P474 **HWK # 7** **Due: March 9, 2011**

1. Density of states in general dimension

a) The density of states of a free electron gas in d-dimensions is defined as

$$D_d(\omega) = L^d \int \frac{d^d k}{(2\pi)^d} \delta(\omega - \epsilon_{\mathbf{k}})$$

with $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$. Using the d-dimensional spherical coordinate representation

$$\int d^d k \to S_d \int dk k^{d-1}$$

find $D_d(\omega)$ for arbitrary positive integer d. Here, $S_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface area of a unit sphere in d-dimensions and Γ is the gamma function.

b) Using $\Gamma(d/2) = \sqrt{\pi}, 1, \sqrt{\pi}/2$ for $d = 1, 2, 3$ respectively check the 2D and 3D case against the textbook results and give explicitly $D_1(\omega)$.

c) Based on the result of part (b) find the low-temperature specific heat of the free electron gas in $d = 1, 2$, assuming that $\mu(T) \simeq \epsilon_F$.

2. Free electrons in the Zeeman field. Consider a *two-dimensional* gas of N electrons in volume L^2 subject to the uniform magnetic field $\mathbf{B} = \hat{z}B$. For the purposes of this problem we shall neglect the effect of the field on the orbital motion of electrons and focus on the Zeeman coupling to their spin. This is described by the Hamiltonian

$$H = \frac{\hat{\mathbf{p}}^2}{2m} - \gamma \mathbf{S} \cdot \mathbf{B},$$

where $\gamma \simeq 2e/m$ is the electron gyromagnetic ratio and $\mathbf{S}$ the electron spin operator. This simply means that the energy of the electron now depends on the orientation of its spin,

$$\epsilon_{\mathbf{k}\sigma} = \frac{\hbar^2(k_x^2 + k_y^2)}{2m} - \sigma\frac{\gamma\hbar B}{2},$$

with $\sigma = \pm 1$ for spin up/down.

a) Describe the ground state of the system by specifying which momentum and spin states are occupied. [*Hint:* Consider small N at first.]

b) Determine ϵ_F and k_F for the two spin orientations.

c) For a given planar density $n = N/L^2$ find the minimum field B needed for the ground state to be completely spin-polarized (i.e. all spins pointing along the field).

3. Problem 7.1 from the textbook