REVIEW # 1 ANSWERS

1.14 (b) $J(x,t) = 0$

1.17 (a) $A = \sqrt{\frac{15}{16a^5}}$ (b) $\langle x \rangle = 0$ (c) $\langle p \rangle = 0$

(d) $\langle x^2 \rangle = \frac{a^2}{7}$ (e) $\langle p^2 \rangle = \frac{5}{2}\frac{\hbar^2}{a^2}$

(f) $\sigma_x = \frac{a}{\sqrt{7}}$ (g) $\sigma_p = \sqrt{\frac{5}{2}}\frac{\hbar}{a}$

2.7 (a) $A = 2\sqrt{\frac{3}{a^3}}$

(b) $\Psi(x,t) = \frac{4\sqrt{6}}{\pi^2}\sqrt{\frac{2}{a}}\sum_{n \text{ odd}} (-1)^{(n-1)/2}\frac{1}{n^2}\sin\left(\frac{n\pi}{a}x\right)e^{-E_n t/\hbar}$

2.11 $\langle x \rangle = \langle p \rangle = 0$, $\langle x^2 \rangle = \frac{\hbar}{2m\omega}$ $\langle p^2 \rangle = \frac{m\hbar\omega}{2}$ for $n=0$

$\langle x^2 \rangle = \frac{3\hbar}{2m\omega}$ $\langle p^2 \rangle = \frac{3m\hbar\omega}{2}$ for $n=1$

$\langle T \rangle = \langle V \rangle = \frac{1}{4}\hbar\omega$ for $n=0$

$= \frac{3}{4}\hbar\omega$ for $n=1$

2.13 (a) $A = \frac{1}{5}$ (b) $|\Psi(x,t)|^2 = \frac{1}{25}\left[9\psi_0^2 + 16\psi_1^2 + 24\psi_0\psi_1\cos(\omega t)\right]$

(c) $\langle x \rangle = \frac{24}{25}\sqrt{\frac{\hbar}{2m\omega}}\cos\omega t$ $\langle p \rangle = -\frac{24}{25}\sqrt{\frac{m\omega\hbar}{2}}\sin\omega t$

(d) $E_0 = \frac{1}{2}\hbar\omega$ with $P_0 = \frac{9}{25}$ or $E_1 = \frac{3}{2}\hbar\omega$ with $P_1 = \frac{16}{25}$

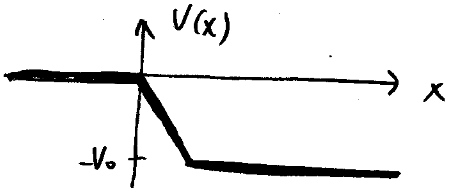
2.19 $J = \frac{\hbar k}{m}|A|^2$, positive x direction

2.33 $T^{-1} = 1 + \frac{2mE}{\hbar^2}a^2$ for $E = V_0$

$T^{-1} = 1 + \frac{V_0^2}{4E(E-V_0)}\sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E-V_0)}\right)$, $E > V_0$

$\boxed{2.35}$ (a) $R = \left(\dfrac{\sqrt{E+V_0} - \sqrt{E}}{\sqrt{E+V_0} + \sqrt{E}} \right)^2 \longrightarrow \dfrac{1}{9}$ for $V_0/E = 3$

(b)



(c) $T = 1 - R = \dfrac{8}{9}$

$\boxed{2.38}$ (a) $P_n = \begin{cases} \frac{1}{2}, & \text{if } n = 2 \\ \frac{32}{\pi^2 (n^2-4)^2}, & \text{if } n \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$, $E_2 = \dfrac{\pi^2 \hbar^2}{2ma^2}$ most probable

(b) $E_1 = \dfrac{\pi^2 \hbar^2}{8ma^2}$, next most probable with $P_1 = \dfrac{32}{9\pi^2} \simeq 0.36$

(c) $\langle H \rangle = \dfrac{\pi^2 \hbar^2}{2ma^2}$