September 2009 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
Day 1: September 3, 2009

Do not write your name on the exam. Instead, sign up on the sign-up sheet, and note the serial number next to your name. Write that number on your exam in place of your name. This will allow us to grade the exams anonymously. We'll match your exam with your name using the sign-up sheet after we finish grading. If you use extra exam booklets, write your serial number on the extra exam books as well.

Today's portion of the exam has 8 questions. Answer *any five* of the eight. Do not submit answers to more than 5 questions—if you do, only the first 5 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 4 hours to complete 5 questions.

You are allowed to use one $8'' \times 11''$ formula sheet (both sides), and a hand-held, non-graphing calculator.

Here is a possibly useful table of physical constants:

absolute zero	0 K	-273°C
air pressure at sea level	1 atm	10^5 N/m^2
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
electron volt	1 eV	1.6×10^{-19} J
mass of the Earth	M_{earth}	6×10^{24} kg
mass of an electron	m_e	0.511 MeV/c^2
mass of a neutron	m_n	940 MeV/c^2
mass of a proton	m_p	938 MeV/c^2
mass of the Sun	M_{sun}	2×10^{30} kg
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
Planck's constant, reduced	$\hbar$	1.1×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of the Sun	R_{sun}	7×10^8 m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}

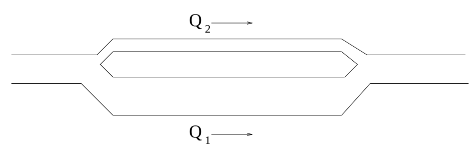
1. When two layers of fluid move past each other with different velocities, there is a viscous force F between the layers given by

$$F = -\eta A \frac{dv_x}{dy}$$

Here η is the coefficient of viscosity, A is the area of contact between the two layers, and dv_x/dy is the derivative of the velocity of flow, assumed to be in the x direction, with respect to the distance between the two layers in the perpendicular y direction.

A. Use this relationship to calculate the pressure difference ΔP between the two ends of a cylindrical pipe of radius R and length L carrying a fluid with viscosity η, as a function of the flow rate Q of the fluid (Q has units of volume per time). Ignore end effects (i.e. assume $L \gg R$). Show that $\Delta P = CQ$, and determine the coefficient C, which we can call the "impedance" of the pipe.

B. Supppose that two pipes are hooked up in parallel, thusly:



The pipe carrying flow Q_1 has a radius of R_1, and the pipe carrying flow Q_2 has a radius of R_2. Derive the effective impedance of this ensemble of pipes, again ignoring end effects. Show that the impedance of the combination has the same relation to the individual impedances of the two pipes as the resistance of two electrical resistors in parallel has to their individual resistances.

2. Electron-Proton collider

HERA is an electron-proton collider located near the city of Hamburg in Germany. The energy of the electron beam is 26 GeV and the energy of the proton beam is 820 GeV. The beams collide head-on. Ignore baryon and lepton number conservation and calculate

A. the maximum number of neutral pions (mass of $\pi^0 = 134.98$ MeV/c^2) that can be produced in one proton-electron collision.

B. What momentum would a beam of electrons incident on protons at rest need to have to produce the same number of pions as in the question above?

3. Consider a planet of mass M orbiting a star of radius r in a circular orbit with radius R. If the orbit of the planet is nearly edge-on when viewed from Earth, then the planet may transit across the face of the star during part of its orbit, occulting some of the star's light and allowing detection of the planet.

A. What is the probability that the inclination of the orbit will be tilted such that a transit occurs?

B. Rather than just monitoring random stars and hoping to see a transit, it is more common to use transit observations to confirm a planet already detected by the "Doppler effect wobble" it produces on the accompanying star. This amounts to observing the Doppler shift in the star's spectral lines due to motion of the star produced by the gravitational tug of the planet going around it. Suppose that the star's apparent velocity is found to wobble sinusoidally with period T and amplitude v_{wobble}. Explain with an explicit calculation how one would calculate the mass of the planet in the case of a planet detected both through its Doppler wobble and through an observed transit, and discuss how you might roughly estimate the density of the planet through these observations. You may assume that, based on its spectral type, you know that the star has solar mass and radius.

C. Why can't you determine the planet's mass uniquely for a planet detected by Doppler wobble alone?

4. An inventor would like to patent a thermodynamic device and is making the following claims:

The device can accept a stream of gas and split it into a hot and cold jet of the same gas. This device is said to contain no moving parts, consumables, or power supply and it is claimed to operate in different modes such that it can:

A. accept an ideal gas (e.g. argon) at 4 atm, 20°C, 2 mol/min and output one stream of 1 mol/min at 1atm, T=-20°C and, simultaneously, a second stream at 1 mol/min, 1atm, T=60°C

B. accept an ideal gas at 4 atm, 20°C, 2 mol/min and output one stream of 1.5 mol/min at 1atm, T=0°C and, simultaneously, a second stream at 0.5 mol/min, 1atm, T=90°C

C. accept an ideal gas at 1.1 atm, 20°C, 2 mol/min and output one stream of 1.6 mol/min at 1atm, T=-30°C and, simultaneously, a second stream at 0.4 mol/min, 1atm, T=220°C

Which of these claims would you reject? Which ones would you accept and why?

5. Neutrinos are massless spin-1/2 particles (ignore their tiny finite masses). There are 6 types of neutrinos (3 flavours of neutrinos and 3 of anti-neutrinos), and each has just one possible polarization state. In the early universe neutrinos and antineutrinos were in thermal equilibrium with zero chemical potential, and filled the universe as a kind of background radiation. Derive a formula for the total energy density (energy per unit volume) of the neutrino-antineutrino background radiation when it was at temperature T. Leave your answer as a constant times a definite integral over a dimensionless variable. Don't forget to determine the value of this constant.

6. A single positive point charge Q is located off-centre at radius R and height $z = 0$ inside a hollow cylindrical conducting shell of infinite length and inner radius a. The outer radius of the shell is b, and the axis of the cylinder is aligned with the z axis. What is the magnitude and direction of the electric field outside the shell, as a function of position?

7. The interior of Jupiter

A. Estimate the pressure at the center of Jupiter from the equation of hydrostatic equilibrium (i.e. pressure balances gravity), assuming that the average density of Jupiter is 1.3 g/cm^3. Jupiter's mass and radius are 1.9×10^{30} g and 7.2×10^9 cm respectively.

B. Estimate the temperature in Jupiter's core assuming that the ideal gas law applies. You may assume that Jupiter's composition is pure hydrogen for the purpose of this calculation.

8. Consider a classical ideal gas trapped in a one-dimensional harmonic potential. Its temperature is much higher than the harmonic oscillator frequency (i.e. the motion of particles can be treated classically if you wish).

A. What is the partition function for an individual particle?

B. What does the spatial density profile look like in the trap?

C. What is the N-body partition function? Can you estimate the chemical potential for the ideal gas with N particles at temperature T?

September 2009 Physics & Astronomy Qualifying Exam
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Day 2: September 4, 2009

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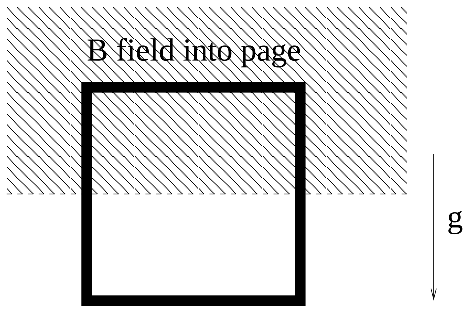
9. An atom with mass m moving in the $x > 0$ half space in three dimensions experiences a weak attractive force when approaching a surface (located at $x = 0$) of a solid state material which fills the other half of the space. Assume the attractive force has a simple form $-kx$, for $x > 0$. Describe the energy eigenstates of this atom when near the surface, including a discussion of which energies are allowed and a discussion of any degeneracies in the energy eigenstates. State the energy E_0 of the ground state. Then calculate the ratio of the number density of states with energies in the range $(2E_0, 2E_0 + dE)$ to the number density of states with energies in the range $(4E_0, 4E_0 + dE)$, where dE is very small.

10. A square loop is cut out of a thick sheet of aluminum (density=2.7 g/cm^3, resistivity $= 2.8 \times 10^{-8}$ Ω m). (You may assume that the length of one side of the loop is large compared to its width or thickness.) It is placed so that its top part lies in a region with a uniform magnetic field B that goes into the page, perpendicular to the plane of the loop. (The magnetic field is in the shaded area in the drawing below.) The bottom part of the loop is not in a magnetic field. The loop is released at rest and allowed to fall.

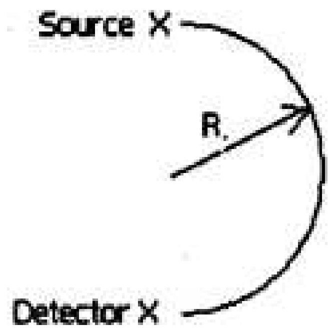


A. Find an expression for the velocity of the loop as a function of time, while the top of the loop is still in the magnetic field.

B. If $B = 1$ Tesla, what is the terminal velocity of the loop while it's still in the B field?

11. 180-degree spectrometer

Cs_{55}^{137} is a common laboratory radioactive source of electrons and gamma rays. Eight percent of the time Cs_{55}^{137} beta decays to the ground state of Ba_{56}^{137}. The net atomic mass difference between the two isotopes is 1.18 MeV/c^2. A 180 degree spectrometer is used to measure the beta decay spectrum. The spectrometer has a radius $R = 3.8$ cm.



A. Write down the reaction for the beta decay.

B. Calculate the maximum momentum of the beta decay electron/positron. Express the result in MeV/c.

C. What is the vector direction of the spectrometer magnetic field relative to the drawing?

D. What is the magnetic field setting of the spectrometer for the maximum energy of the electron/positron to arrive at the detector? Provide a numerical answer with units.

12. Assume that there is a single blue photon in a cavity of the size of 10 micrometers. The total energy of oscillating electric fields due to this photon is one half of the photon energy.

A. Estimate the magnitude of the electric fields associated with this single photon. Compare your result to the typical electrical field experienced by an orbiting electron in a hydrogen atom.

B. At what cavity size does the electrical field in the cavity become comparable to that in a hydrogen atom?

13. $n = 2$ orbitals of the hydrogen atom.

A. List the 8 possible states of the $n = 2$ orbitals of the hydrogen atom in the common eigenbasis of $\vec{L}^2$, L_z, $\vec{S}^2$, and S_z, and also in the common eigenbasis of $\vec{L}^2$, $\vec{S}^2$, $\vec{J}^2$, and J_z.

B. The 8 states of the $n = 2$ orbitals would be degenerate if not for spin-orbit coupling, and hyperfine splitting. Let us ignore the hyperfine splitting but treat the spin orbit coupling using first order perturbation theory. Determine the shifts in the energies of the 8 states of the $n = 2$ orbitals under the perturbation $V = \beta \vec{L} \cdot \vec{S}$, where $\vec{L}$ is the orbital angular momentum operator of the electron, $\vec{S}$ is the electron's spin operator, and β is a constant.

C. What is the physical origin of the hyperfine structure?

14. Order of magnitude estimation

A. How many humans have ever lived? (Useful facts: the population of Earth was 1 billion in the year 1800, after which point scientific gains in agriculture resulted in a large increase in population growth. Before the discovery of agriculture seven thousand years ago the population was flat at 4 million. For accounting purposes the human species can be reckoned to be 100,000 years old.)

B. How large of an auditorium, in terms of floor space, would you need to host a plenary conference attended by every graduate students in physics and/or astronomy on the planet?

15. An atomic transition takes place between an $\ell = 1$ and an $\ell = 0$ energy level, where ℓ is the orbital angular momentum quantum number, in an atom with no net electron spin. When no magnetic field is applied, the frequency of the emitted light is F. In a sunspot, there is a strong vertical magnetic field. When viewed from directly above the sunspot, this emission line is split into two frequencies F_1 and F_2 (with $F_2 > F_1$). When the sunspot is viewed off to the side, however, a third line with frequency F_3 is seen. Calculate the strength of the magnetic field and the value of F_3 in terms of F_1 and F_2, and explain in detail why only two lines are seen when viewing the sunspot from directly above.

16. A white dwarf

A. A photon emitted by a white dwarf star loses part of its energy due to work necessary to overcome the gravitational field of the star. Derive a relationship between the observed frequency ν of a photon emitted with frequency ν_0, the white dwarf mass M and its radius R.

B. The maximum possible value for the mass of a white dwarf (M_{wd}^{max}) can be derived from the condition that the sum of the thermal plus gravitational energy $U + W$ is equal to zero. Demonstrate that

$$M_{wd}^{max} \sim \left(\frac{5hc}{2G}\right)^{3/2} \frac{3}{16\pi} \left(\frac{1}{um_p}\right)^2$$

assuming

$$U \sim \frac{3}{4} N_e hc \left(\frac{3n_e}{8\pi}\right)^{1/3}$$

with u being the ratio of the number of nucleons to the number of protons in the star ($u = A/Z$), n_e the number of electrons per unit volume, N_e the total number of electrons in the star and m_p the proton mass. Assume that the density ρ is constant.