PhD(Astro) Qualifying examination - 2015

13:30 - 17:30, 4 September 2015

Do not open the exam until instructed to do so
but you may read this cover sheet

Instructions:

A one-page (8.5 $\times$ 11 inch) hand-written double-sided sheet of notes is allowed.

A scientific calculator is allowed and expected.

Do not write your name on the exam. Instead, write your student number on each exam booklet. This will allow us to grade the exams anonymously. Well match your name with your student number after we finish grading. If you use extra exam booklets, write your student number on the extra exam books as well.

All answers must be written in the exam books.

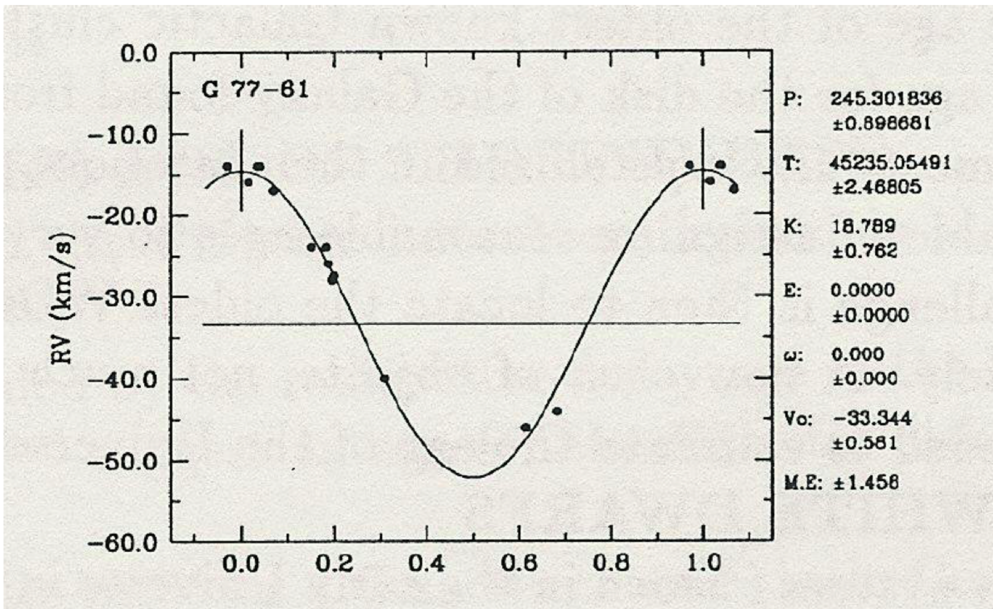
Start every question on a new page.

There are 8 questions to choose from. You will only select and provide answers for 5 of them; you may not attempt any portion of the other 3 questions. The five questions you choose all have equal value, and thus you may wish to time budget about 45 minutes per question.

On the front of your exam booklet you should clearly indicate which 5 questions you wish graded if you have any writing in your answers for more than 5 questions. If not, the first 5 questions started in your exam booklets will be graded.

Please return this examination with your exam booklet.

1. [20 points] The figure below shows a radial-velocity diagram for a binary star system, where only one of the two stars is visible (a 0.8 solar mass main-sequence star). K is the velocity amplitude and Vo its mean line-of-sight value (both in km/s). The invisible companion is a 0.2 solar-mass white dwarf star. The Period P and Julian Day of zero-longitude pass are given in days for this assumed circular, edge-on orbit. Assuming the main-sequence star (with current radius $\simeq$1 solar radius $R_{\odot} = 700,000$ km $= 0.005$ AU) is about to evolve into a red giant, roughly how many times its current radius must it expand to before you expect it to begin to deliver material to the white dwarf, potentially generating a nova or supernova?



2. [20 points] A supernova explosion ejects $E_0 = 10^{51}$ ergs of energy into the interstellar medium, which has number density $n_0 = 1\,\mathrm{cm}^{-3}$ (assume this is pure hydrogen). Assume that the gas density and and temperature are uniform throughout the remnant and equal to their post-shock values. Deal only with the adiabatic expansion phase of the SNR, during which $R = (25E_0/3\pi\rho_0)^{1/5}t^{2/5}$ (that is, neglect the initial free-expansion phase of SNR development).

   (a) Derive a value for the undisturbed ISM mass density ρ_0. The post-shock density is given by $\rho_d = 4\rho_0$. Assuming the post-shock gas is fully ionized, find a value for the downstream number electron and ion number densities $n_{e,d} = n_{i,d}$.

   (b) The post-shock temperature T_d is given by $3\mu m_p V_s^2/16k_{\mathrm{B}}$ and here $V_s = \dot{R}$. Find an expression for T_d in terms of time t. (Assume $\mu = 0.5$ for ionized H.)

   (c) Find an expression for the volume V of the remnant as a function of time t.

   (d) The power per unit volume emitted in bremsstrahlung radiation is given by the expression $dE/dtdV = 1.4 \times 10^{-27} T_d^{1/2} n_{e,d} n_{i,d} \bar{g}_{\mathrm{B}} Z^2\,\mathrm{erg\,s^{-1}\,cm^{-3}}$, where $\bar{g}_{\mathrm{B}} \simeq 1.2$ and here $Z = 1$. Use your results from the first three parts to derive an expression for the emitted power dE/dt as a function of time t.

   (e) Integrate this to find the fraction of the original explosion energy E_0 that has been radiated away by bremsstrahlung after 10000 years. Based on this, would you consider the assumption of adiabatic expansion (no energy radiated away from the SNR) to be acceptable?

3. In this problem you need to show that the dry adiabatic lapse rate for a geometrically thin, hydrostatic atmosphere is $\alpha_{\rm ad} = g/c_p$. Here, g is the magnitude of the surface gravitational acceleration and c_p is the atmosphere's specific heat at constant pressure. You will also need to use your result to evaluate $\alpha_{\rm ad}$ for given conditions, and then answer conceptual questions related to the problem. Recall that the lapse rate $\alpha = -dT/dz$ for temperature T and altitude z. Throughout the problem, assume that the gas is perfect (and ideal by extension).

   (a) [5 points] Show that $\alpha = A\,(d\ln T/d\ln P)$, where A is a combination of constants that are appropriate for the problem. Hint: Use hydrostatic equilibrium.

   (b) [5 points] Now demonstrate that $(d\ln T/d\ln P)|_{\rm ad} = (\gamma - 1)/\gamma$. Hints: The adiabatic index γ is the ratio of specific heats. Because you are calculating the dry adiabatic lapse rate, you should consider the case of a perfect gas that follows a given adiabat.

   (c) [5 points] Combine the above relations to show $\alpha_{\rm ad} = g/c_p$. Hint: Recall that $c_p - c_v = \mathcal{R}/\mu$ for gas constant $\mathcal{R}$ and mean weight μ.

   (d) [5 points] If the Earth's atmosphere were made entirely of CO_2, what would be the corresponding dry adiabatic lapse rate? Give your answer in K/km. Justify your approximations.

4. [20 points] Derive the density structure of a star $\rho(r)$ assuming hydrostatic equilibrium and an equation of state

$$P = K\rho^2$$

   where P is the pressure and K is a constant.

5. (a) [10 points] Suppose that galaxies are randomly distributed in space (i.e. with no clustering), with an average number density n. What is the probability that a randomly-placed spherical volume V will contain no galaxies? How would you expect galaxy clustering to affect the probability?

   (b) [10 points] The Andromeda galaxy (M31) is observed to have a redshift of $z = -0.0010$ (in other words, a blueshift). Its present distance is estimated to be 778 kpc. Assuming that M31 is falling towards us for the first time, find a lower limit to the combined mass of our galaxy and M31. (Hint: apply Kepler's third law)

6. In this problem assume that there is some unknown number N_ν of neutrino (lepton) flavours. You can assume that all the neutrinos are massless and that they decoupled from all other particles when the radiation temperature cooled to 1.5 MeV.

   (a) [10 points] At temperatures below 0.511 MeV (after electron-positron annihilation was complete), the neutrino and photon temperatures settled into a ratio of

$$T_\nu/T_\gamma = (4/11)^{1/3}$$

   Explain and show by calculation the origin of this ratio.
   Why is this result only approximately correct?

   (b) [10 points] The energy density of the photons alone is given by

$$\rho_\gamma = a_{bb}T^4,$$

   where $a_{bb} = \pi^2 k^4/15c^3\hbar^3$ is the black-body constant.

With a radiation fluid that is composed of only photons and neutrinos after electron-positron annihilation, the total radiation energy density can be written as a factor $1+F$ times that of the photons alone,

$$\rho_{rad} = \rho_\gamma + \rho_\nu = a_{bb}T^4(1+F).$$

Using youre knowledge of the Fermi-Dirac and Bose-Einstein distribution functions, and incorporating the (in principle unknown) number of neutrino flavours N_ν, derive an expression for F.

7. (a) [10 points] In the theory of optical propagation through turbulence one commonly encounters exponentials of fluctuating quantities. Suppose that x is a Gaussian random variable with mean μ and standard deviation σ. Show that the mean value of $\exp(x)$ is $\langle \exp(x) \rangle = \exp(\mu + \sigma^2/2)$.

   (b) [10 points] In isotropic scattering, the incident photon is equally likely to be scattered into any direction whatsoever. Determine the probability distribution of the angle θ between the incident and scattered photons. Calculate the mean value of θ.

8. (a) [10 points] A distant star-forming galaxy with a redshift $z = 2.5$ is observed to have an AB magnitude of 22.5 at a wavelength of 8750 Å. Determine the specific flux in Jy and also in erg s^{-1} cm^{-2} Å^{-1}. (Hint: $m_{AB} = 0$ corresponds to a flux of 3631 Jy).

   (b) [10 points] For a diffraction limited telescope having aperture diameter D and effective focal length f, observing at a wavelength λ, what is the largest pixel size (centre-to-centre pixel spacing in the telescope focal plane) that can be used without aliasing? Explain.