Autumn 2020 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
September 3, 2020
12:30-14:45 PDT

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded.

You have 2.25 hours to complete 3 questions. **This is a closed book exam. During the exam, you shall not use Google or other online/off-line resources, other than your own formula sheet. At the end of the exam, please scan and upload your papers to the corresponding Assignment on Canvas. Your paper should be uploaded by 15:00 PDT. In case of emergency you can email your work to feizhou@phas.ubc.ca.**

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2 f =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
molecular weight of N_2		28
molecular weight of O_2		32
weight of Helium atom He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^{6} m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

Session One: 12:30 –14:45pm, Sept 3, 2020

1. Just as motion of charged particles can result in radiation, motion of mass can lead to gravitational radiation. The power L emitted in the form of gravitational radiation by a mass distribution depends on the third time derivative of the quadrupole moment of its mass distribution:

$$L = A \left\langle \sum_{j,k} \left(\frac{d^3 Q_{jk}}{dt^3} \right)^2 \right\rangle .$$

Here the sum is over elements of the quadrupole moment, the brackets indicate an average over many periods of rotation, and A is a proportionality constant that can be written in terms of fundamental constants.

A. Use dimensional analysis to produce an order of magnitude estimate of the value of the constant A.

B. Consider a rotating neutron star with radius 10 km and rotational frequency of 100 Hz. If the neutron star is perfectly spherical then its quadrupole moment is constant and no radiation occurs. Suppose however that there is a bulge on its equator with a height of 10 cm, covering an area of 100 m $\times$ 100 m on the surface. Assuming that the material in the bulge has nuclear density, estimate the bulge's mass, and then do an order of magnitude estimate of the power radiated as gravitational waves by this star as it rotates.

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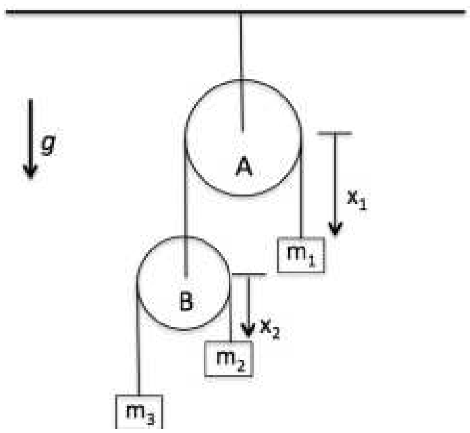
2. An atom is trapped in the ground state of a 3D harmonic potential $V_c(\mathbf{r}) = \frac{1}{2} m\omega^2 r^2$ where ω is the trap frequency and m is the mass. In modern laboratories, such confining potentials can be easily induced by lasers.

(a) Now an additional potential is adiabatically turned on so that effectively $V_a(\mathbf{r}) = \infty$ for the half space of $z > 0$. How much work has been done on the *atom* during this adiabatic switching on?

(b) After keeping $V_{tot} = V_c + V_a$ for a while, one decides to suddenly switch off all the potentials to release the particle into free space. How much work has been done on the *atom* after the potentials are off?



3. Consider the rope and pulley system that is depicted in the figure. Assume that the pulleys and ropes are ideal, that is, in particular, they are weightless and frictionless and the ropes are infinitely flexible and do not stretch. Assume that at some time the system is released from rest, in the configuration shown. Find the relationship between the masses m_1, m_2, and m_3 so that, when the system is released, the mass m_1 starts moving downward. Does its acceleration increase, decrease or stay constant with time?

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4. In answering the questions below, make reasonable estimates for any needed parameters/quantities not already provided. Consider an idealized Sun and Earth as blackbodies in otherwise empty space. The Sun has a surface temperature $T_S = 6000K$, and heat transfer processes on the Earth are effective enough to keep the Earth's surface temperature uniform. The radius of the Earth is $R_E = 6.4 \times 10^6$ m, the radius of the Sun is $R_S = 7.0 \times 10^8$ m, and the Earth-Sun distance is $d = 1.5 \times 10^{11}$ m. The mass of the Sun is $M_S = 2.0 \times 10^{30}$ kg.

(a) Find the temperature of the Earth.

(b) Find the radiation force on the Earth.

(c) Compare these results with those for an interplanetary granule in the form of a spherical, perfectly black body with a radius $R = 0.1$ cm, moving in a circular orbit around the Sun at a radius equal to the Earth-Sun distance d.

(d) For what size particle would the radiation force calculated in part (c) be equal to the gravitational force from the Sun at a distance d?

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Note: if you are in the PhD in astronomy program, stop! This is the physics version of the exam. Please download the astronomy version instead.

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Session Two: 15:45pm–18:00pm, Sept 3, 2020

5. A spin-1/2 particle carries a known magnetic moment $\frac{1}{2}\mu_0$ and its coupling with a magnetic field $\mathbf{B}$ is $\frac{1}{2}\sigma \cdot \mathbf{B}\mu_0$, where $\sigma_\alpha, \alpha = x, y, z$ are three Pauli matrices. The spin is initially pointing along the z (i.e. $\mathbf{e}_z$) direction. (A spin state $|\mathbf{n}\rangle$ pointing along a direction specified by a unit vector $\hat{\mathbf{n}}$ is defined as $\frac{\hbar}{2}\sigma \cdot \hat{\mathbf{n}}|\mathbf{n}\rangle = +\frac{\hbar}{2}|\mathbf{n}\rangle$, i.e. with spin projection along the direction $\hat{\mathbf{n}}$ being $+\hbar/2$.).

(a) Now you need to prepare a new spin state pointing along the direction of $\mathbf{e}_x$ by applying a constant magnetic field $B_1\mathbf{e}_y$ along the y-direction over duration t_0 (again $\mathbf{e}_{x(y)}$ is the unit vector along the $x(y)$-direction). Find the magnitude of B_1 as a function of pulse duration t_0 required to achieve the desired spin state.

(b) Now assume you have a pair of spins in a triplet state $\frac{1}{\sqrt{2}}[|\mathbf{e}_z\rangle_1|-\mathbf{e}_z\rangle_2 + |-\mathbf{e}_z\rangle_1|\mathbf{e}_z\rangle_2]$ where $|\pm\mathbf{e}_z\rangle$ are spin states pointing along the direction of $\pm\mathbf{e}_z$, respectively, and the indices $1, 2$ label two spins in the pair. You further apply a pulse of $B_2\mathbf{e}_y$, a constant magnetic field along the y-direction over a duration of t_0 where $B_2 = \frac{\pi\hbar}{\mu_0 t_0}$. What is the final spin state for this pair? Express your results in terms of $|\pm\mathbf{e}_z\rangle_{1,2}$.

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6. A capacitor consists of two coaxial conducting cylindrical tubes of length L, with inner and outer radii of R & 2R respectively. Assume L is large so that fringe effects are negligible. The region between the conductors is filled with an inhomogenous linear dielectric for which the permittivity ϵ is independent of the distance s from the axis but varies as a function of position z along the axis and the azimuthal angle ϕ, i.e. $\epsilon = \epsilon(\phi, z)$. There are no free charges inside the dielectric. The centre of the cylinder is at the origin of the coordinate system.

(a) Write down the differential equation for the potential V inside the dielectric. Show that a potential that depends only on s, $V = V(s)$ satisfies the differential equation and find the most general form.

(b) With the centre of the capacitor at the origin of the coordinate, if the permittivity of dielectric varies as

$$\epsilon(\phi, z) = \epsilon_0[1 + \cos(\frac{\pi z}{L}) \cos^2 \phi], \tag{1}$$

calculate the capacitance C.

7. A ball of mass m is released from rest and drops from a height of h onto a hard surface, which it bounces off with perfect elasticity. Losing no energy, the ball bounces forever. Ignore possible sideways movement of the ball, and model the "bounce" as happening instantaneously (i.e. the impulse imparted to the ball during the bounce is a delta function).

A. At random times an observer records the height z of the ball above the surface. Calculate the probability distribution for the recorded height of the ball, $P(z)$, assuming classical mechanics, and draw a graph of this probability as a function of z.

B. Now consider this system quantum mechanically. Suppose the ball is in its lowest energy state, with total energy E_0. Sketch qualitatively the probability of finding the ball at some height, as a function of z. Label the z axis clearly, indicating any special values, and sketch as well the classical probability distribution from Part A on the same graph for comparison.

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8. Consider a 1D chain consisting of $N(\gg 1)$ segments as illustrated in the sketch. Let the length of each segment be a when the long dimension is parallel to the chain (or horizontal) and zero when the segment is vertical. Each segment has just these two states, horizontal and vertical. The distance between the chain ends is fixed.

(a) For a given length $L = Nl$ $(0 < l < a)$ of the chain, what is the total number of microstates accessible by the system and what is the entropy of the system as a function of l?

(b) Apply the first law of thermodynamics to the system and obtain an expression for tension force F that is needed to maintain the length Nl (assuming the joints turn freely), using your result from Part (a).

(c) Under which conditions does your answer lead to Hooke's Law, and what is corresponding expression for the spring constant?