Spring 2023 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
May 12, 2023
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead. *Note that the medical physics exam is not being offered today.*

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A sphere of mass m and radius r hangs from a massless string of length L. A very light but steady breeze blows in the horizontal direction with speed v_0. The ball experiences a drag force that depends on its velocity relative to the air, the mass density of the air (ρ), the cross-sectional area of the ball, and a dimensionless coefficient of drag C_D.

A. Calculate the angle θ_0 relative to vertical at which this pendulum will hang motionless.

B. Now assume that the sphere is slightly displaced from θ_0 to $\theta_0 + \epsilon$, and then released. Calculate the damped motion $\theta(t)$ in the limit of small ϵ.

2. Consider a 2D quantum mechanical simple harmonic oscillator with potential given by $V(r) = \frac{1}{2}m\omega^2 r^2$. The eigenfunctions of this system, expressed in polar coordinates, can be shown to be of the form $\Psi(r, \theta) = R_{\ell k}(r)e^{i\ell\theta}$, where ℓ is any integer, k is any non-negative integer ($k = 0, 1, 2, 3, ..$), and $R_{\ell k}(r)$ is the radial eigenfunction. The energy eigenvalues are then given by $E = A(Bk + |\ell| + C)$. Determine the values of the coefficients A, B, and C. (Hint: this problem can be solved without solving for the radial eigenfunctions!)

3.

Consider a system of two identical electrons which are confined to move on a circle of radius R. The electrons are charged and R is small enough that their electromagnetic repulsion is important.

The dynamics of the electrons is described by the Hamiltonian

$$H = \frac{|\vec{p}_1|^2}{2m} + \frac{|\vec{p}_2|^2}{2m} + \frac{e^2}{4\pi\epsilon_0|\vec{x}_1 - \vec{x}_2|}$$

where we have assumed that spin-dependent interactions are negligible. Once constrained to the circle, the coordinates are $\vec{x}_i(t) = (R\sin\phi_i(t), R\cos\phi_i(t), 0)$.

In a semi-classical approach to the problem of finding the low energy states of this system we would start by finding a classical solution for the positions of the electrons and study the quantum mechanics of small deviations from that classical solution. Let us assume that this procedure will give an accurate result.

This system (known as the "Wigner molecule") can either have spin 1 or spin 0, depending on how the spins of the electrons are aligned. It will have quantized rotational and vibrational excitations.

Find the rotational and vibrational energy spectra of this molecule as a function of $R, \hbar, e, m$. How do they depend on the spin of the molecule?

Hint: consider reparametrizing the angular positions in terms of $\xi_A = \frac{1}{2}(\phi_1 + \phi_2)$ *and* $\xi_B = \frac{1}{2}(\phi_1 - \phi_2)$.

4. Two stars with equal masses M orbit each other in a circular orbit. The initial distance between the stars at time t_0 is R_0. The system will emit gravitational radiation with a power of

$$L \approx A\frac{M^3}{R^5}$$

where A is a proportionality constant. Calculate the time dependence of the distance between the stars and the time derivative of the orbital frequency as a function of time. You may assume the masses don't change over time.

Spring 2023 Physics Qualifying Exam
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Part 2
May 12, 2023
12:30-14:45 PDT

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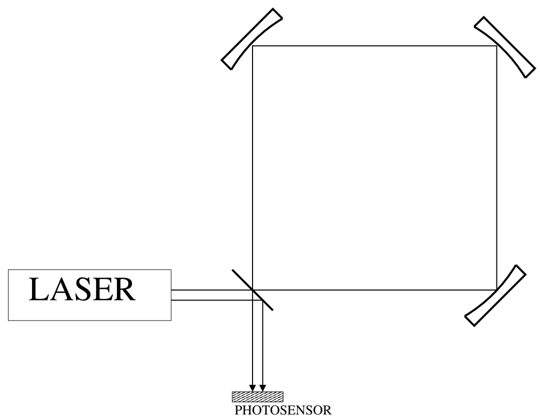
5. Consider an interferometer consisting of a laser with wavelength λ, a beam splitter, and three mirrors, arranged at the four corners of a square of side length L. One beam bounces around the four sides of the square, where it is then combined with light that bounces only off the beamsplitter. Suppose that the setup is tuned so that there is destructive interference between the two beams at the sensor.

The entire apparatus is then sent spinning counterclockwise (in the same direction as the beam) around the centre of the square with a small angular velocity ω. If the power of the laser is P, calculate the power of the detected light at the photosensors to lowest non-zero order in ω.



6.

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j; \quad \sigma_i = \pm 1$$

Consider the Ising spins in a 1D lattice which interact with each other with nearest neighbour interactions of strength J (see above Hamiltonian: i, j are site indices and $\langle ij \rangle$ are two neighbouring sites.) At zero temperature, the ground states are two-fold degenerate, corresponding to either spin-up (+1) or spin-down (-1) at all sites.

A. Now assume there are a total of $L \gg 1$ spins in a 1D chain at a finite temperature T. Evaluate the probability of finding exactly one single domain wall within the 1D spin chain of L spins. A single domain wall is a state where all of the spins point in one direction on one side of the wall and in the opposite direction on the other side. A domain wall can be located at any link connecting two neighbouring spins along the chain. Assume that the domains are not interacting.

B. Now assume that L is infinite. The equilibrium state at low temperatures can be thought as a dilute gas of these domain walls. Estimate the density of domain walls using your results from part A. (You can assume the lattice spacing constant is equal to 1 when calculating the density.)

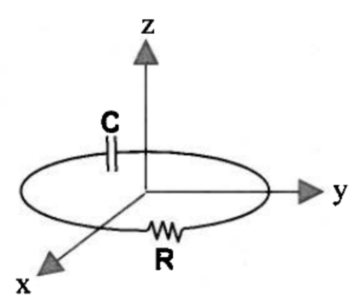
7. A chain of 10^4 identical (1D) harmonic oscillators, each with characteristic frequency ω, is isolated from the environment but weakly connected within the chain. Each harmonic oscillator can exchange energy with its neigbours but it otherwise unaffected by the rest of the chain. Starting from the state with each harmonic oscillator in its ground state, $10^5 * \hbar\omega$ of energy is added to the chain, and the system is allowed to thermalize (within the chain). Then, the energy of each oscillator is measured. You may provide answers that are exact only in the large-number limit.

A. What is the temperature of the chain?

B. What is the most likely energy for a given oscillator to have?

C. What is the probability to find a given oscillator with the most likely energy?

8.



A conducting wire, placed in a magnetic field $\vec{B} = B_0 \sin(\omega t)\hat{z}$, forms a circular loop and lies in the $z = 0$ plane with a radius of 0.1 m. The magnitude of the field is $B_0 = 0.2$ Tesla, and $\omega = 10^3$ rad/s. The wire is connected in series to a 5 Ω resistor and a 100 μF capacitor, as shown. Determine the current in the conducting wire. Justify any approximations you make.