Spring 2022 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
May 13, 2022
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A particle traveling along the positive x axis of reference frame S with speed $0.5c$ decays into two identical particles, $a \to b + b$, both of which continue to travel on the x axis. Given that $m_a = 2.5m_b$, find the speed of either b particle in the rest frame of particle a. Then, by making the necessary transformation on the result, find the velocities of the two b particles in the original frame S.

2. Two identical elastic balls 1 and 2 are initially at two ends of a one-dimensional frictionless track that is 1 m long, and move toward each other with speeds v_1 and v_2 respectively. The balls can collide with each other and the two ends of the track elastically. Under which conditions can both balls reach their initial positions at the same time with their initial speeds? (Hint: draw a plot showing the world lines (position vs. time) of the two balls.)

3. A basketball has a diameter of 24 cm when inflated, and the wall of the basketball is 3 mm thick. When inflated to a gauge pressure of 55 kPa and placed on a scale at sea level, the scale reading is 625 g.

A. What reading would the scale give if all of the air were removed from this basketball? (You may assume that the ball collapses to its minimal possible volume.)

B. The inflated ball is set spinning at 3 rotations per second. What is the quantum limit on how accurately the angle of the spin axis can be aligned to vertical?

4. The electric potential inside a thin spherical shell of radius $r = a$ centred on the origin $(x, y, z) = (0, 0, 0)$ is:

$$V = v_0(x^2 - y^2) + v_1 x$$

A. What is the potential outside of the shell?

B. Find the charge distribution on the thin spherical shell at radius $r = a$.

Fall 2021 Physics Qualifying Exam
for Advancement to Candidacy
Part 2
May 13, 2022
12:30-14:45 PDT

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5. Consider an atom of mass m in a 3D central potential of the form $U = (1/2)kr^2$.

A. Neglecting any internal energy of the atom, what are the first three eigenenergies of the atom in the central potential, E_0, E_1, and E_2?

B. At what temperature T will the probability of finding the atom to have energy E_0 be the same as finding it to have energy E_2? Express your answer in terms of k, m, and k_B.

C. In an atomic gas experiment, atoms at an initial temperature 1 mK are downloaded to a shallow micro-trap which provides a harmonic confining potential $V(r) = (1/2)kr^2$. Afterwards, the micro-trap is adiabatically turned into a much steeper harmonic trap with constant "k" being ten times bigger, i.e. $k \to 10k$. Estimate the final temperature of atoms in the final trap.

6. Einsteinium-252 is a nucleus containing 99 protons and 153 neutrons, and has an atomic mass of 253.085 amu. Its radius is estimated to be 7.9×10^{-15} m, and the electric charge is uniformly distributed throughout the volume. The nucleus's mass consists of four contributions: the rest masses of the nucleons, the kinetic energies of the nucleons, the Coulomb self-energy of the nucleus, and a (negative) nuclear binding energy between the nucleons. Model the motion of the nucleons as non-relativistic Fermi gases protons and neutrons, and calculate each of the four contributions to the atomic mass.

7. An electron is initially prepared in a quantum mechanical state with zero average momentum and a wavefunction given by

$$\psi(x, t = 0) \propto \exp\left[-\frac{x^2}{2a^2}\right]$$

The potential everywhere is zero. A detector is positioned at distance $x = L$ from the electron's initial position (where $L \gg a$).

A. After approximately how long is the electron likely to have passed the detector?

B. Give the probability of observing the electron at $x = L$ as a function of time.

8. The solar constant expresses the intensity of radiation from the sun at the Earths orbit, and is approximately 1400 W/m^2. A "sail" made from a reflective mylar film 1 μm in thickness (the density of mylar is 1400 kg/m^3) is deployed in a spacecraft in a distant orbit around the earth, rotating such that it remains perpendicular to (normal to) the light from the sun at all times. The sail is initially orbiting the sun in the Earth's orbit, but far enough from the Earth that the effects of Earth's gravity may be neglected. What are the radial and tangential components of the sail's velocity when it crosses the orbit of Mars? (The radius of the Earth's orbit is 1.5×10^8 km, while Mars' orbit is at 2.3×10^8 km.)