Mixed-State Quasiparticle Transport in High-T_c Cuprates

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(Received 21 August 1998)

Theory of quasiparticle transport in the mixed state of a d-wave superconductor is developed under the assumption of disordered vortex array. A novel universal regime is identified at fields above $H^* = c^* H_{c2}(T/T_c)^2$, characterized by a *field-independent* longitudinal thermal conductivity κ_{xx}^e. It is argued that this behavior is responsible for the high-field plateau in κ_{xx}^e experimentally observed in high-T_c cuprates. [S0031-9007(99)08562-2]

PACS numbers: 74.60.Ec, 74.25.Fy, 74.72.–h

Properties of the mixed state of high-T_c cuprate superconductors keep surprising the scientific community in ways never imagined before. On one hand, the collective behavior of vortices produces a multitude of "vortex phases" attributable to an intricate interplay between the thermal fluctuations, dimensional crossover, and pinning forces on vortices. At low temperatures, on the other hand, unexpected and fascinating properties are observed related to the d-wave symmetry of the order parameter and the consequent relativistic "Dirac" spectrum of the low-energy quasiparticle excitations.

A recent example of such an unexpected behavior is the experimental observation of the high-field plateau in the longitudinal thermal conductivity κ_{xx} in $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BiSCCO) and $YBa_2Cu_3O_{6+x}$ (YBCO) by Krishana, Ong, and co-workers [1,2] and by others [3–5]. In particular, it is found that after an initial steep drop at low fields (but well into the mixed state) κ_{xx} becomes *field-independent* above $H^* = c^* H_{c2}(T/T_c)^2$ and remains so up to the highest attainable fields $\sim$14 T. The implication is that both electronic and phononic contributions to κ_{xx} separately become independent of field, with the initial drop attributed solely to electronic κ_{xx}^e [2,4]. These observations stand in a sharp contrast to the behavior of $\kappa_{xx}(H)$ found in conventional s-wave superconductors [6], where vortices are strong scatterers of both electrons and phonons. Understanding the physics of the plateau phenomenon therefore presents a considerable challenge to the theory of quasiparticle transport in cuprates. Its broader significance lies in the fact that simultaneous measurement of the field-dependent longitudinal (κ_{xx}) and transverse (κ_{xy}) thermal conductivities, now feasible experimentally, contains a great wealth of information on the quasiparticle dynamics, and in principle affords deduction of inelastic scattering rate, thought to be important for the pairing mechanism in cuprates.

The initial interpretation of the plateau involved a field-induced transition to a fully gapped state, such as $d_{x^2-y^2} + id_{xy}$ (or "$d + id'$"), which would effectively freeze out the quasiparticle transport at low energies [1]. Laughlin [7] presented a compelling theoretical argument in support of such a scenario with a weakly first order transition, but pointed out that opening of a gap with physically reasonable magnitude could not in itself account for the complete suppression of the electronic transport in BiSCCO at temperatures up to 30 K. Furthermore, we find that sudden opening of a sizeable gap leads to a *jump* in $\kappa_{xx}(H)$, as opposed to a kink observed in [1,3]. Experimentally, there appears to be little additional evidence for the $d + id'$ state, except perhaps for the apparent bound states found by scanning tunneling microscopy (STM) in the vortex cores of YBCO [8] which should not exist in a pure $d_{x^2-y^2}$ state [9].

The existing theoretical treatments [7,10–12] so far side-stepped the important issue of the effect of magnetic field on the quasiparticle mean free path (MFP), determined primarily by scattering of quasiparticles by the Abrikosov vortices. A detailed theory of the MFP due to vortices in s-wave superconductors was developed long ago by Cleary [13]. The d-wave problem, however, is fundamentally different [9] and, except for heuristic treatments given in Refs. [14,15], no analogous theory exists at present. Here we develop such a theory and show that field-independent longitudinal thermal conductivity arises quite naturally in a pure $d_{x^2-y^2}$ state above a crossover field $H^*(T)$. This behavior reflects an exact compensation between the enhancement of the quasiparticle density of states (DOS) in the presence of nonuniform superflow (the "Volovik effect") [16] and the concomitant reduction in the quasiparticle mean free path ℓ due to increased scattering from vortices in a disordered vortex array. The limiting high-field value of $\kappa_{xx}^e(H)$ is universal in the similar sense as the impurity induced universal microwave conductivity $\sigma(\omega \to 0)$ predicted by Lee [17]. The approach to this limiting value depends on the distribution of vortices in the sample and will therefore be material and sample dependent.

The basic physics of the new universal regime can be understood from the following simple argument. In the elementary Boltzmann-type treatment the electronic thermal conductivity of a normal metal is given by

$$\kappa_{xx}^e = \frac{1}{3} v_F c_v \ell , \tag{1}$$

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where c_v is the electronic specific heat and ℓ is the MFP. We now argue that this relation remains valid in the mixed state of a d-wave superconductor at fields above H^*. As first pointed out by Volovik [16], the superfluid velocity field around vortices induces a Doppler shift in the excitation spectrum of quasiparticles, which in turn leads to a finite DOS at the Fermi level. For $H \gtrsim H^*$ the system behaves effectively like a normal metal and we expect (1) to hold. The residual DOS scales as $N_H(0) \sim \sqrt{H}$ and gives rise to the well-known low temperature specific heat [16]

$$c_v \simeq k_0 N_F T \sqrt{H/H_{c2}}\,, \tag{2}$$

where N_F is the normal-state DOS, and k_0 is a constant of order unity. This type of behavior is indeed observed experimentally [18].

We now estimate the vortex contribution ℓ_H to the total MFP $\ell^{-1} = \ell_0^{-1} + \ell_H^{-1}$. The central assumption we make is that in the regime of experimental interest the vortex lattice is *disordered* in the sense that it possesses no long range translational order, and will therefore scatter quasiparticles [19]. At low energies ℓ_H is dominated by the quasiparticle scattering from the superfluid velocity field $\mathbf{v}_s(\mathbf{r})$. Scattering from the vortex cores is down by a factor $(\xi/a_v)^2 = H/H_{c2} \ll 1$ [22], where $a_v = \xi\sqrt{H_{c2}/H}$ is the average intervortex distance, $\xi = v_F/\pi\Delta_0$ is the coherence length, and Δ_0 is the maximum gap. In the (disordered) vortex lattice $\mathbf{v}_s(\mathbf{r})$ varies on the length scale set by a_v. Since a_v is the only relevant length scale in the problem [23], one expects on general grounds that

$$\ell_H \simeq k_1 a_v \propto \sqrt{H_{c2}/H}\,, \tag{3}$$

where k_1 is a field-independent constant. This conclusion is indeed confirmed by an explicit calculation of the quasiparticle propagator outlined below, as well as by a calculation of the vortex transport scattering cross section σ_{tr} [22], carried out along the lines of the classical treatment by Cleary [13]. We note that Eq. (3) is also consistent with the result of Ref. [15] based on a heuristic argument involving the Andreev reflection.

In sufficiently strong fields Eq. (3) implies that $\ell_H \ll \ell_0$ [24] and the total MFP will be dominated by vortices: $\ell \approx \ell_H$. On substituting (2) and (3) into (1) we arrive at the desired result that for $H \gg H^*$, κ_{xx}^e will approach the *field-independent* universal value

$$\kappa_{xx}^{eH}/T = k'\pi N_F v_F^2/3\Delta_0\,, \tag{4}$$

with $k' = k_0 k_1$. Evaluation of k' below shows that κ_{xx}^{eH} is identical to the "universal" thermal conductivity κ_{00}^e due to impurities predicted by various authors [11,25] and observed experimentally by Taillefer and co-workers [26].

The argument based on Eq. (1) captures the essential physics of the new universal regime and provides a very natural explanation of the plateau phenomenon observed in cuprates [1–5] in terms of fundamental properties of the Dirac fermions. We now present a more rigorous treatment of $\kappa_{xx}^e(H)$, based on the Kubo formula for the heat current response. Besides supplying the unknown constant k', such calculation provides the necessary confidence in our result (4) and helps understanding the approach to the universal limit with the increasing H.

In the absence of field the quasiparticle propagator is a 2×2 matrix in the Nambu space (taking $\hbar = 1$)

$$\hat{G}_0(\omega, \mathbf{k}) = \frac{\omega + \hat{\tau}_3 \epsilon_{\mathbf{k}} + \hat{\tau}_1 \Delta_{\mathbf{k}}}{\omega^2 - \epsilon_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2}\,, \tag{5}$$

where $\hat{\tau}_i$ are the Pauli matrices. In intermediate magnetic fields the coupling of quasiparticles to the superflow around the vortices is well described by the semiclassical replacement $\omega \to \omega - \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r})$ in Eq. (5) [11,27]. In a simple London model, which will be sufficient for our purposes, the superfluid velocity field is given by [28]

$$\mathbf{v}_s(\mathbf{r}) = \frac{\pi\lambda^2}{m} \int \frac{d^2k}{(2\pi)^2} \frac{i\mathbf{k} \times \hat{z}}{1 + \lambda^2 k^2} \sum_i e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{R}_i)}. \tag{6}$$

Here λ is the London penetration depth, $\hat{z}$ is a unit vector along the field direction, and $\{\mathbf{R}_i\}$ denotes vortex positions. In a disordered vortex array, on the length scales large compared to a_v, propagation of quasiparticles will be described by the Greens function *averaged* over $\{\mathbf{R}_i\}$:

$$\hat{G}(\omega, \mathbf{k}) = \langle \hat{G}_0(\omega - \mathbf{k} \cdot \mathbf{v}_s, \mathbf{k}) \rangle_{\{\mathbf{R}_i\}}\,. \tag{7}$$

If we now define a probability density

$$\mathcal{P}(\eta) = \langle \delta[\eta - \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r})] \rangle_{\{\mathbf{R}_i\}}\,, \tag{8}$$

we may rewrite (7) as

$$\hat{G}(\omega, \mathbf{k}) = \int d\eta\, \mathcal{P}(\eta) \hat{G}_0(\omega - \eta, \mathbf{k})\,, \tag{9}$$

and all the information on the vortex array is now encoded in $\mathcal{P}(\eta)$. Making the most natural ansatz that the vortex positions are random and uncorrelated, it can be shown [15] that $\mathcal{P}(\eta) = (2\pi\sigma_H^2)^{-1/2} e^{-\eta^2/2\sigma_H^2}$ with

$$\sigma_H^2 = k_\alpha k_\beta \langle v_s^\alpha(\mathbf{r}) v_s^\beta(\mathbf{r}) \rangle_{\{\mathbf{R}_i\}} \simeq \frac{\pi^2}{8} \left(\frac{H}{H_{c2}}\right) \Delta_0^2 \ln \kappa\,. \tag{10}$$

Here $\kappa = \lambda/\xi$, and the last equality follows from Eq. (6).

Evaluation of the propagator $\hat{G}$ is somewhat complicated by the fact that the η integral in Eq. (9) cannot be expressed in terms of elementary functions. This difficulty can be avoided by taking $\mathcal{P}$ to be a Lorentzian $\mathcal{P}(\eta) = \pi^{-1}\sigma_H/(\eta^2 + \sigma_H^2)$ rather than a Gaussian, in which case the integration is elementary and one obtains

$$\hat{G}(\omega, \mathbf{k}) = \hat{G}_0(\omega - i\sigma_H, \mathbf{k})\,. \tag{11}$$

Scattering from vortices therefore results in a self-energy correction to the Greens function, which, for the particular case of Lorentzian distribution, is simply a constant $\Sigma_H(\omega, \mathbf{k}) = i\sigma_H$. Had we kept the Gaussian distribution, the self-energy would be similar at low energies, but would tend to zero for $|\omega^2 - \epsilon_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2| \gg \sigma_H^2$. This additional structure results in slight mathematical complications but does not change the result for $\kappa_{xx}^e(H)$ qualitatively. We therefore choose to proceed with the simpler form (11) and treat it either as a reasonable low-energy approximation to Eq. (9) or as a slightly different physical problem with a vortex distribution $\{\mathbf{R}_i'\}$ resulting in the Lorentzian distribution $\mathcal{P}(\eta)$.

Equation (11) may be used to calculate the field-induced DOS, $N_H(\omega) = -(2\pi)^{-1}\sum_{\mathbf{k}} \mathrm{Tr}\,\hat{A}(\omega,\mathbf{k})$, where $\hat{A}(\omega,\mathbf{k}) = 2\,\mathrm{Im}\,\hat{G}(\omega,\mathbf{k})$ is the spectral function. One obtains $N_H(0) \propto \sigma_H \propto \sqrt{H}$, in agreement with Volovik's result [16]. Interpretation of σ_H as the scattering rate due to vortices allows estimation of the vortex MFP $\ell_H = \bar{v}/\sigma_H$, where $\bar{v}$ is the average quasiparticle velocity. Taking $\bar{v} = \sqrt{v_F v_\Delta}$ [10], we find $\ell_H = a_v(8v_\Delta/v_F \ln\kappa)^{1/2} \simeq 0.5a_v$ (for $v_F/v_\Delta = 7$ and $\kappa = 70$), in agreement with our naive estimate (3). We finally note that since the scattering rate σ_H is proportional to $\sim\sqrt{n_v}$ (with $n_v = H/\Phi_0$ the vortex density), Eq. (11) will be in general difficult to derive from conventional diagrammatic techniques which typically employ low-density approximation and therefore yield scattering rates proportional to n_v. Physically, the $\sqrt{n_v}$ nonanalyticity reflects the long ranged $1/r$ nature of the $\mathbf{v}_s$ field associated with a single vortex.

We are now in the position to evaluate the thermal conductivity tensor from the standard linear response theory. Following Ambegaokar and Tewordt [29], we find for a d-wave superconductor in 2D,

$$\frac{\kappa_{ij}^e}{T} = \frac{1}{32\pi m^2}\int_{-\infty}^{\infty}\frac{d\omega}{T}\left(\frac{\omega}{T}\right)^2 \mathrm{sech}^2\left(\frac{\omega}{2T}\right)K_{ij}(\omega)\,, \tag{12}$$

where (neglecting vertex corrections)

$$K_{ij}(\omega) = \int \frac{d^2k}{(2\pi)^2}\,k_i k_j\,\mathrm{Tr}\{\hat{\tau}_3\hat{A}(\omega,\mathbf{k})\hat{\tau}_3\hat{A}(\omega,\mathbf{k})\}. \tag{13}$$

At temperatures low compared to Δ_0 it is permissible to linearize the quasiparticle spectrum in the Dirac cones near the four nodes of the gap function $\Delta_{\mathbf{k}} = \Delta_0\cos(2\theta)$ [10,17]. Taking $\epsilon_{\mathbf{k}} = v_F k_1$ and $\Delta_{\mathbf{k}} = v_\Delta k_2$ in the new coordinate system (k_1,k_2) with the origin at the node, one can explicitly perform the integral in (13) to obtain

$$K_{xx}(\omega) = \frac{4k_F^2}{\pi v_F v_\Delta}\left[1 + \left(\frac{\omega}{\sigma} + \frac{\sigma}{\omega}\right)\arctan\frac{\omega}{\sigma}\right]. \tag{14}$$

Here $v_\Delta = 2\Delta_0/k_F$ and we have replaced σ_H by the total scattering rate $\sigma = \sigma_0 + \sigma_H$, with σ_0 describing both elastic and inelastic processes in the superconductor at $H = 0$. The proper description of these processes would presumably require a self-energy $\Sigma_0(\omega,\mathbf{k})$ which depends strongly on both of its arguments. However, absent the detailed microscopic theory of such processes, we follow [30] and simply model their effect by a phenomenological constant σ_0, which we interpret as a process-specific average of $\Sigma_0(\omega,\mathbf{k})$.

For arbitrary σ and T the expression (12) for κ_{xx}^e must be evaluated numerically. However, the leading terms may be readily obtained in a perturbative expansion. We obtain, for $\sigma \gg 2T$ (high field regime):

$$\frac{\kappa_{xx}^e}{T} \simeq \frac{\pi v_F^2 N_F}{6\Delta_0}\left(1 + \frac{7\pi^2}{15}\frac{T^2}{\sigma^2}\right); \tag{15}$$

and for $\sigma \ll 2T$ (low field regime):

$$\frac{\kappa_{xx}^e}{T} \simeq \frac{v_F^2 N_F}{2\Delta_0}\left[\frac{9\zeta(3)}{4}\frac{T}{\sigma} + \frac{\ln 2}{2}\frac{\sigma}{T}\right]. \tag{16}$$

The first term in Eq. (15) reproduces Eq. (4) for $k' = 1/2$ and represents the universal thermal conductivity $\kappa_{xx}^{eH} \equiv \kappa_{00}^e$ [11,25], which does not depend on the details of the vortex distribution, as long as there is no long range order. This result justifies the usage of the simple Boltzmann approach (1) in the mixed state of a d-wave superconductor. The leading correction in (15) is nonuniversal; e.g., one may show that the power will change to $(T/\sigma)^4$ for a Gaussian $\mathcal{P}$.

Equations (15) and (16) provide a simple tool for extracting the zero-field scattering rate σ_0 from the experimental data. Assuming that the phonon contribution κ_{xx}^p to the thermal conductivity κ_{xx} is field independent [2,4], the total drop in $\kappa_{xx}(H)$ between $H = 0$ and the plateau is simply related to σ_0 by

$$\sigma_0 = \begin{cases} c_1 T(1 + \delta\kappa_{xx})^{-1}, & \sigma_0 \ll T\,, \\ c_2 T(\delta\kappa_{xx})^{-1/2}, & \sigma_0 \gg T\,. \end{cases} \tag{17}$$

Here $\delta\kappa_{xx} = [\kappa_{xx}(0) - \kappa_{xx}(\infty)]/\kappa_{00}^e$, $\kappa_{xx}(\infty)$ denotes the plateau value, $c_1 = 27\zeta(3)/4\pi \simeq 2.58$, and $c_2 = \sqrt{7\pi^2/15} \simeq 2.15$. Applying these relations to the data on underdoped YBCO [2] and using $\kappa_{00}^e/T = 0.019\ \mathrm{W/K^2\,m}$ [26], we obtain $\sigma_0 \approx 0.4$ meV at 10 K, increasing to about 7 meV at 50 K and 19 meV at 60 K (just below $T_c = 63$ K).

To illustrate the behavior of $\kappa_{xx}^e(H)$ over the wide range of fields and temperatures in Fig. 1 we have evaluated Eq. (12) with kernel (14) numerically, adopting a phenomenological expression for the scattering rate $\sigma_0/\Delta_0 = \gamma_0 + \gamma_3(T/\Delta_0)^3$. Here γ_0 represents the residual scattering rate due to impurities and the γ_3 term models the inelastic scattering rate [30] which is known to collapse rapidly below T_c [14]. The similarity of Fig. 1 to the experimental data on the underdoped YBCO [2] is striking: the positive curvature of $\kappa_{xx}(H)$ is enhanced with decreasing T and the curves approach a field-independent value at high fields. The characteristic crossing of the curves near $T/\Delta_0 \simeq 0.15$, which reflects the crossover in $\sigma_0(T)$ from inelastic scattering at high T (solid lines) to elastic scattering at low T (dashed lines) is also present.

In some samples of BiSCCO, the plateau is reached abruptly, with a discontinuity in the slope of $\kappa_{xx}(H)$

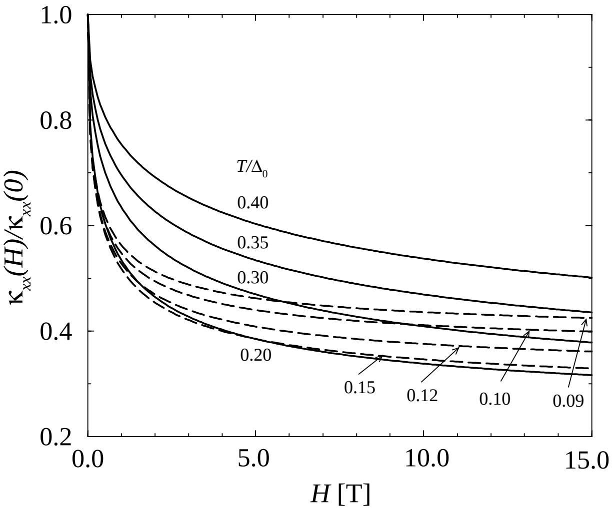

FIG. 1. Normalized electronic thermal conductivity $\kappa^e_{xx}(H)$ as a function of H at selected temperatures as calculated from Eq. (12). Parameters used are $H_{c2} = 160$ T, $\kappa = 70$, $\gamma_0 = 0.1$, and $\gamma_3 = 4$.

at H^* [1,3,5], indicative of a phase transition. This "kink" behavior is not reproduced by the present simple model, which, however, can be presumably appropriately modified once the nature of the phase transition is understood. We take a position here that it is the plateau phenomenon which is universal and significant (since it is observed in more than one compound) while the approach to it is a material-specific issue of secondary importance.

Aubin and co-workers [3] further report that the behavior of $\kappa_{xx}(H)$ in BiSCCO qualitatively changes below 1 K, in that it *increases* with magnetic field, approximately as $\kappa_{xx}(H) \sim \sqrt{H}$. From Eq. (1) the most natural interpretation is that, below 1 K, ℓ becomes independent of H. There are two possible reasons for this. The vortex array may order at low T, in which case the Bloch theorem prevents quasiparticles from being scattered by vortices [11]. The second possibility stems from the computation of the transport scattering cross section of a d-wave vortex [22], which indicates that $\sigma_{\rm tr}(\omega)$ vanishes as ω^3 for quasiparticle energies $\omega \lesssim \Delta_0/k_F\lambda$. Thus, at low energies, vortices become "transparent," and even a disordered vortex array will not scatter quasiparticles.

An immediate consequence of our picture is that *all* of the high-T_c compounds with the $d_{x^2-y^2}$ gap should exhibit the plateau phenomenon with the universal value $\kappa^{eH}_{xx}/T = \pi v_F^2 N_F/6\Delta_0$ in sufficiently high fields. We predict that the existence of plateau will correlate with the absence of long range order in the vortex array, and will fade away at the lowest temperatures. The formalism developed here also permits calculation of the Hall conductivity κ^e_{xy}. However, this constitutes a more complicated problem since one has to consider the particle-hole asymmetry in the vortex scattering rate [13] as well as corrections to the linearized Dirac spectrum [10].

The author is indebted to N.P. Ong for sharing his insights and his data prior to publication; and to Y. Ando, A.V. Balatsky, K. Behnia, A.J. Berlinsky, P.J. Hirschfeld, C. Kallin, A.J. Millis, L. Taillefer, and Z. Tešanović for valuable discussions. This work was supported by NSF Grant No. DMR-9415549 and was done in part at the Aspen Center for Physics.

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