Fall 2021 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
September 1, 2021
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please download the version appropriate for your program instead.

**Do not write your name on your exam papers.** Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

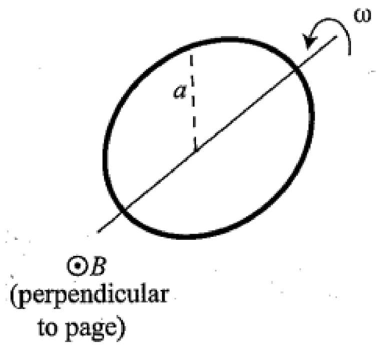
You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

| | | |
|---|---|---|
| absolute zero | 0 K | -273°C |
| atomic mass unit | 1 amu | $1.66 \times 10^{-27}$ kg |
| Avogadro's constant | $N_A$ | $6.02 \times 10^{23}$ |
| Boltzmann's constant | $k_B$ | $1.38 \times 10^{-23}$ J/K |
| charge of an electron | $e$ | $1.6 \times 10^{-19}$ C |
| distance from earth to sun | 1 AU | $1.5 \times 10^{11}$ m |
| Laplacian in spherical coordinates | $\nabla^2\psi =$ | $\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$ |
| mass of an electron | $m_e$ | 0.511 MeV/c$^2$ |
| mass of hydrogen atom | $m_H$ | $1.674 \times 10^{-27}$ kg |
| mass of a neutron | $m_n$ | $1.675\times10^{-27}$ kg |
| mass of a proton | $m_p$ | $1.673\times10^{-27}$ kg |
| mass of the sun | $M_{sun}$ | $2 \times 10^{30}$ kg |
| molecular weight of $H_2O$ | | 18 |
| Newton's gravitational constant | $G$ | $6.7 \times 10^{-11}$ N m$^2$ kg$^{-2}$ |
| nuclear magneton | $\mu_N$ | $5 \times 10^{-27}$ J/T |
| permittivity of free space | $\epsilon_0$ | $8.9 \times 10^{-12}$ C$^2$ N$^{-1}$/m$^2$ |
| permeability of free space | $\mu_0$ | $4\pi \times 10^{-7}$ N/A$^2$ |
| Planck's constant | $h$ | $6.6 \times 10^{-34}$ J·s |
| radius of the Earth | $R_{earth}$ | $6.4 \times 10^6$ m |
| radius of a neutron | $R_{neutron}$ | $3 \times 10^{-16}$ m |
| speed of light | $c$ | $3.0 \times 10^8$ m/s |
| Stefan-Boltzmann constant | $\sigma$ | $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ |
| Stirling's approximation | $N!$ | $e^{-N}N^N\sqrt{2\pi N}$ |

1.

A conducting loop of radius $a$, resistance $R$, and moment of inertia $I$ is rotating around an axis in the plane of the loop, initially at an angular frequency $\omega_0$. A uniform static magnetic field $B$ is applied perpendicular to the rotation axis (see figure). (a) Calculate the rate of kinetic energy dissipation, assuming it all goes into Joule heating of the loop resistance. (b) In the limit that the change in energy per cycle is small, derive the time dependence of the angular velocity $\omega$. In particular, how long will it take for $\omega$ to fall to $\frac{1}{e}$ of its initial value? You may ignore any effects relating to self-inductance.



2. There is a theorem, called Earnshaw's theorem, which states that it is not possible to build a device which levitates an electric charge in a gravitational field in vacuum by using static electric fields alone. However you configure the fields, if the fields are non-zero, the system is at best meta-stable. Find a proof of Earnshaw's theorem. Assume the simplest case of equilibrium of a single point charge.

3. Consider an equilibrium magnetic system in fixed magnetic field $B = 0$. The free energy $G(m, T)$ of the system as a function of magnetization density $m$ can be written as:

$$G(m, T) = a + \frac{b}{2}m^2 + \frac{c}{4}m^4 + \frac{d}{6}m^6$$

In some relevant range of temperatures $T$ , the coefficients $b$ and $d$ can be taken to be positive constants, $b, d > 0$, while $c$ depends on the temperature and it goes through 0 at some temperature $T^*$ in this range,

$$c(T) = c_0(T - T^*) \ , \quad c_0 > 0$$

In this regime of temperatures, the free energy $G$ describes a phase transition, in which the system transitions from the state with no magnetization, $m = 0$, to the magnetized state with $m = m_0 \neq 0$ at some temperature $T_0$. Find the temperature $T_0$ at which the phase transition occurs. What is the order of this phase transition? Find the magnitude of the magnetization $m_0$ appearing at the transition temperature $T_0$. Calculate the latent heat of the transition.

4. A flat plate with area $A$ and negligible thickness sits inside an classical ideal gas with pressure $P$, temperature $T$, and molecular mass $m$. Calculate the rate at which molecules strike the plate. You may ignore edge effects. Do a full exact calculation, not just an order of magnitude estimate.

Fall 2021 Physics Qualifying Exam
for Advancement to Candidacy
Part 2
September 1, 2021
12:30-14:45 PDT

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5. Neutrons experience an attractive interaction described by a potential energy $V(r)$, where $r$ is the distance between the neutrons. An approximate form for $V(r)$ is:

$$V(r) = \begin{cases} +\infty & r < 0.5 \text{ fm} \\ -V_0 & 0.5 < r < 3 \text{ fm} \\ 0 & 3 \text{ fm} < r \end{cases}$$

Here $V_0$ is a positive constant, and 1 fm$= 10^{-15}$ m. Determine the minimum value of $V_0$, in MeV, for which a bound state with orbital angular momentum $\ell = 0$ exists. If $V_0$ is large enough that such a bound state exists, what is the *total* angular momentum of the system, and why?

Hint: the Laplacian operator in spherical coordinates is given on the first page of this exam. The substitution $u(r) = r\psi(r)$ can simplify the problem considerably.

6. Consider a particle of mass $m$ trapped between two walls. We'll solve this problem in 1D, so the two walls effectively make an 1D infinite square well potential confining the particle. The distance between the walls is $D$.

A. What is the quantum mechanical energy of the particle in the $n^{th}$ energy level?

B. Using your answer to Part A, what is the force on the walls when the particle is in the $n^{th}$ level?

C. What is the average force on the walls due to a classical particle, whose kinetic energy matches your answer to Part A, bouncing back and forth between the walls? (Again assume this is a 1D problem.)

7. A comet is spotted in an orbit around the sun. When spotted, the comet is $10^{11}$ m from the sun, heading 20 degrees away from the direct path to the sun at a speed of $10^4$ m/s. What is the closest approach of the comet to the sun in its orbit?

8. A car begins moving on a horizontal road, with a door accidentally left open with an initial angle $\phi_0$ (where $\phi = 0$ indicates the door is closed). The motion of the car is described by known function $x(t)$. The rectangular door has a mass $m$, length $L$, negligible thickness and uniform mass density, and is allowed to rotate a full 360-degree turn around its vertical hinge. You may ignore air resistance in this problem.

A. Find a differential equation for the angle of the door with the car, in terms of the known $x(t)$ of the car.

B. If the door is allowed to swing into the car (remember, the hinge allows full rotation) and the car is moving with constant acceleration, the door may oscillate about an equilibrium position. For small oscillations, what is the frequency of oscillation?