Astronomy Comprehensive Exam, Fall 2023
Session 1

Aug 25, 2023

Note: if you are in the PhD in physics program, stop! This is the astronomy version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer ***any three*** of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours 15 minutes to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.5110 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.673×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. (a) Find the temperature dependence of the energy density associated with photons at temperature T in 3D space, assuming equilibrium. (You need the temperature dependence and the dependence on the speed of light; the dimensionless prefactor doesn't play much role below).

   (b) For photons, the entropy density can be directly related to the energy density from part A. Using dimensional analysis, can you find the temperature and speed dependence of the entropy as well?

   (c) In complex quantum systems, photon-like excitations can emerge, which propagate with a tunable "speed of light" v. If this "speed of light" adiabatically changes from v to $2v$, what is the change of the emergent photon temperature in this case? (You may assume that the excitations have a thermal spectrum.)

2. Order of magnitude estimation:

   (a) Estimate the total energy stored in ocean waves on the surface of the earth at any moment in time.

   (b) Estimate how close (in orders of magnitude) a modern-day computer processor is to fundamental thermodynamic limits on energy efficiency.

3. A spring of uniform stiffness and composition has a mass of m and an unstretched length of L. Its spring constant is measured to be some value k. The spring is allowed to hang under its own weight. Let f denote the fraction of the spring's mass that is below some location x, measured from the lower end of the spring. Calculate $x(f)$. What is the total length of the spring when hanging under its own weight, i.e. what is $x(1)$?

4. A particle moves in a potential well given by

$$V(x) = (Ax^2 - B)\exp[-\alpha x^2],$$

   where A, B, and α are positive constants. At time $t = 0$ the particle is at $x = 0$ and has velocity v_0 in the $+x$ direction. For the following, treat the problem relativistically.

   (a) For what range of v_0 will the particle escape to infinity?

   (b) If the particle escapes to infinity, what velocity will it have in the limit that $t \to \infty$?

   (c) Suppose that a laser beam with power P, measured in the lab frame, is applied in the $-x$ direction. What power does the particle measure in its rest frame at $t = 0$?

Astronomy Comprehensive Exam, Spring 2023

Session 2

Aug 25, 2023

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Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

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You have 2 hours 15 minutes to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

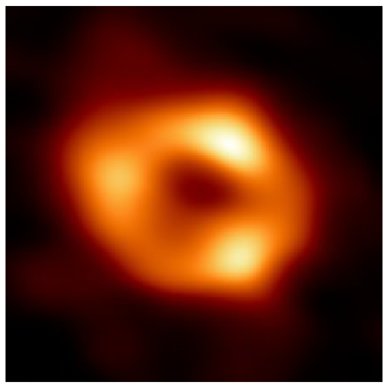
absolute zero	0 K	$-273.16°$C
astronomical unit	au	$149,597,870.7$ km
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
luminosity of the sun	$L_\odot$	3.8×10^{26} W
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	$1.675{\times}10^{-27}$ kg
mass of a proton	m_p	$1.673{\times}10^{-27}$ kg
mass of the earth	$M_\oplus$	5.97219×10^{24} kg
mass of the sun	$M_\odot$	2×10^{30} kg
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
parsec	pc	3.086×10^{16} m
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the earth	$R_\oplus$	$6,371.0$ km
radius of Jupiter	$R_{♃}$	$69,911$ km
radius of the sun	$R_\odot$	$696,342$ km
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Thomson cross section	σ_T	6.65×10^{-29} m^2

1. The Event Horizon Telescope (EHT) employs very-long baseline interferometry (VLBI), using radio telescopes located all over the earth. The EHT observes at mm wavelengths using radio receivers that have a typical bandwidth on the order of 10 GHz.

   (a) The EHT recently observed the black hole at the centre of our galaxy, at a wavelength of 1.3 mm. The resulting image is shown below. Approximately what angular resolution (in microarcsec) can be achieved by the EHT at this wavelength? What physical distance (in au) does this correspond to at the galactic centre? How does that compare with the Schwarzschild radius of the $4.3 \times 10^6 M_\odot$ black hole?

   (b) In VLBI, the signals received by each telescope are recorded, along with time marks from an atomic clock. The recordings are taken to a central facility, synchronized in time to remove light propagation delays, and then combined to produce interference patterns. Roughly how large a timing error can be tolerated when combining the signals?

   (c) Explain what we see in the EHT image.



2. Consider a self-gravitating gas cloud that has uniform temperature T and mean molecular weight μ.

   (a) Find a differential equation for the density ρ as a function of radius r.

   (b) Find a power-law solution to this equation. Calculate the mass of the cloud as a function of its radius R. What happens in the limit that $R \to \infty$?

3. Suppose that our universe is spatially flat (Euclidean), homogeneous, and expanding at a constant rate. By this we mean that the Hubble parameter H is not a function of time.

   (a) Find an expression for the scale factor a as a function of time. As is customary, you can denote present value of a parameter by the subscript 0.

   (b) We observe a distant galaxy and measure its redshift to be z. For how long has the light that we observe been traveling?

   (c) Find the comoving distance to the galaxy. (The proper distance is the product of the comoving distance and the scale factor.)

   (d) The luminosity of the galaxy, in its rest frame, is L. Assuming that the galaxy emits isotropically, derive an expression for the flux (power/per unit area) received at the Earth.

4. A star having luminosity L_* and radius R_* is surrounded by a spherical shell of dust that has inner radius R_1 and outer radius R_2. As light propagates a small distance ds within the shell, a fraction αds of the light is absorbed and then reradiated isotropically.

(a) To a distant observer, the shell has the appearance of a glowing disk. Assuming that $R_1 \gg R_*$ and $R_2 - R_1 \gg 1/\alpha$, estimate

   i. The luminosity of the shell, as seen from the outside.

   ii. The intensity $I(\vartheta)$ as seen by an observer at distance $d \gg R_2$. Here ϑ is the angular distance from the centre of the disk.

(b) Describe qualitatively how the result would differ if $R_2 - R_1 \ll 1/\alpha$. What would the observer see?