# Astronomy Comprehensive Exam, Fall 2022

## Session 1

August 26, 2022

**Note: if you are in the PhD in physics program, stop! This is the astronomy version of the exam. Please ask the proctor for the version appropriate for your program instead.**

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer ***any three*** of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours 15 minutes to complete 3 questions.

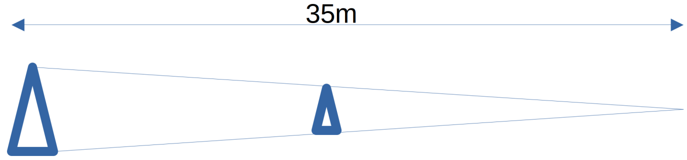
You are allowed to use two $8.5'' \times 11''$ formula sheets (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

| | | |
|---|---|---|
| absolute zero | 0 K | $-273.16°$C |
| atomic mass unit | 1 amu | $1.66 \times 10^{-27}$ kg |
| Avogadro's constant | $N_A$ | $6.02 \times 10^{23}$ |
| Boltzmann's constant | $k_B$ | $1.38 \times 10^{-23}$ J/K |
| charge of an electron | $e$ | $1.6 \times 10^{-19}$ C |
| distance from earth to sun | au | $1.5 \times 10^{11}$ m |
| Laplacian in spherical coordinates | $\nabla^2 f =$ | $\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$ |
| mass of an electron | $m_e$ | 0.511 MeV/c$^2$ |
| mass of hydrogen atom | $m_H$ | $1.674 \times 10^{-27}$ kg |
| mass of a neutron | $m_n$ | $1.675{\times}10^{-27}$ kg |
| mass of a proton | $m_p$ | $1.673{\times}10^{-27}$ kg |
| mass of the sun | $M_\odot$ | $2 \times 10^{30}$ kg |
| molecular weight of $H_2O$ | | 18 |
| Newton's gravitational constant | $G$ | $6.7 \times 10^{-11}$ N m$^2$ kg$^{-2}$ |
| nuclear magneton | $\mu_N$ | $5 \times 10^{-27}$ J/T |
| permittivity of free space | $\epsilon_0$ | $8.9 \times 10^{-12}$ C$^2$ N$^{-1}$/m$^2$ |
| permeability of free space | $\mu_0$ | $4\pi \times 10^{-7}$ N/A$^2$ |
| Planck's constant | $h$ | $6.6 \times 10^{-34}$ J·s |
| radius of the earth | $R_\oplus$ | $6.4 \times 10^6$ m |
| radius of a neutron | $R_n$ | $3 \times 10^{-16}$ m |
| speed of light | $c$ | $3.0 \times 10^8$ m/s |
| Stefan-Boltzmann constant | $\sigma$ | $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ |
| Stirling's approximation | $N!$ | $e^{-N}N^N\sqrt{2\pi N}$ |

1. A wind turbine has three blades, 35 m long, with a mass of 5200 kg each. The blades have a hollow cross section and taper linearly in width (see diagram).

   When the wind is blowing steadily, the blades rotate with a period of 5 s. The wind suddenly stops, and the angular rotation is observed to decrease exponentially with a time constant of 3 s. Assuming that the turbine converts mechanical energy to electric energy with 80% efficiency, estimate the power output of the turbine during the period when the wind was blowing.




2. A closed sphere contains an ideal gas, which has a pressure of $10^5$ Pa at a temperature of 300 K. At what temperature will the radiation pressure pushing outward on the walls of the sphere be equal to the pressure from the gas? Assume that the gas is in thermal equlibrium with the walls, and that the walls (inside and outside) emit as blackbodies.

3. A bead of mass $m$ moves along a circular wire of radius $R$ in zero gravity.

   (a) Treating the problem quantum mechanically, solve for the energy eigenstates of the system.

   (b) If the system is heated to some temperature $T$, calculate the expectation value of the magnitude angular momentum of the system, $\langle |L| \rangle$, in the high-temperature limit.

4. A 1 mm diameter ring is made from superconducting wire that itself is 0.1 mm diameter. The density of the wire is 10000 kg/m$^3$, and the ring carries a circulating current of 1 A. Two 1 m$\times$1 m metal plates are located at $\pm 0.005$ m on the $z$ axis, parallel to the $xy$ plane, and a voltage $V$ is applied between them.

   The ring is initially oriented with its normal vector $\hat{n} = \hat{z}$ parallel to the $z$ axis, and is fired at velocity $\vec{v} = v\hat{x}$ along the $x$ axis so it passes between the capacitor plates, entering at $\{x = -0.5\text{ m}, y = 0, z = 0\}$. When $V = 0$, the ring leaves the capacitor in the same orientation as it began, and still at $y = 0, z = 0$. When $V$ is small but not 0, what are the $y$ and $z$ coordinates of the ring when it leaves the capacitor, and what is the $\hat{n}$? Because $V$ is small, you should express your answer to first order in $V$. Your answer may be approximate, but you should state any approximations you make. You may ignore the influence of gravity entirely. *Hint: what is the B field in the frame of the ring?*

# Astronomy Comprehensive Exam, Fall 2022
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| luminosity of the sun | $L_\odot$ | $3.8 \times 10^{26}$ W |
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| mass of a neutron | $m_n$ | $1.675\times10^{-27}$ kg |
| mass of a proton | $m_p$ | $1.673\times10^{-27}$ kg |
| mass of the earth | $M_\oplus$ | $5.97219 \times 10^{24}$ kg |
| mass of the sun | $M_\odot$ | $2 \times 10^{30}$ kg |
| Newton's gravitational constant | $G$ | $6.7 \times 10^{-11}$ N m$^2$ kg$^{-2}$ |
| parsec | pc | $3.086 \times 10^{16}$ m |
| permittivity of free space | $\epsilon_0$ | $8.9 \times 10^{-12}$ C$^2$ N$^{-1}$/m$^2$ |
| permeability of free space | $\mu_0$ | $4\pi \times 10^{-7}$ N/A$^2$ |
| Planck's constant | $h$ | $6.6 \times 10^{-34}$ J·s |
| radius of the earth | $R_\oplus$ | $6.4 \times 10^6$ m |
| radius of the sun | $R_\odot$ | $7 \times 10^8$ m |
| speed of light | $c$ | $3.0 \times 10^8$ m/s |
| Stefan-Boltzmann constant | $\sigma$ | $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ |
| Thomson cross section | $\sigma_T$ | $6.65 \times 10^{-29}$ m$^2$ |

1. A planet orbits a star of mass $M$ in a circular orbit of radius $r_0$. Suddenly, the star explodes as a supernova, ejecting a fraction $1 - f$ of its mass outward at high velocity and leaving behind a remnant of mass $fM$.

   (a) For what range of $f$ is the planet ejected from the system?

   (b) Suppose instead that the star slowly sheds mass in a stellar wind. It mass at time $t$ is $f(t)M$, where $f(t)$ is intitially 1 and slowly decreases. Find the orbital radius of the planet as a function of $f$.

2. A cloud of neutral hydrogen is observe to have a redshift $z = 2.0$.

   (a) Recall that the ground state of atomic hydrogen is spit into two hyperfine states, corresponding to electron and proton spins parallel and antiparallel, separated by 1420.4 MHz. Transitions between these two state give rise to the 21 cm line. The lifetime of the excited state is $\tau = 3.4 \times 10^{14}$ s.

   Assuming that the cloud is optically thin, and in thermal equilibrium with the cosmic background radiation, estimate the fraction of atoms that will be in the excited state.

   (b) If the cloud has mass $M = 10,000 M_\odot$, estimate its luminosity in the 21-cm line. Assume that the cloud is 75% neutral hydrogen and 25% helium, by mass.

3. Suppose that if the light from a stellar population is dominated by short-lived giant stars and that new star formation has ceased. Show that the luminosity is approximately proportional to $t^{-\gamma}$, with $\gamma \simeq 0.5$. (Hint: If the light is dominated by giants, the luminosity will be proportional to the rate at which stars evolve off the main sequence. Assume a power-law initial mass function $dN/dM \propto M^{-(1+x)}$ with $x \simeq 1.5$.)

4. Two black holes of equal mass $m$ are in circular orbits around their common centre of mass. The separation between the two black holes is $2a$. According to General Relativity, the gravitational-wave luminosity is given by

$$L = \frac{1}{5} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle,$$

   where $i, j = 1, 2, 3$, and a summation is implied over all indices that appear twice in a product. We are using geometrized units in which $G = c = 1$, dots indicate time derivatives, and the brackets indicate an average over the orbit. Here $Q_{ij}$ is the reduced quadrupole moment of the system (the trace-free part of the moment of inertia tensor),

$$Q_{ij} = \sum_{\text{masses}} m \left( x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right).$$

   where $r^2 = x_i x_i$ and $\delta_{ij}$ equals 1 if $i = j$ and 0 otherwise.

   Find an expression for the luminosity in terms of $m$ and $a$, putting in the missing factors of $G$ and $c$.