Spring 2021 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
April 29, 2021
12:30-14:45 PDT

If you are in the PhD in astronomy program, stop! This is the physics version of the exam. Please download the astronomy version instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2 f =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

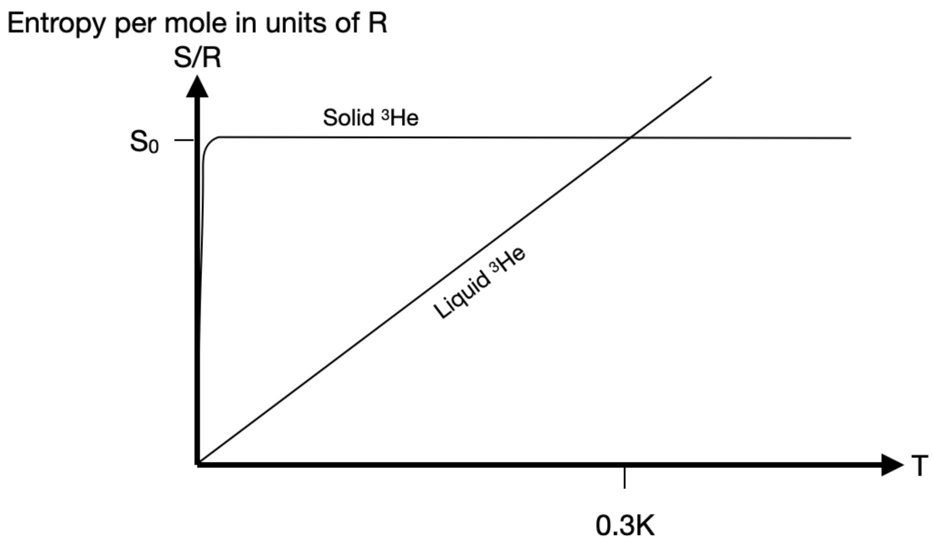
1. He-3 atoms are fermions, and at ultra-low temperatures their solid and liquid phases give experimental access to a wide variety of thermodynamic states. In liquid form, ^{3}He atoms form a Fermi liquid, whose entropy well below the Fermi temperature is linear in T. In solid form, ^{3}He atoms are described thermodynamically just as a collection of weakly interacting and disordered nuclear spins (spin 1/2), down to extremely low temperatures where the spins order and the entropy drops to zero. The result is the graph below, showing the entropy of solid and liquid He-3 at very low temperature. The density of liquid ^{3}He is 0.114 g/cm^3, and that of solid ^{3}He is 0.12 g/cm^3 (both nearly temperature independent).

A. What is S_0, the entropy of a mole of solid ^{3}He at all but the lowest temperatures, treating solid ^{3}He as an ensemble of weakly interacting spins?

B. Starting from a liquid state at 0.1 K, a sample of ^{3}He is compressed adiabatically to a solid. Draw the path of that process on the graph below, and label it as "Path 1".

C. Starting from a liquid state at 0.1 K, a sample of ^{3}He is compressed isothermally to a solid. Draw the path of that process on the graph, labelling it "Path 2".

D. Calculate the heat absorbed or expelled in Path 1, and in Path 2. Specify whether the heat is absorbed or expelled by the ^{3}He in the process.



2. Einsteinium-253 (atomic mass 253.085 amu) decays to berkelium-249 (atomic mass 249.075 amu) and helium-4 (atomic mass 4.003 amu), with a half-life of 20.5 days. Its density is 8.8 g/cm^3. Consider a sphere of this isotope with a mass of 1 mg. Assuming that all of the heat produced by radioactive decay is radiated from the surface as black body radiation, calculate the surface temperature of the sphere. If the thermal conductivity of the material is 10 W/(m·K), calculate the temperature in the center of the sphere. (You may assume that all of the decay products stop inside the sphere, and that none escape.)

3.

Two spin-1/2 particles, with spins $\vec{s}_1$ and $\vec{s}_2$, interact by an exchange interaction $H = J(s_{1x}s_{2x} + s_{1y}s_{2y})$, where $J > 0$.

A. What are the ground state and excited states of the system? Show the whole spectrum and spin wave functions (in terms of the up-down spin states of each spin).

B. Now applying a Zeeman magnetic field B along the z-direction. Find the spin polarization as a function of the B field. Spin polarization is defined as $\langle s_{1z} + s_{2z} \rangle$ averaged over the spin ground state in the presence of the B field.

4. An H^+ ion is placed in a linear trap, that is, where trapping potentials along the y and z axes are much tighter than along the x axis so the ion is effectively confined to one dimension. The trapping potential along x is $V = ax^2$, where a has units of V/m^2.

A. What is the value of a (in V/m^2) such that the ground state wavefunction for the H^+ ion is spread out by approximately 3 mm along the x axis?

B. Sketch the 8th and 9th excited state wavefunctions $\psi(x)$ for the H^+ ion. No need to draw these exactly, but your goal is to get the rough shape right. Aspects your drawing should highlight:

- Approximate location of nodes and antinodes
- Curvature of wavefunction (concave vs convex) at different locations along x

Feel free to describe in words any aspects of the wavefunction your drawing is trying to capture.

C. The H^+ ion is put into a superposition of the 8th and 9th excited states. As time moves forward, the wavefunction will precess and the probability density will oscillate. What is the period of the oscillation?

Spring 2021 Physics Qualifying Exam
for Advancement to Candidacy
Part 2
April 29, 2021
15:45-18:00 PDT

If you are in the PhD in astronomy program, stop! This is the physics version of the exam. Please download the astronomy version instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

You are allowed to use one 8.5″ × 11″ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2 f =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^{6} m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

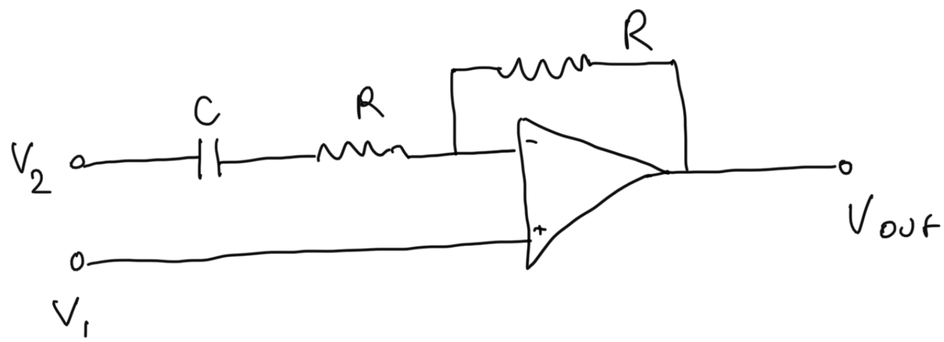
5. Two rockets travel in open space along the x axis, far from any gravitational fields, with identical speeds and constant acceleration a. At $t = 0$, the first rocket is at $x = 0$ and the second rocket is ahead of it at $x = L$, as measured in the rest frame of the rockets. (You may assume that $aL \ll c^2$.)

A. The first rocket fires a laser at the rocket in front of it. The laser's wavelength, measured in the first rocket's rest frame, is λ. What is the wavelength of the laser light that the second rocket detects, as measured in its rest frame?

B. Consider a coordinate system in which the rockets are moving at speed v at $t' = 0$, where the coordinate systems in the moving frame and the rockets' rest frame coincide at $t = t' = 0, x = x' = 0$. What is the distance between the two rockets as measured in the second (primed) coordinate system?

C. Einstein's equivalence principle says that a constant acceleration cannot be distinguished from a constant gravitational field. Apply this principle to your result from part A to calculate the gravitational redshift in wavelength of a 500 nm laser fired from the bottom of a 100 m tall tower as measured by a sensor at its summit.

6. A standard operational amplifier (op-amp) is connected up with the following circuit. Recall that the voltage output of an op-amp is a huge factor (you can assume it to be infinity) times the voltage difference between the input terminals. The inputs of an op-amp present a very large (essentially infinite) impedance so that no current flows through them into the op-amp. Assume the op-amp is properly powered, with large positive and negative supply voltages. V_2 is held at +1 V with respect to ground, and a sine signal is applied to V_1 with respect to ground: $V_1 = 0.001V\sin(\omega t)$. What is V_{out} in the limit that $\omega \ll 1/RC$? What is it in the limit that $\omega \gg 1/RC$?



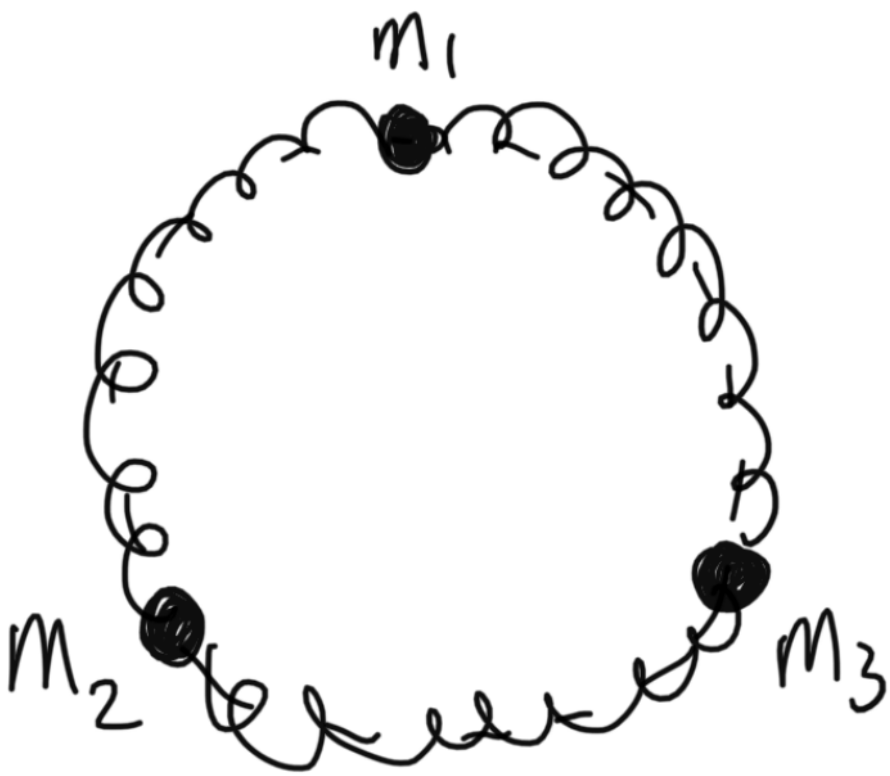
7.

Three identical point masses m are constrained to move on a circle, as shown in the figure below. The masses are connected with identical springs each with spring constant k, that obey Hookes Law. There is no friction, gravity or motion outside the circle.

A. Find all the natural or characteristic frequencies of oscillation for this system.

B. Solve for the normal modes, labelling each with a Roman numeral (I, II, III, ...) then sketch or describe in words the motion for each mode, and list what its frequency will be.

C. Suppose mass #1 is displaced slightly to the left (by dx) at $t = 0$, but none of the masses are moving and the other two are not displaced. At $t = 0$, mass #1 is released. Decompose the initial displacements at $t = 0$ into the normal modes you found in Part B, then solve for the positions of each mass for $t > 0$.



8. Consider an extremely thin sheet of oscillating electric current, lying in the x-y plane. Suppose that this electric current density is given by

$$\vec{J}(x, y, t) = \hat{e}_x A \delta(z) \cos(kx - \omega t)$$

(where $\hat{e}_x$ is a unit vector in the x-direction).

Show that there must also be an electric charge density and find an explicit expression for it. Find expressions for the electric and magnetic fields everywhere. Calculate the magnitude and direction of any energy radiated by this system.