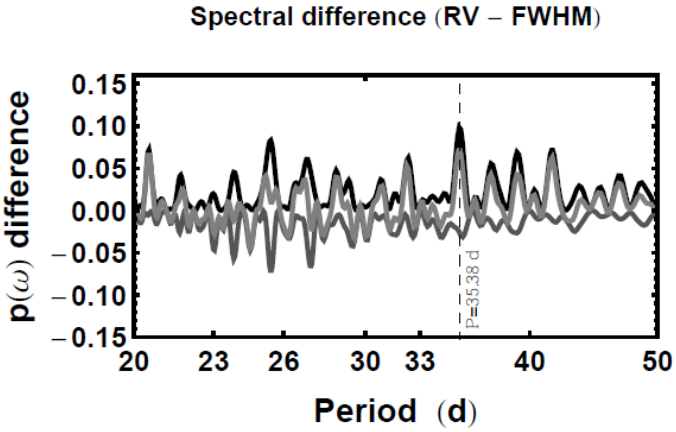
Solar and sidereal day aliases seen near P = 0.94 & 1.06 d.

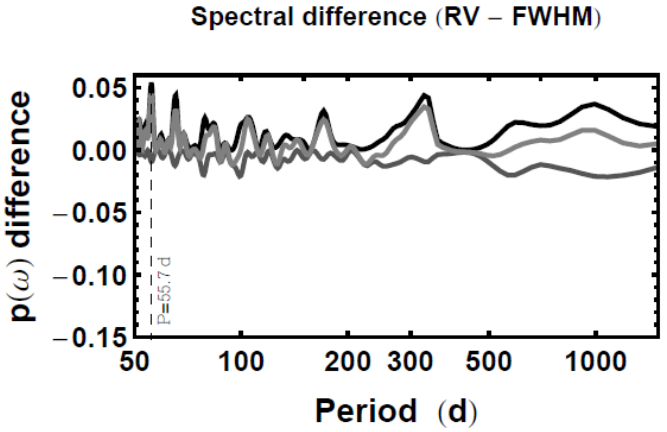








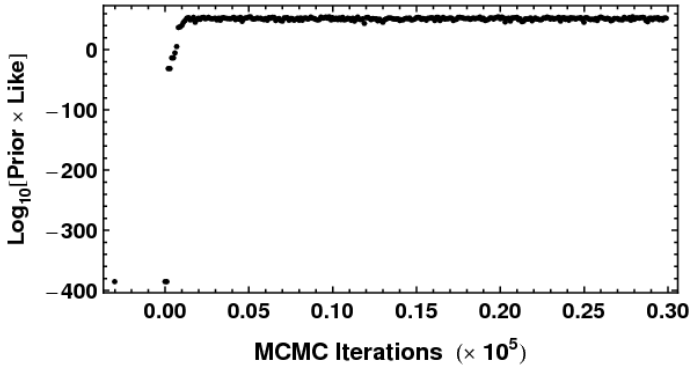


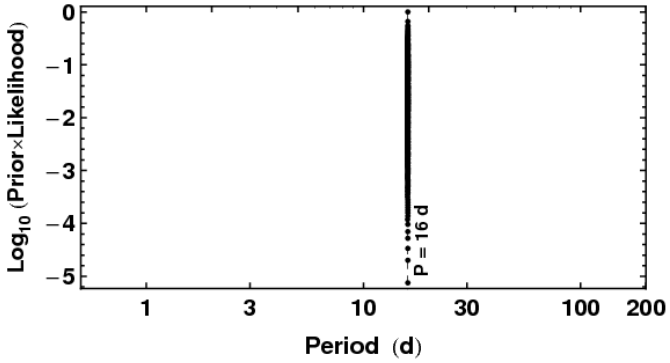


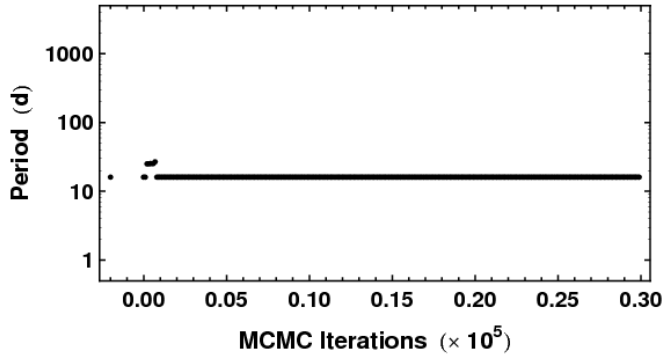
Model: 1 apodized Kepler signal + log(R'hk) regression fit (Test data)

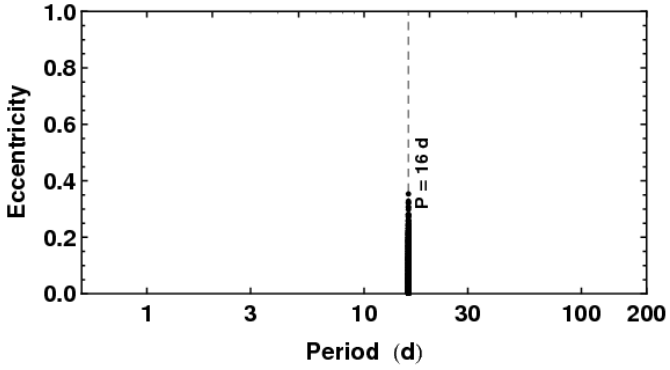
The model parameters explored using fusion MCMC.

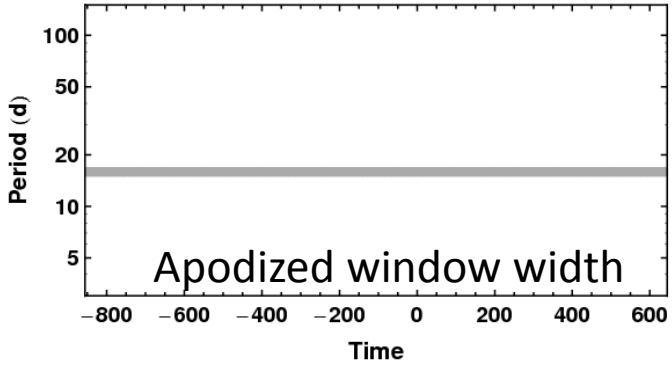
The figure shows Various plots of the MCMC parameter estimates.

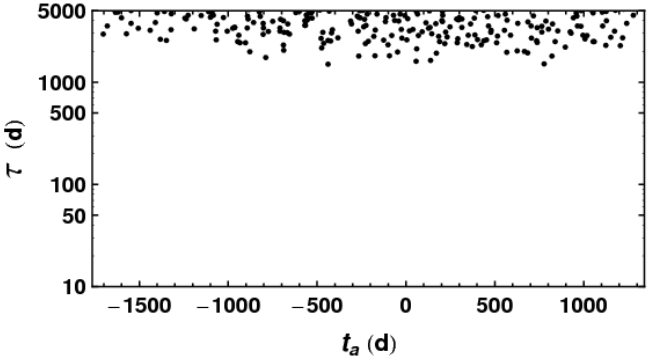












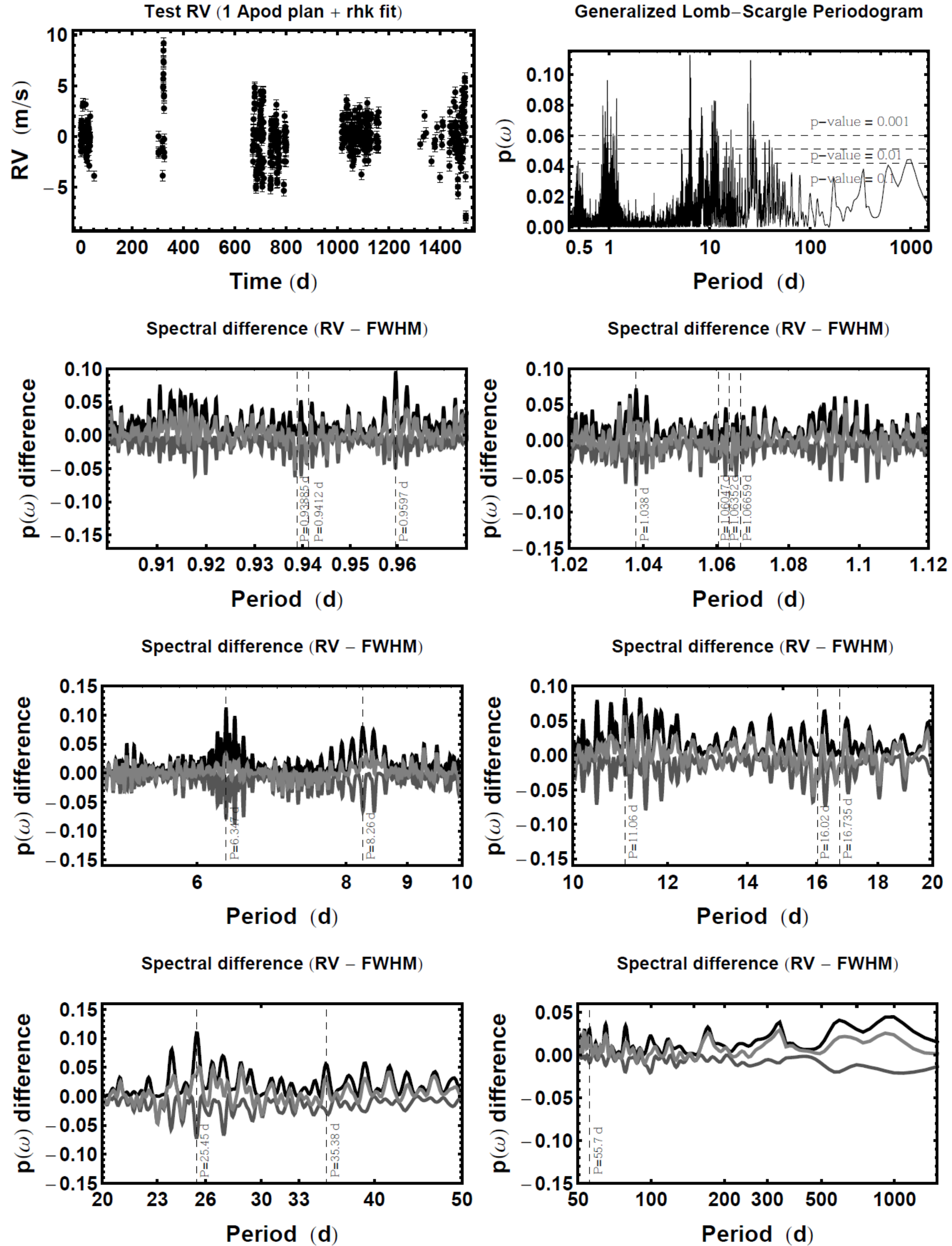
Lower left panel: apodization interval for each signal shown by gray trace for MAP values of τ and t_a. Lower right panel: apodization time constant, τ, versus t_a for the 16 d signal.

GLS & Spectral difference
of residuals from
1 apodized Kepler + rhk fit

Dominant 16 d signal and aliases have been removed including those near P = 0.94 & 1.06 d.

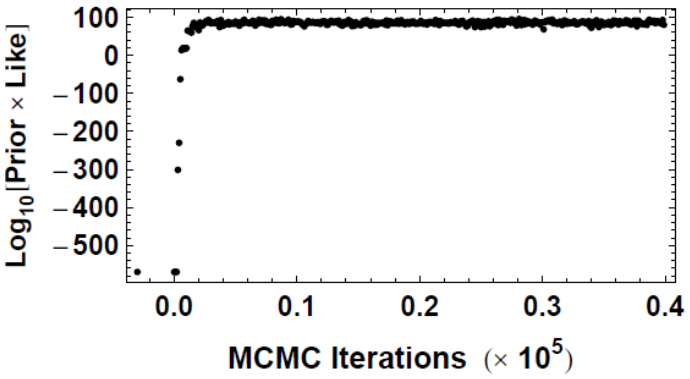
Largest GLS residual peak at P = 6.3 d has p-value << 0.001

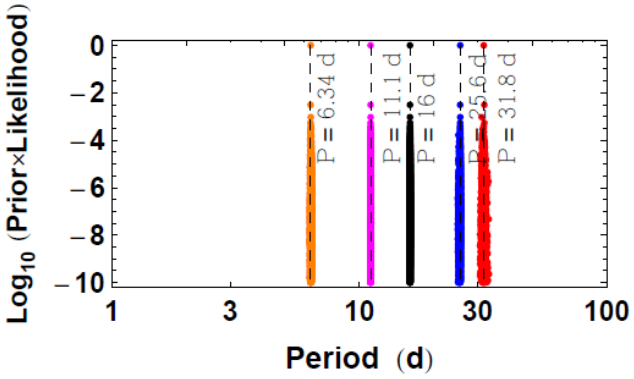
Note: the FWHM control indicates 6.3 d is stellar activity

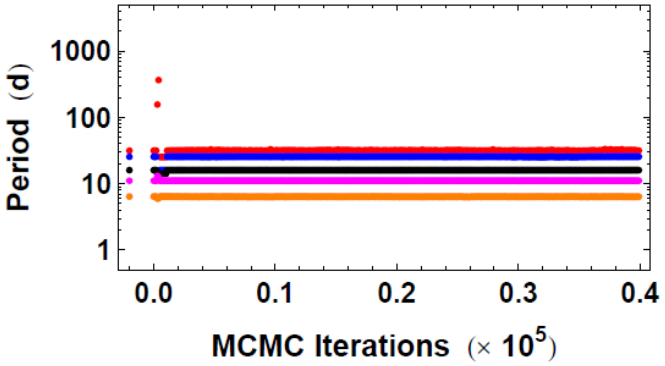


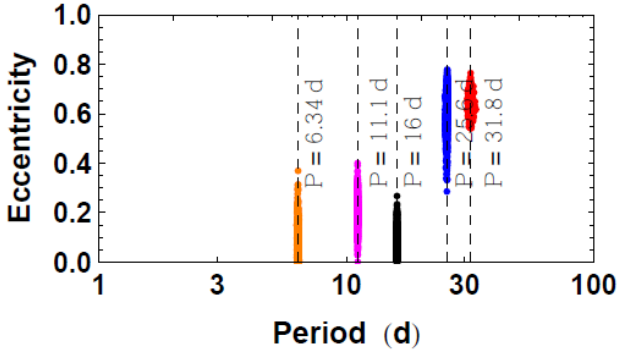
Model: 5 apodized Kepler signals + log(R'hk) regression fit (Test data)

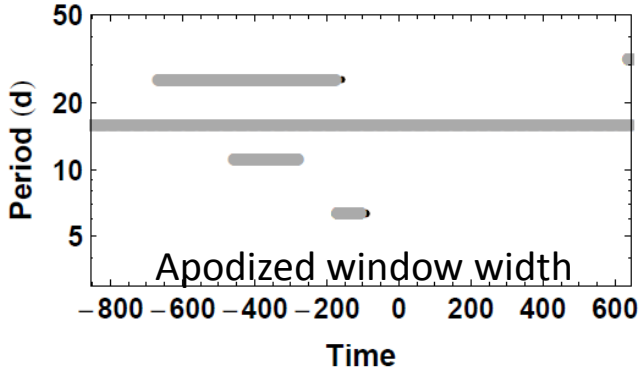
Only the 16 d signal has an apodization time constant τ (d) consistent with a planet.

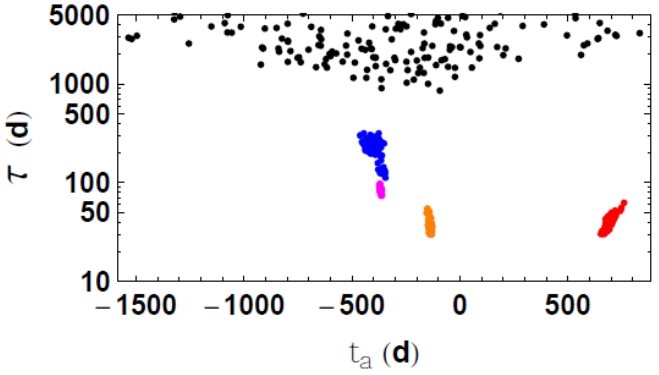










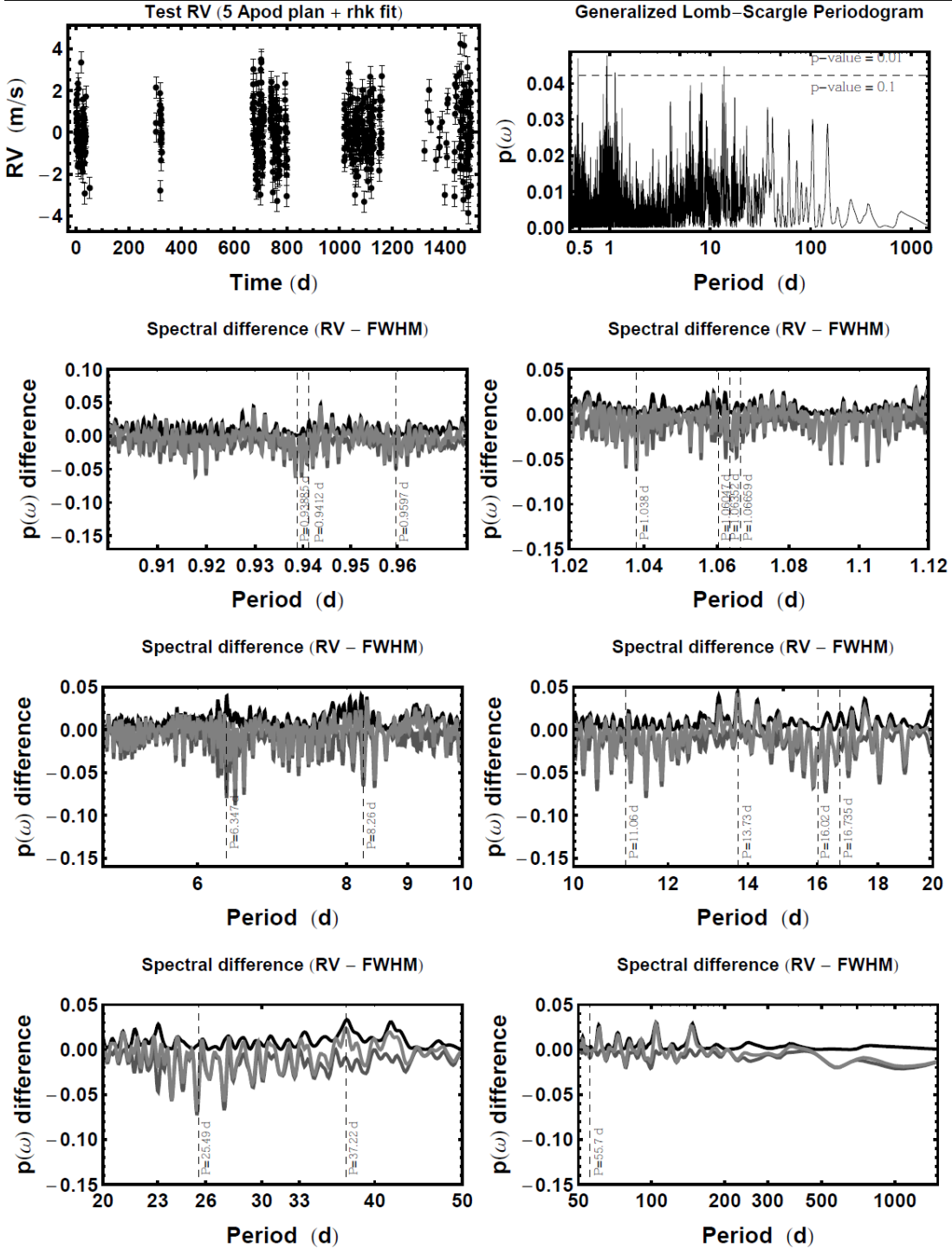


Free *Mathematica* fusion MCMC code for simple 2 planet Kepler model and program details available under resources at: http://www.cambridge.org/pl/academic/subjects/statistics-probability/statistics-physical-sciences-and-engineering/bayesian-logical-data-analysis-physical-sciences-comparative-approach-mathematica-support

GLS & Spectral difference

of residuals from
5 apodized Kepler + rhk fit

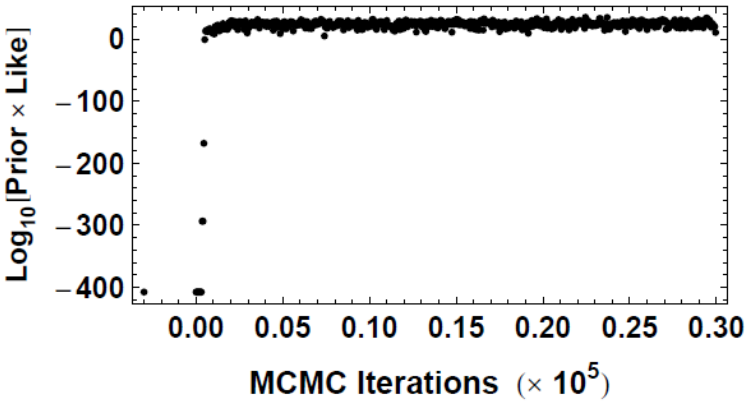
Largest GLS residual peak has p-value between 0.1 & 0.01

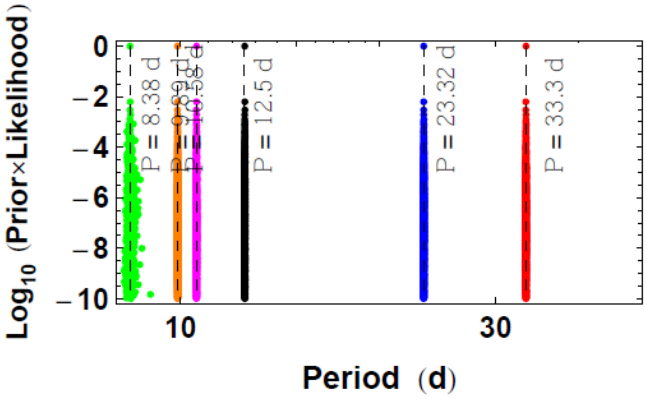


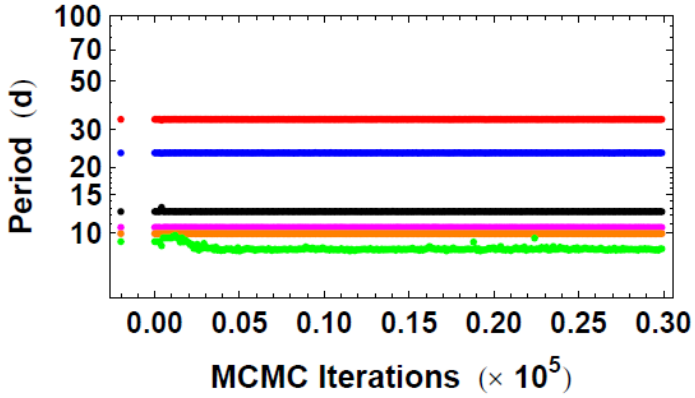
RV 1 Results

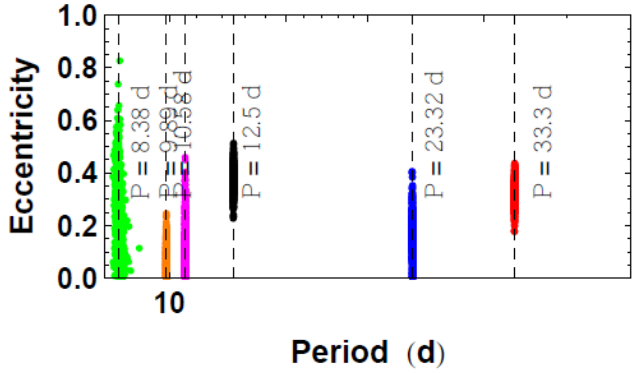
RV 1 Model: 6 apodized Kepler signals + log(R'hk) regression fit

Results indicate
3 planets with
P= 9.89, 23.4, 33.3d
+
3 stellar activity (SA) signals



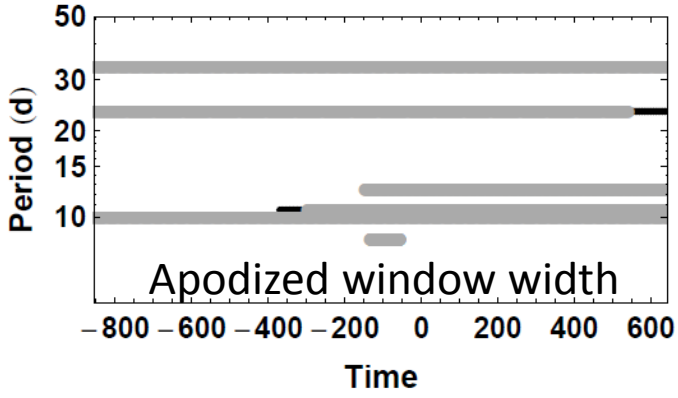


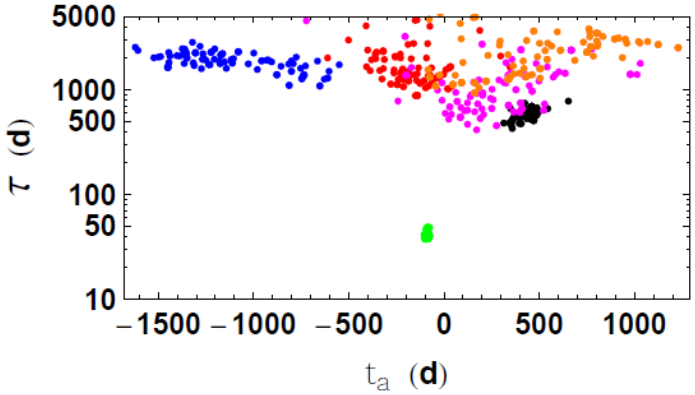




True planets signals

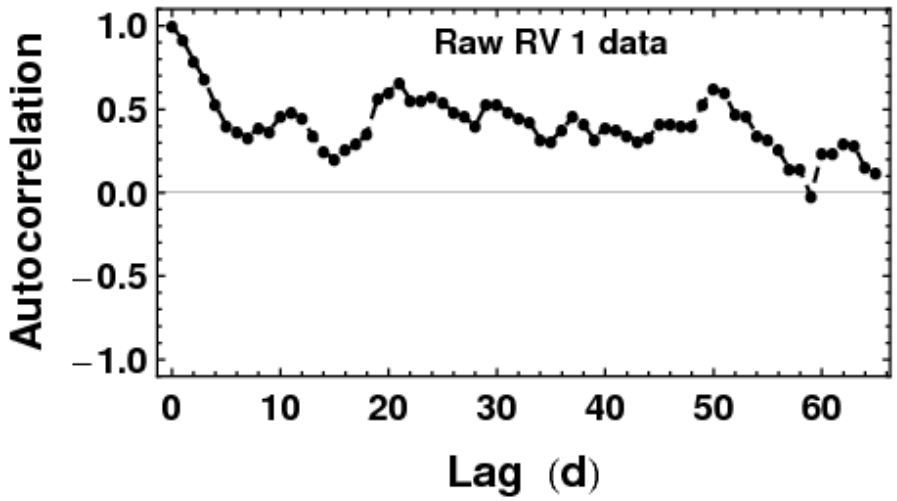
P (d)	ecc	K (m/s)
9.89	0.1	1.45
23.4	0.12	1.67
33.3	0.08	2.05
112.5	0.21	0.38
273.2	0.16	0.22

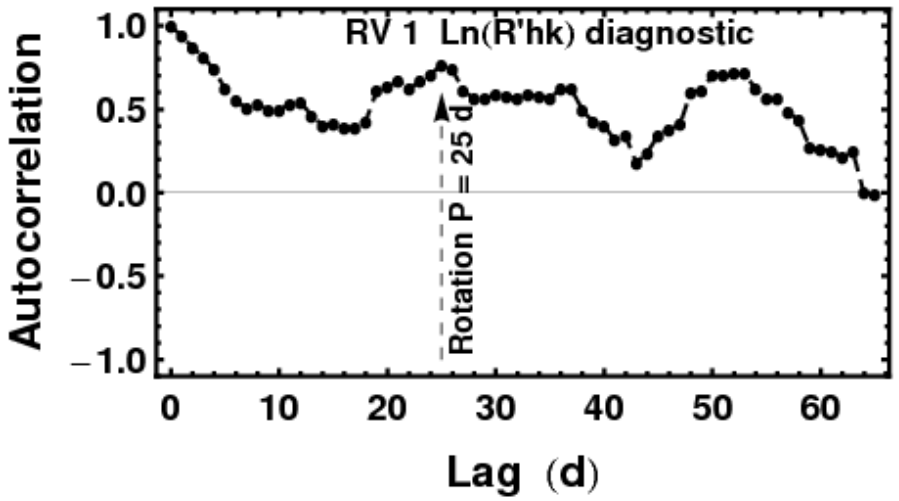


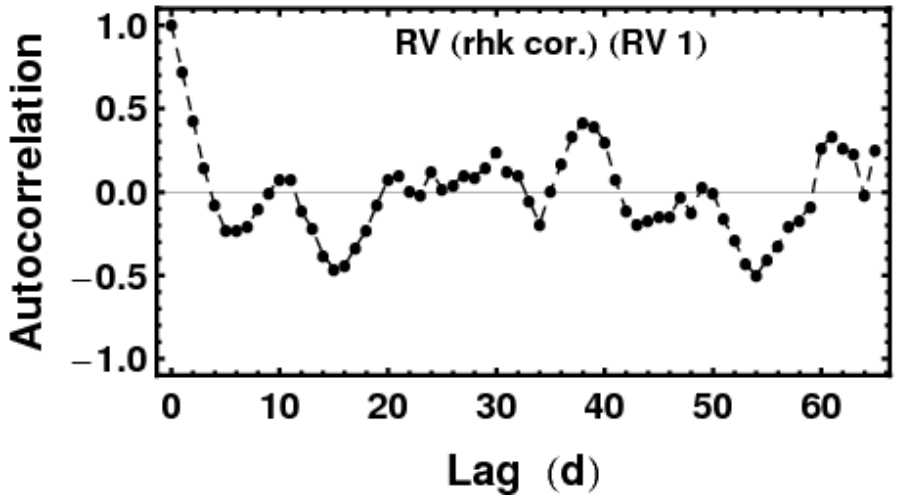


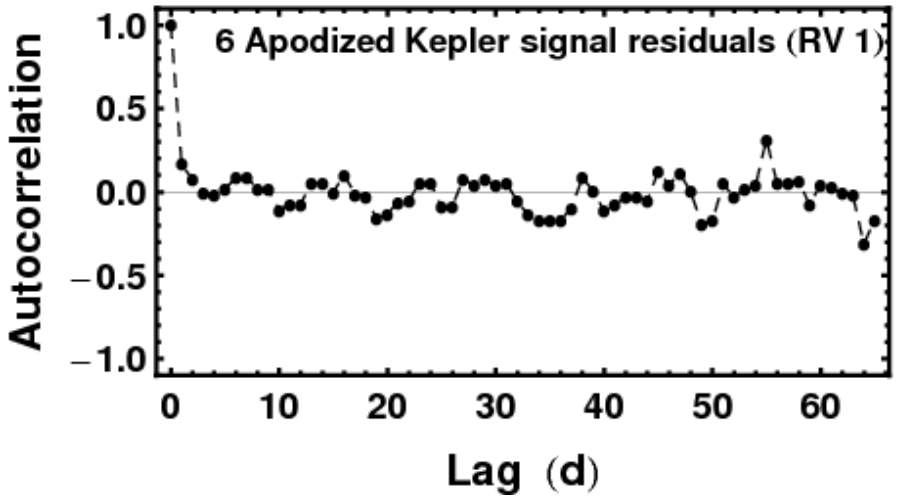
Kep6ApodPlan_RV1rhkCor_1May15_M7rev_corNRMC_ProbPvsIterProbvsPEccvsPCol.eps

Correlated Noise









By the time the 6 apodized Kepler signals and Log(R'hk) regression are removed, the autocorrelation of the residuals is looking close to white noise.

RV 2 Results

RV 2 Model: 8 apodized Kepler signals + log(R'hk) regression fit

Results indicate

3 planets

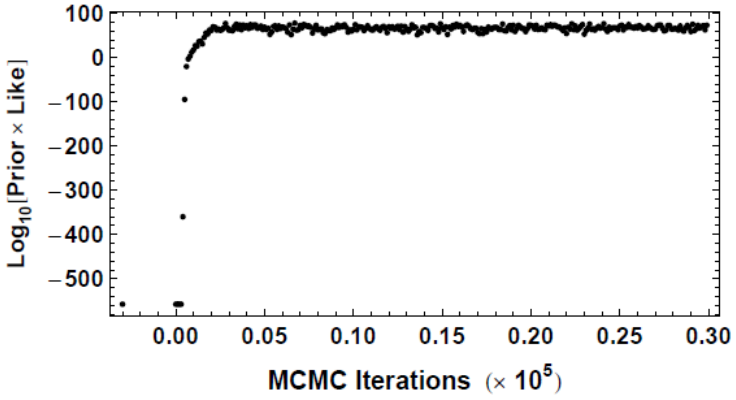
P= 3.77, 10.6, 75.5d (10.6d listed as a probable due to many nearby SA signals.)

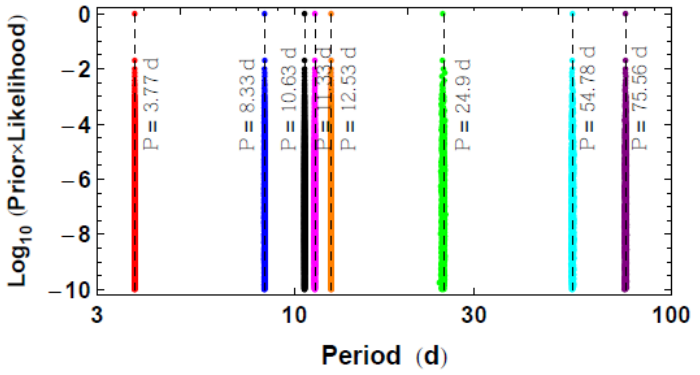
+

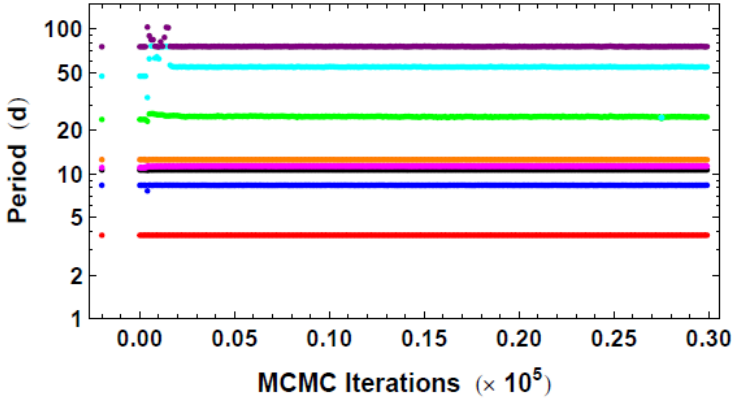
5 SA signals

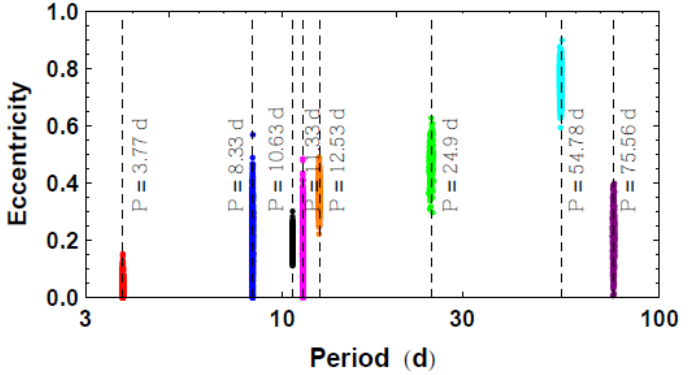
True planets signals

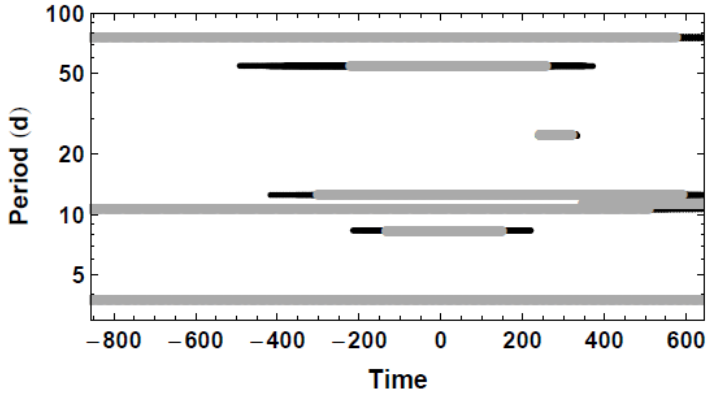
P (d)	ecc	K (m/s)
3.77	0.05	2.75
5.79	0.11	0.27
10.6	0.14	2.85
20.2	0.08	0.34
75.3	0.19	1.35

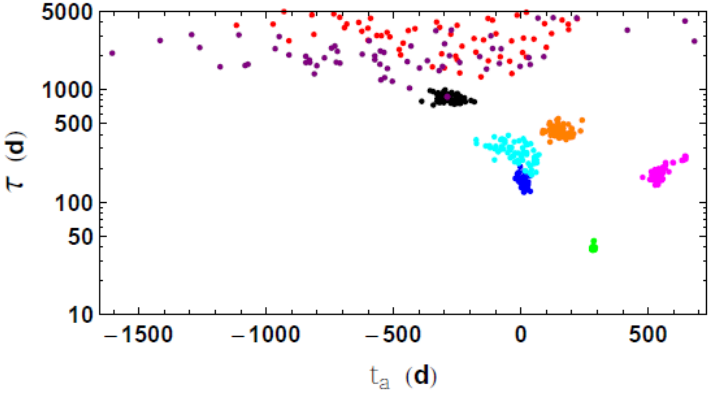












Kep8ApodPlan_RV2rhkCor_5May15_M7rev_corNRMC_ProbPvsIterProbvsPEccvsPCol.eps

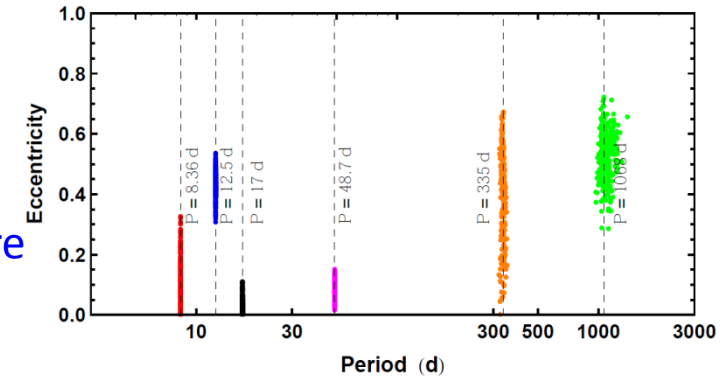
RV 3 Results

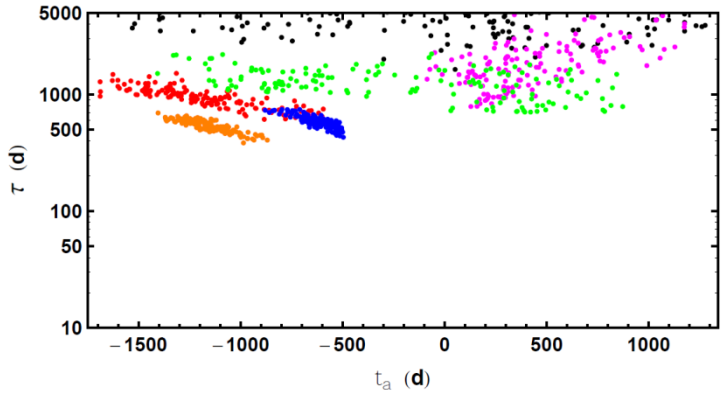
RV 3 Models

6 apodized Kepler signals

Results indicate

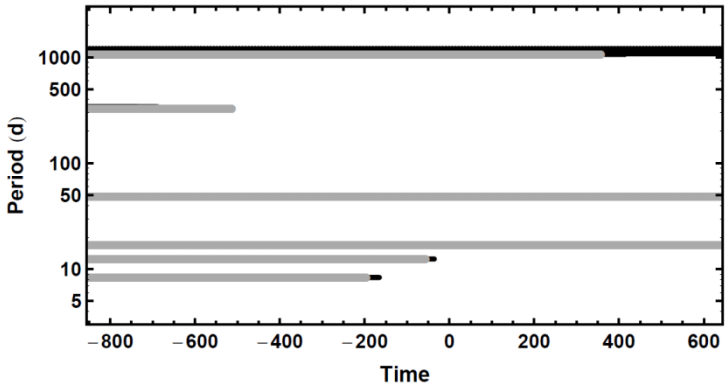
3 planets with
P= 17, 48.8,~1100d
(17 d listed probable due to weak signature in FWHM control)
(1100 d credited as harmonic of 2315)
+
3 SA signals





True planets signals

P (d)	ecc	K (m/s)
1.12	0.0	0.96
17.0	0.15	3.68
26.3	0.08	0.38
48.7	0.06	5.14
201.5	0.2	0.42
596	0.13	1.91
2315	0.15	3.87



RV 3 Models

6 apodized Kepler signals

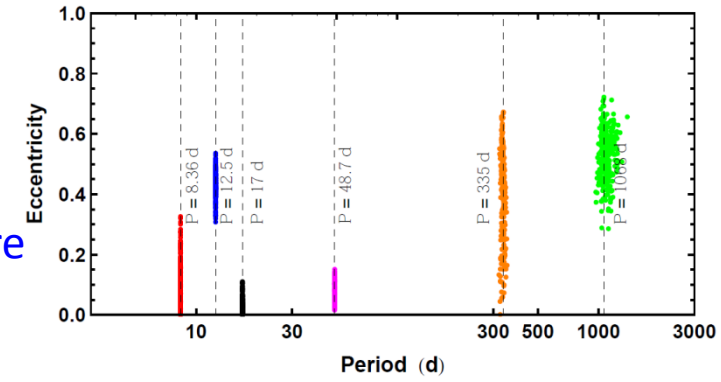
3 apodized Kepler signals
+ 3 straight Kepler signals

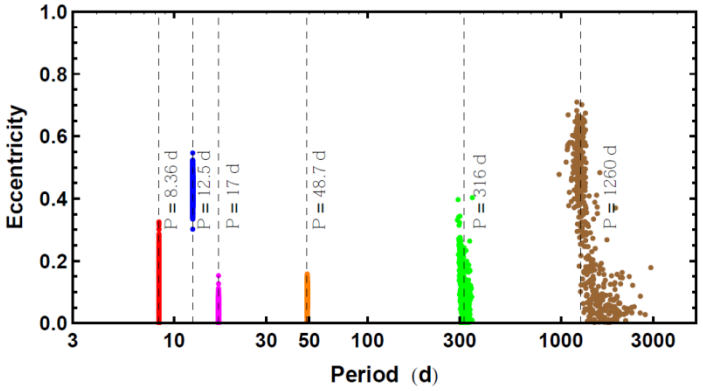
Results indicate

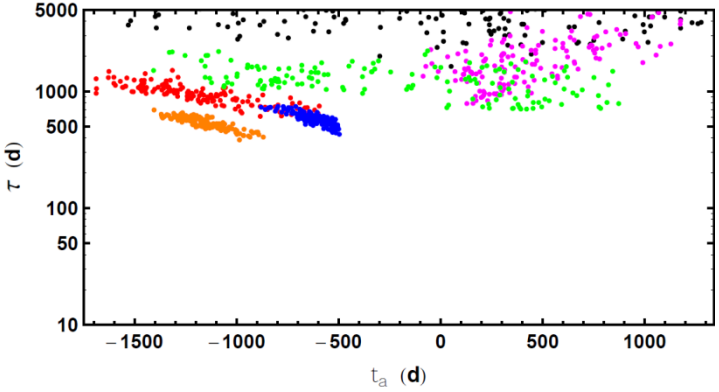
3 planets with
P= 17, 48.8,~1100d
(17 d listed probable due to weak signature in FWHM control)
(1100 d credited as harmonic of 2315)
+
3 SA signals

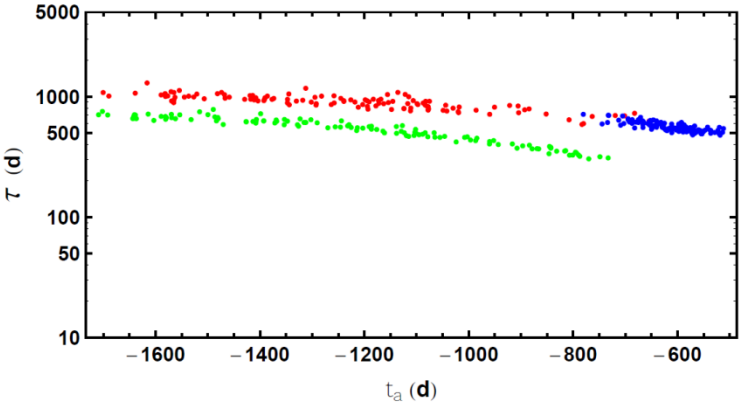
True planets signals

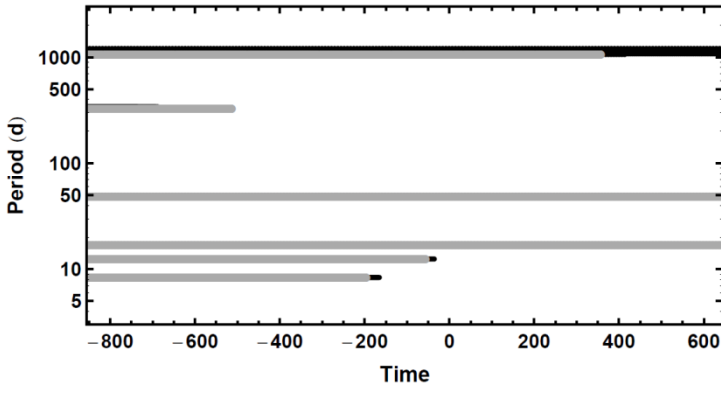
P (d)	ecc	K (m/s)
1.12	0.0	0.96
17.0	0.15	3.68
26.3	0.08	0.38
48.7	0.06	5.14
201.5	0.2	0.42
596	0.13	1.91
2315	0.15	3.87

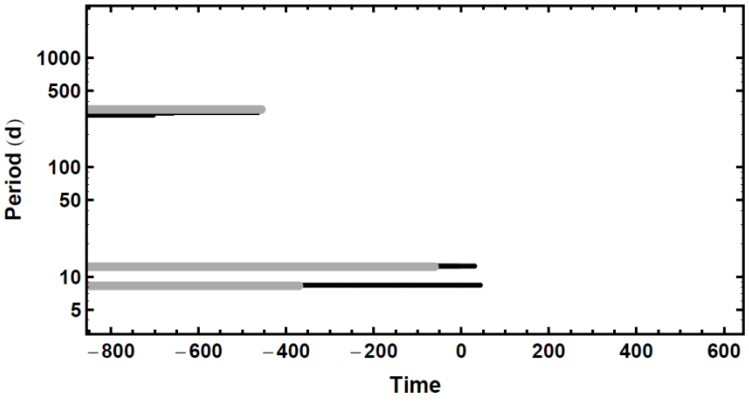












RV 4 Results

RV 4 Model: 8 apodized Kepler signals + log(R'hk) regression fit

No definite planets

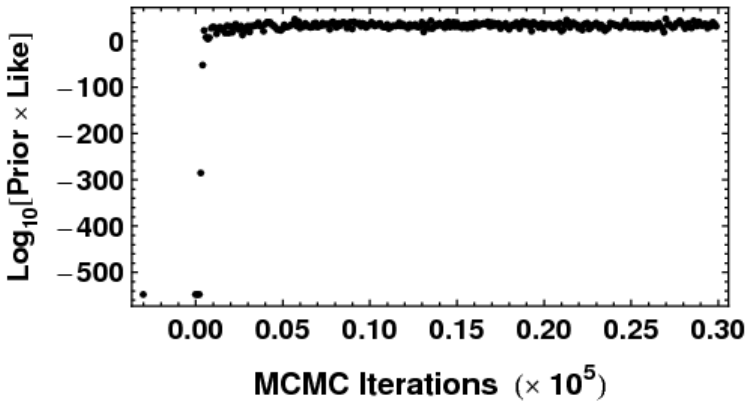
Possible planets at P = 0.946 & 11.75 d based on apodization.

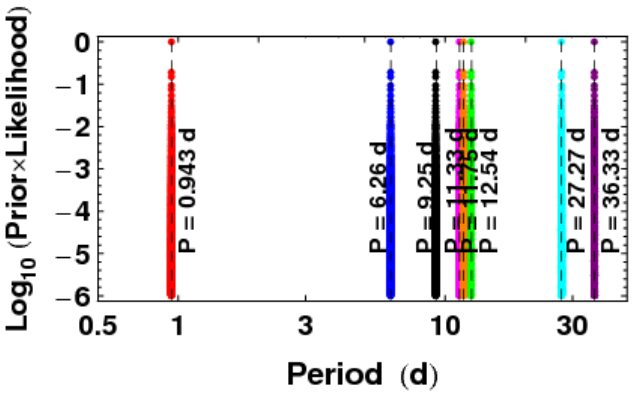
Bayes factor finds against a real P = 0.946 d planet.

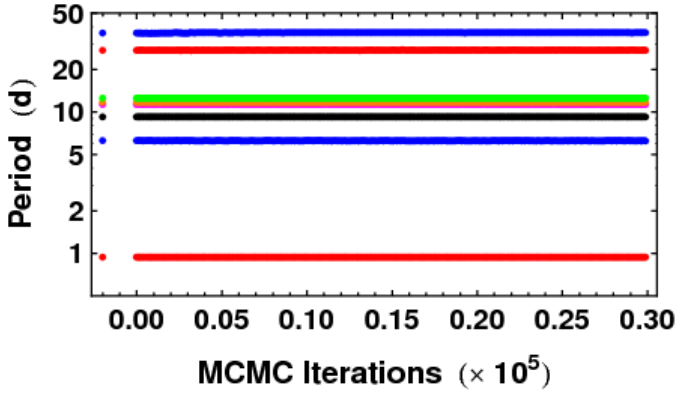
P = 11.75 only a possible because of weak FWHM Control counterpart, see differential GLS periodogram.

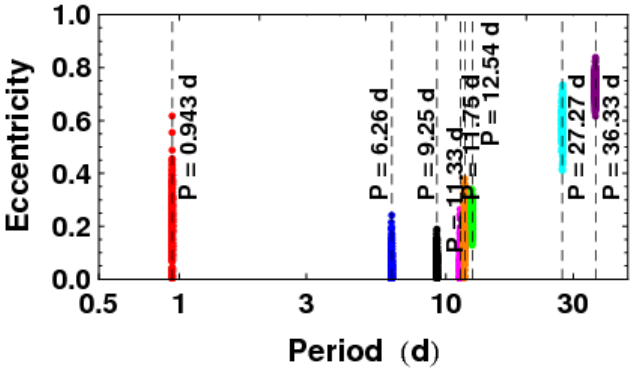
True planets signals

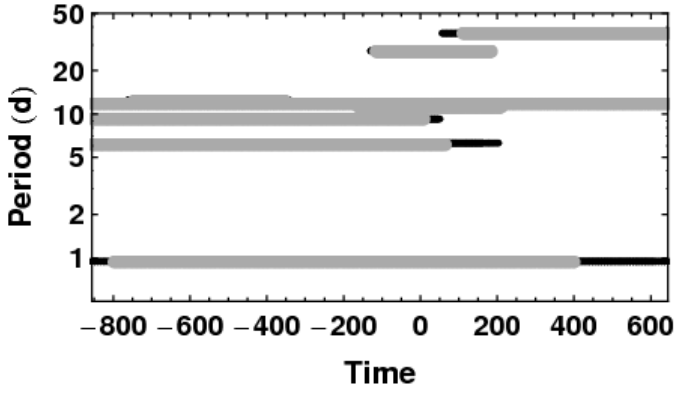
P (d)	ecc	K (m/s)
None		

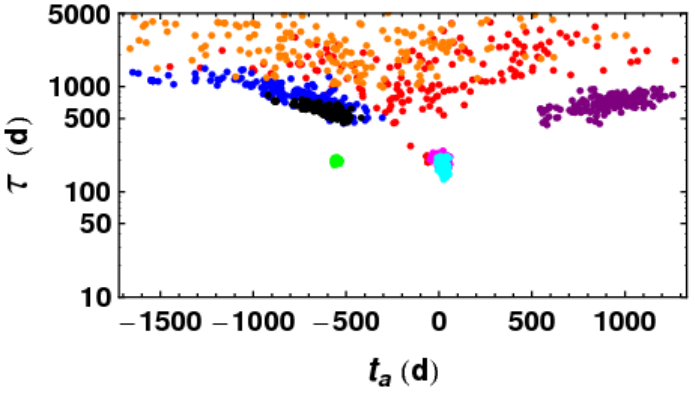






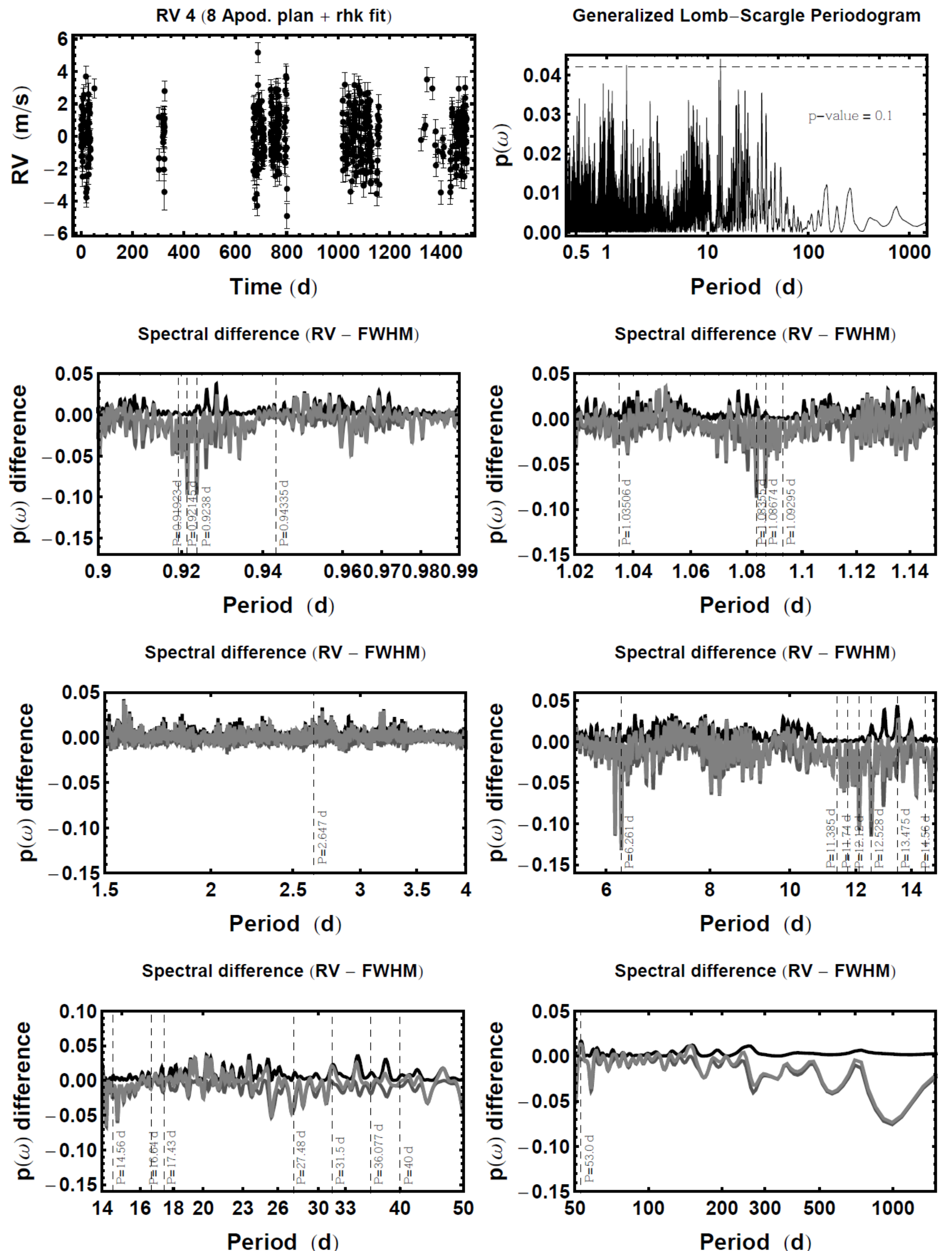






Kep8ApodPlan_RV4rhkCor_9May15_M7rev_corNRMC_ProbPvsIterProbvsPEccvsPCol.eps

GLS & Spectral difference
of residuals from
8 apodized Kepler + rhk fit



Significant power at P = 11.75 d in FWHM (rhk corr.) control

RV 5 Results

RV 5 Model: 6 apodized Kepler signals + log(R'hk) regression fit

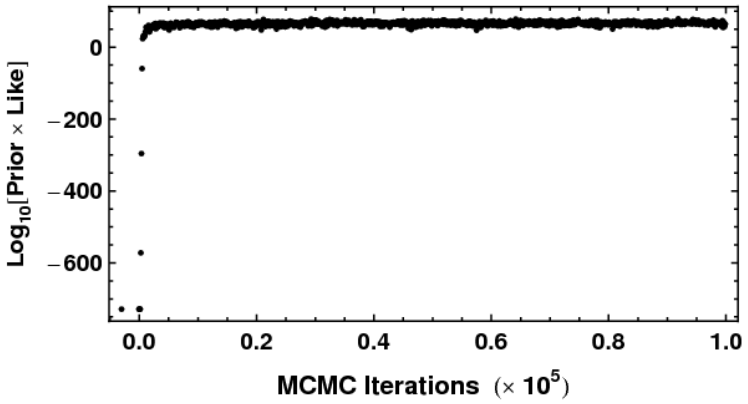
No definite planets

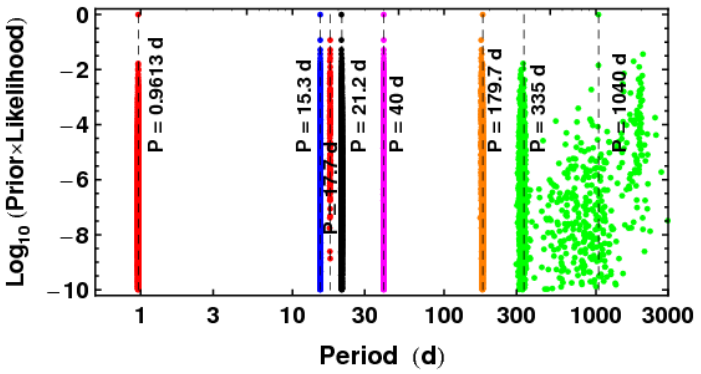
Possible planet at P = 0.96 d based on apodization width.

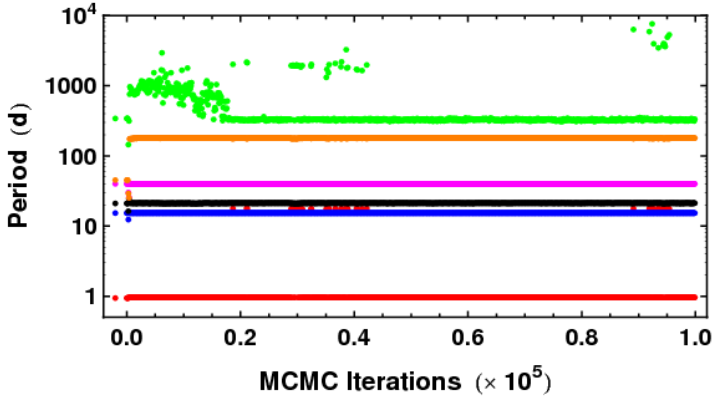
Bayes factor finds against a real P = 0.96 d planet.

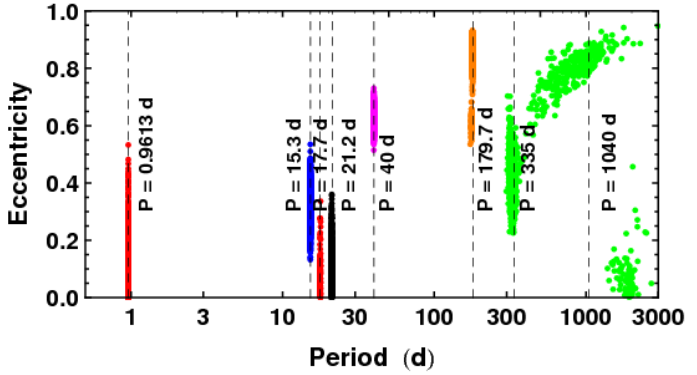
True planets signals

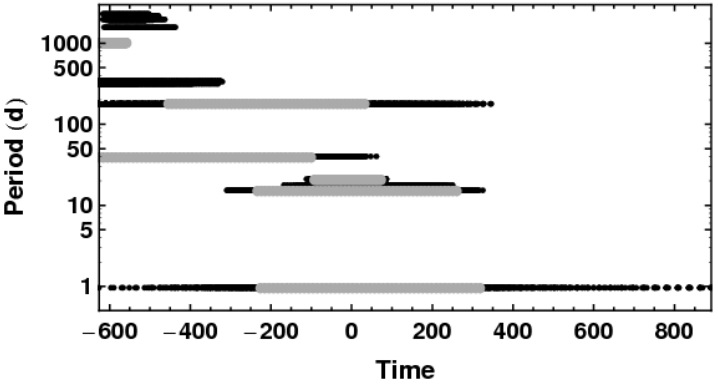
P (d)	ecc	K (m/s)
14.7	0.17	0.65
26.2	0.25	0.44
34.7	0.03	0.69
173.2	0.05	0.59
283.1	0.3	0.41
616.3	0.03	0.55

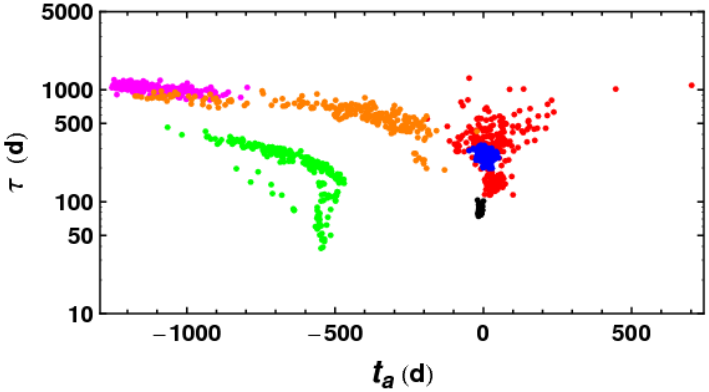












Kep6ApodPlan_RV5rhkCor_16May15_M7rev_corNRMC_ProbPvsIterProbvsPEccvsPCol_Sel.pdf

Summary Statistics

Data set	# Signals extracted	Initial σ_I (m/s)	Regression residual (m/s)	Final Residual σ_R (m/s)	Mean meas. error (m/s)	σ_I/σ_R
Test	5	8.55	2.7	1.35	0.5	6.3
RV 1	6	5.6	3.0	1.44	0.67	3.8
RV 2	8	8.58	4.0	1.42	0.67	6.0
RV 3	6	10.85	5.5	1.79	0.67	6.1
RV 4	8	8.3	3.3	1.52	0.67	5.5
RV 5	6	8.92	2.6	1.18	0.67	7.6

Conclusion: we are able to dig into the effective noise level set by stellar activity by a factor of ~ 6.
Still have a long way to go!!

Conclusions on Apodized Kepler model

1. Conceptually simple approach based on assumption that stellar activity signals vary on time scales shorter than the duration of the data set.
   For very short data sets this assumption would break down.

2. Relatively fast to compute (15 min for a one apodized Kepler model implemented in *Mathematica* and scales linearly with number of signals.)

3. Performed well for K > 1 m/s and resulted in no false detections.

4. Can be employed with other likelihood models (like Student's t) to help with outliers.

5. Next step to see if some combination of the 3 best techniques performs better and try out other apodization functions.