P502 **Problem set # 0** **Due: Sept. 20, 15:30**

1. (10 points) A&M 2/26; *"Joule Heating"*

Consider a metal at uniform temperature in a static uniform electric field **E**. An electron experiences a collision, and then, after time t, a second collision. In the Drude model, energy is not conserved in a collision, for the mean speed of an electron emerging from a collision does not depend on the energy that the electron acquired from the field since the time of the preceding collision.

(a) Show that the average energy lost to the ions in the second of two collisions separated by a time t is $(eEt)^2/2m$. The average is over all directions in which electron emerged from from the first collision.

(b) Show that the average energy loss to the ions per electron per collision is $(eE\tau)^2/m$, and hence that the average energy loss per cubic centimeter per second is $(ne^2\tau)/mE^2 = \sigma E^2$. Deduce that the power loss in a wire of length L and cross section A is RI^2, where I is the current flowing and R is the resistance of the wire.

2. (15 points) A&M 1/53; *"Free electron gas in 2D"*

(a) What is the relation between n and k_F in two dimensions?

(b) What is the relation between k_F and r_s in two dimensions?

(c) Prove that in 2D the free electron density of states $g(\epsilon)$ is a constant independent of ϵ for $\epsilon > 0$ and 0 for $\epsilon < 0$. Determine the constant.

(d) Show that because $g(\epsilon)$ is constant, every term in the Sommerfeld expansion for n vanishes except the $T = 0$ term. Deduce that $\mu = \epsilon_F$ at any temperature.

(e) Deduce that when $g(\epsilon)$ is as in (c), then

$$\mu + k_B T \ln(1 + e^{-\mu/k_B T}) = \epsilon_F.$$

(f) Based on the above estimate the amount by which μ differs from ϵ_F. Comment on the numerical significance of this "failure" of the Sommerfeld expansion, and on the mathematical reason for this "failure".

3. (5 points) A&M 1/82; *"Bravais Lattices"*

In each of the following cases indicate whether the structure is a Bravais lattice. If it is, give three primitive vectors; if it is not, describe it as a Bravais lattice with as small as possible a basis.

(a) Base-centered cubic (simple cubic with additional points in the centers of the horizontal faces of the cubic cell).

(b) Side-centered cubic (simple cubic with additional points in the centers of the vertical faces of the cubic cell).

(c) Edge-centered cubic (simple cubic with additional points at the midpoints of the lines joining nearest neighbors).

4. (10 points) A&M 5/109; *"Structure Factors"*

(a) The sodium chloride structure can be regarded as an fcc Bravais lattice of cube side a, with a basis consisting of a positively charged ion at the origin and a negatively charged ion at $(a/2)\hat{\mathbf{x}}$. The reciprocal lattice is a body-centered cubic, and the general reciprocal lattice vector has the form $\mathbf{K} = (4\pi/a)(\nu_1\hat{\mathbf{x}} + \nu_2\hat{\mathbf{y}} + \nu_3\hat{\mathbf{z}})$ with all ν_i's either integers or integers $+\frac{1}{2}$. If the atomic form factors for the two ions are f_+ and f_-, show that the structure factor is $S_{\mathbf{K}} = f_+ + f_-$ if the ν_i are integers, and $f_+ - f_-$ if the ν_i are integers $+\frac{1}{2}$. Why does S vanish in the latter case when $f_+ = f_-$?

(b) The zincblende structure is also a face-centered cubic Bravais lattice of cube side a, with a basis consisting of a positively charged ion at the origin and negatively charged ion at $(a/4)(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$. Show that the structure factor is $f_+ \pm if_-$ if the ν_i are integers $+\frac{1}{2}$, $f_+ + f_-$ if ν_i are integers and $\sum_i \nu_i$ is even, and $f_+ - f_-$ if ν_i are integers and $\sum_i \nu_i$ is odd.

(c) Suppose that a cubic crystal is known to be composed of closed-shell (and hence spherically symmetric) ions, so that $f_\pm(\mathbf{K})$ depends only on magnitude of $\mathbf{K}$. The positions of the Bragg peaks reveal that the Bravais lattice is face-centered cubic. Discuss how one might determine, from the structure factors associated with the Bragg peaks, whether the crystal structure was likely to be of the sodium chloride or zincblende type.