Chaos Dynamics and the Logistic Map

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Abstract

The logistic map is a specific type of 1-D map that creates a chaotic system which can be characterized using numerical methods. The bifurcation points of this system will be determined, as will its Lyapunov exponent. This chaotic system ceases its chaotic behaviour at a particular value of its control parameter and then resumes its chaotic behaviour as this control parameter is incresed. This short time of period 3 orbit will also be investigated.

1 Introduction

Chaos resulted from the realization that analytic solutions could not be found for every mechanical system. There are some systems were even slight permutations in the initial conditions can cause completely different behaviour for the system as time progresses. This is not to say that the physics describing the system is wrong. In fact, the physics predicts this chaotic motion, it is just extremely sensitive to the initial state of the system. This sort of divergence of behaviour of two system with almost identical initial conditions is called *exponential divergence*. Chaos was first described by Poincaré who showed that the motion of the solar system exhibited chaos and had no analytical solution. This discovery was essentially forgotten until Edward Lorenz in 1960 was working on turbulence relating to weather systems and discovered that by using data with differing numerical precision, he numerically calculated radically different behaviour. This showed how some problems are very sensitive to initial conditions because using numbers with differing precision was the same as using slightly different initial conditions. Because numerical calculations are used to study chaos, it is only in the past couple decades that computer power has advanced enough in order for the study of chaos to come into its own. This project will study a simple subset of the chaos field - the logistic map.

2 The Logistic Map

The logistic map is a 1-D map where the value of the next step x_{j+1} is only dependent on the current step value x_j and their relationship is given by equation 1.

$$x_{j+1} = rx_j(1 - x_j) \tag{1}$$

$$0 < x_j < 1, 1 < r < 4$$

In this equation, r is called the control parameter and it plays a critical role in determining the type of behaviour that x_j will have as j is increased and reaches an equilibrium behaviour.