August 2013 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
Day 1: August 29, 2013

Do not write your name on the exam. Instead, write your student number on each exam booklet. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading. If you use extra exam booklets, write your student number on the extra exam books as well.

Today's portion of the exam has 8 questions. Answer *any five* of the eight. Do not submit answers to more than 5 questions—if you do, only the first 5 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 4 hours to complete 5 questions.

You are allowed to use one $8'' \times 11''$ formula sheet (both sides), and a hand-held, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
air pressure at sea level	1 atm	10^5 N/m^2
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to moon	d_{moon}	3.8×10^{8} m
distance from earth to sun	1 AU	1.5×10^{11} m
electron volt	1 eV	1.6×10^{-19} J
Larmor formula		$P_{rad} = \mu_0 (\ddot{\vec{m}})^2 / 6\pi c^3$
mass of an electron	m_e	0.511 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
mass of a proton	m_p	938 MeV/c^2
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Sun	R_{sun}	7×10^{8} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N} N^N \sqrt{2\pi N}$

1. Order of magnitude estimation:

Small meteors striking the earth will burn up in the 10-km thickness of the atmosphere, while larger ones will survive to reach the surface. Consider a solid iron meteor (density = 8000 kg/m^3, heat of fusion = 250 J/g, heat of vaporization = 6000 J/g). Do an order of magnitude estimate of the minimum size for such meteors that will survive to strike the Earth's surface. Hint: the drag force on a very fast object moving through a fluid is $F_D = \frac{1}{2}\rho_{fluid}v^2 C_D A$, where A is the cross-sectional area of the object, v is the speed of the object, ρ_{fluid} is the density of the fluid, and the drag coefficient C_D is a constant of order 1. You may assume that the initial velocity of the meteor is of the same order as the Earth's orbital velocity around the Sun.

2.

A. A sphere of uniform density with mass 0.1 kg and radius 5 cm is set spinning with a rotational speed of 10 revolutions per second. What is the quantum mechanical limit to the angular accuracy with which the direction of the spin can be aligned?

B. An atomic transition produces a spectral line with wavelength 400 nm when the atom decays from some state A to the ground state. The spectral line is measured with a diffraction grating containing 10,000 lines. What is the longest lifetime for state A that it is possible to measure using this spectrometer?

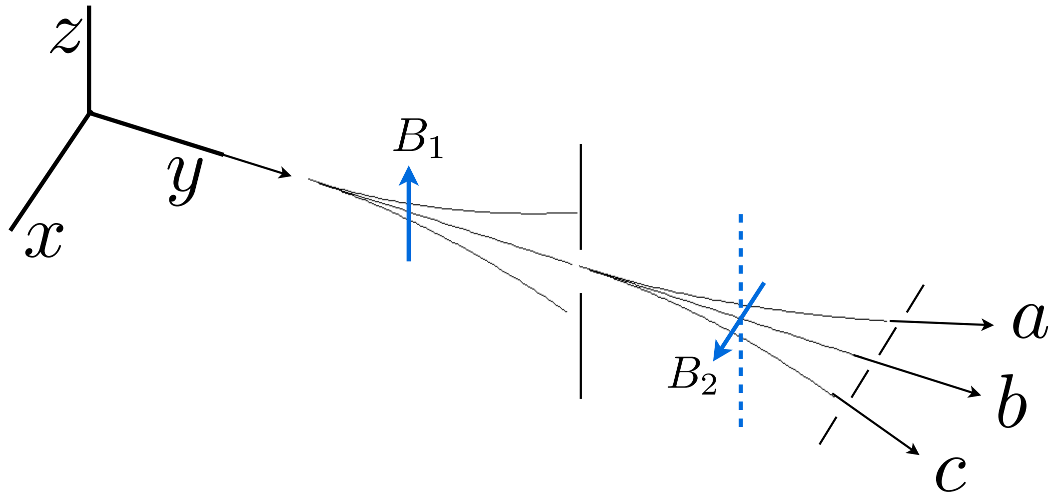

Figure 1: Positronium beam passing through a double Stern-Gerlach apparatus

3. A beam of ground state positronium particles (an "atom" composed of an electron and its anti-particle a positron) enters the double Stern-Gerlach apparatus shown below. Assume the particles are in an equal mixture of the two spin configurations — that is, there is an equal number of para-positronium with a total spin $S = 0$ and ortho-positronium with a spin $S = 1$. Assume also that the beam is completely unpolarized. The positronium first encounters a magnetic field along the z axis with a gradient increasing in the z direction (in the direction of the arrow) and the beam splits into three. The un-deflected beam then flies through a hole in the beam-stop and encounters a magnetic field directed *perpendicular* to the first one and along the x axis. The beam splits again.

3A) What fraction of the total beam flux will pass through the first beam-stop? What fraction of the total beam flux will pass through to each of the outputs labeled a, b, and c? Write down the spin state for the positronium passing through each of these outputs in the z basis.

3B) Suppose the positronium is disassociated at the outputs a, b, and c so that the spin of the electron and positron can be measured independently and the electron and positron spin projections along the z axis are measured. What is the chance of finding both the electron and the positron in the *same* spin state for each of the outputs.

4. An electron is constrained to move in a ring with radius R (in the XY plane).

A) Find the eigenstates and eigenvalues for the electron. (Ignore the electron's spin.)

B) Now an electric field of strength E is applied along the X-direction. Find the leading (non-zero) correction to the ground state energy.

C) Calculate the polarizability of the system (the ratio of the electric dipole moment to the electric field strength).

5. Two spin-1/2 particles located at sites 1 & 2 respectively interact with an exchange interaction $H = J\vec{S}_1 \cdot \vec{S}_2$, where J is the coupling constant.

A) For two spins in an initial state of $|\text{up}\rangle\ |\text{down}\rangle$, what are the probabilities of finding two spins in $|\text{up}\rangle|\text{up}\rangle$, $|\text{down}\rangle|\text{down}\rangle$ and $|\text{down}\rangle\ |\text{up}\rangle$ states at time t later.

B) Now assume the exchange interaction is modulated periodically, $J = J(t) = J_0 \cos\omega t$. Find out the corresponding probabilities in Part A.

6. Consider N independent spin-1/2 particles at temperature T, each having a magnetic moment μ. An external magnetic field B is further applied.

A) How many microscopic configurations or microstates are there for a given total energy E of the N-spin system?

B) Use Boltzmann statistics to calculate the probability of finding the N-spin system with energy E.

C) Find the most probable E for the N-spin system.

D) Calculate the distribution function of E when N is very, very large.

7. Consider a uniformly charged, solid sphere with mass M of radius R, carrying a total charge Q and spinning with angular velocity ω about the z axis. An external magnetic field $\vec{B} = B_0\hat{x}$ is applied. Calculate the frequency of precession of the spin axis.

8. A long, solid, and well insulated rod has one end attached to a cold temperature reservoir so that the entire rod is initially at the reservoir temperature. The rod is characterized by a length d, a known specific heat C, density ρ, and thermal conductivity κ. If the other end of the rod is suddenly connected to a high temperature reservoir at a temperature that is ΔT higher, then

A. At $t = 0$, find the heat intensity (power per unit area) that flows into the low temperature reservoir.

B. At thermal equilibrium, find the heat intensity that flows into the low temperature reservoir.

C. Do an order of magnitude estimate of the time it takes for this equilibrium to be established.

August 2013 Physics & Astronomy Qualifying Exam
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Day 2: August 30, 2013

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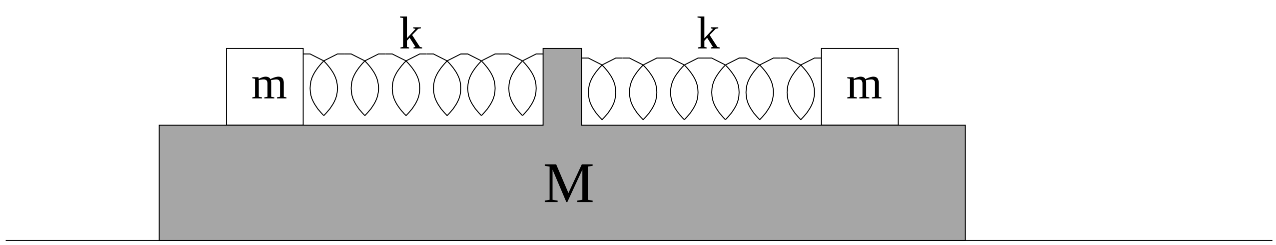
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Sun	R_{sun}	7×10^8 m
specific heat of water	C	4186 J/kg/°C
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N} N^N \sqrt{2\pi N}$

9. The reversible Brayton engine cycle, $A \to B \to C \to D \to A$, consists of two isentropic changes from $A \to B$ and $C \to D$ as well as two isobaric state changes from $B \to C$ and $D \to A$. During the step $D \to A$, the gas absorbs heat Q_{DA}. During the step $B \to C$, the gas expels heat Q_{BC} which can be considered 'wasted'. Let the efficiency of the engine be W_{ABCD}/Q_{DA}, and let the working substance be an ideal gas. Calculate the efficiency of the engine in terms of the gas's heat capacities C_P and C_V and the ratio of pressure at the isobaric steps.

10. A long uniform rope with mass M and length L is suspended vertically above a scale, with the lower end of the rope almost touching the surface of the scale. When the upper end of the rope is released, the rope falls onto the scale's surface. Each segment of the rope comes to rest immediately (ideal non-elastic) as soon as it hits the scale. Sketch the reading on the scale as a function of time, and calculate the maximum scale reading during this process.

11. Consider, as depicted in the figure below, a platform of mass M sitting on a frictionless surface and free to move in the horizontal (right-left) direction. Two identical blocks, both of mass m, are connnected to a post which is fixed to the platform by two identical springs with spring constants k. The blocks are free to move on the platform, also in the horizontal direction and friction at all of the interfaces can be neglected. Find the normal mode frequencies of the system in terms of the constants k, M and m. Describe the nature of the motion of each of the normal modes.



12. High-permeability materials are often used for magnetic shielding applications. The relative permeability of iron is 5000 and the saturation field is 2 T. Do an order of magnitude estimation of the following:

A) Estimate the magnetic field in the center of an iron tube with 2 cm diameter, 30 cm length, and 1 mm wall thickness subject to the Earth's magnetic field (0.05 mT), oriented transverse to the axis of the tube.

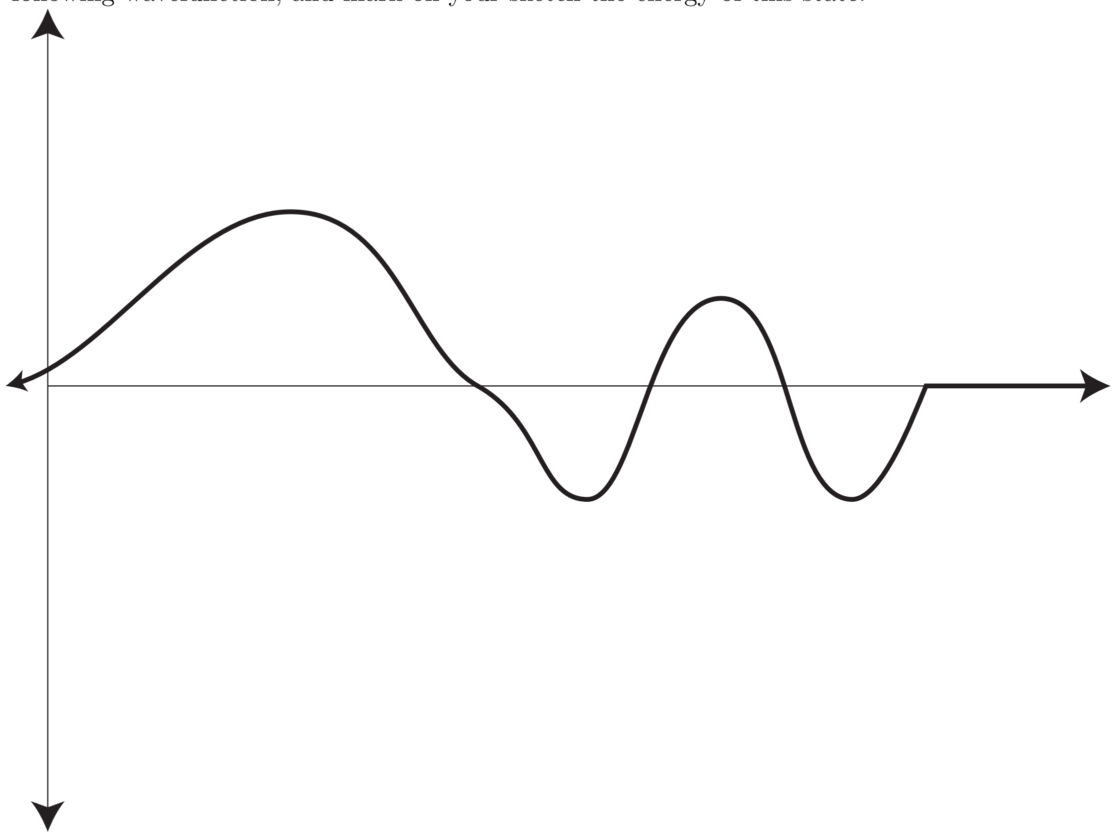
B) An MRI magnet creates a 7 T magnetic field, with a bore the size of a person. You wish to create a magnetic shield that will eliminate stray fields from this magnet beyond a radius of 3 m. Assuming you have available a material of density $\rho = 8$ g/cm^3 with infinitely high relative permeability, but saturation magnetization of 0.5 T, what is the minimum mass of the shield?

13. Liquid helium-3 (an isotope of helium with atomic weight 3) at very low temperature is a good approximation of a Fermi gas. Given that the density of liquid He-3 at 100mK is 0.08 g/cm^3:

A) Would you characterize He-3 at 100 mK as a highly degenerate, weakly degenerate, or classical Fermi gas?

B) Do an order of magnitude estimation of the specific heat of He-3 at this temperature.

14. Sketch a 1D potential V(x) that would have as one of its bound states the following wavefunction, and mark on your sketch the energy of this state:



Explain clearly in words the most important features of your potential and why they must be present.

15. A free electric dipole $\vec{p}$ sits in the middle of a dielectric sphere of radius a, and points in the $+z$ direction. The sphere is a linear isotropic dielectric with dielectric constant $\epsilon > \epsilon_0$. Calculate the potential for $r > a$.

16.

Active galactic nuclei (AGN), presumably powered by supermassive black holes, often eject back-to-back relativistic jets of material. The motion of material in these jets can be measured by observing the passage of lumps of material along the jet using radio telescopes.

Suppose that an AGN is at some large, known distance d from us, with the jet pointing (generally) towards Earth at some angle θ. (You may safely assume that $d \gg$ the size of the AGN+jet.) Let β be the speed of material in the jet, in units of c. Demonstrate that for a range of θ the apparent velocity of motion of the material in the transverse direction can appear to exceed c. Determine the range of θ for which the illusion of such superluminal motion can occur, and calculate the largest possible apparent transverse velocity for a given β.

(Remarkably many such systems have actually been observed. Even more remarkably this effect was actually predicted in advance!)