Autumn 2018 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
Day 1: August 30, 2018

Do not write your name on the exam. Instead, write your student number on each exam booklet. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading. If you use extra exam booklets, write your student number on the extra exam books as well. Write all answers in the blank exam booklet(s), not on this printout!

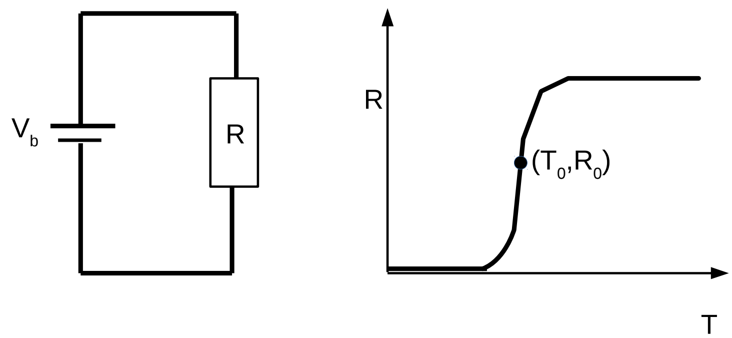
Today's portion of the exam has 8 questions. Answer *any five* of the eight. Do not submit answers to more than 5 questions—if you do, only the first 5 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 4 hours to complete 5 questions.

You are allowed to use one 8.5″ × 11″ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2 f =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	$1.675{\times}10^{-27}$ kg
mass of a proton	m_p	$1.673{\times}10^{-27}$ kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

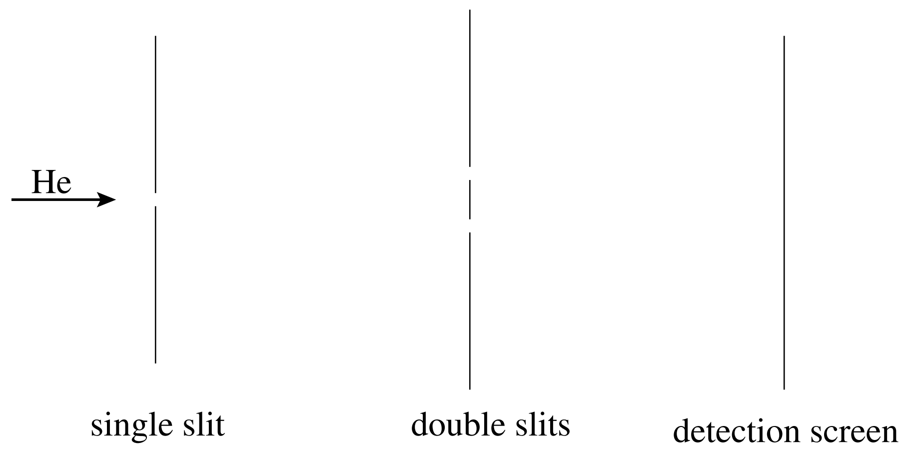


1. A transition edge sensor is a thin superconducting film that is used to detect tiny temperature changes. It is hooked up to a constant voltage source V_b (see drawing on left) and is in thermal contact with a cold bath of temperature T_{bath}. The sensor has a heat capacity C, and let T denote its temperature. Heat flows from the sensor to the bath at a rate given by $P(T) = K(T^2 - T_{bath}^2)$. V_b is chosen so that the current flowing through the film is large enough to balance it on the transition between its normal and superconducting state (see drawing on right). At some temperature T_0 the sensor therefore has a finite resistance R_0 and is in a state of equilibrium. If the temperature of the sensor increases from T_0 to $T_0 + \delta T$, where δT is very small, the resistance of the sensor will change quickly, due to the steep (but finite) slope α of the superconducting transition curve.

A. Write down an expression for T_0.

B. Supppose that at $t = 0$ the sensor quickly absorbs some energy, increasing its temperature from equilibrium by a small δT. Calculate the temperature of the sensor as a function of time for $t > 0$. You should linearize the relevant differential equation for T by using a Taylor series expansion.

(drawing not to scale)



2. A beam of He atoms moving at 1350 m/s is normally incident onto a screen with a narrow slit of width d. A second screen with two slits in it sits 64 cm downstream of the first. The two slits are 1 μm wide and their centres are 8 μm apart. A "detection screen" sits a further 64 cm downstream, and records the hit locations of each helium atom. The atoms form an interference pattern on this screen.

Calculate the spacing of the interference bands on the detection screen. How small must d be in order for the interference pattern to be seen?

3. A neutron star is in a binary system with another star. It has an orbital period of 42 minutes around the centre of mass and its orbit is circular. The orbital velocity of the neutron star around the centre of mass is 11 km/sec while the companion has an orbital velocity of 770 km/sec. Find the masses of the companion star and the neutron star.

4. A neutron star is composed primarily of neutrons packed to nuclear densities, and the mass of a neutron star is two times the mass of the Sun.

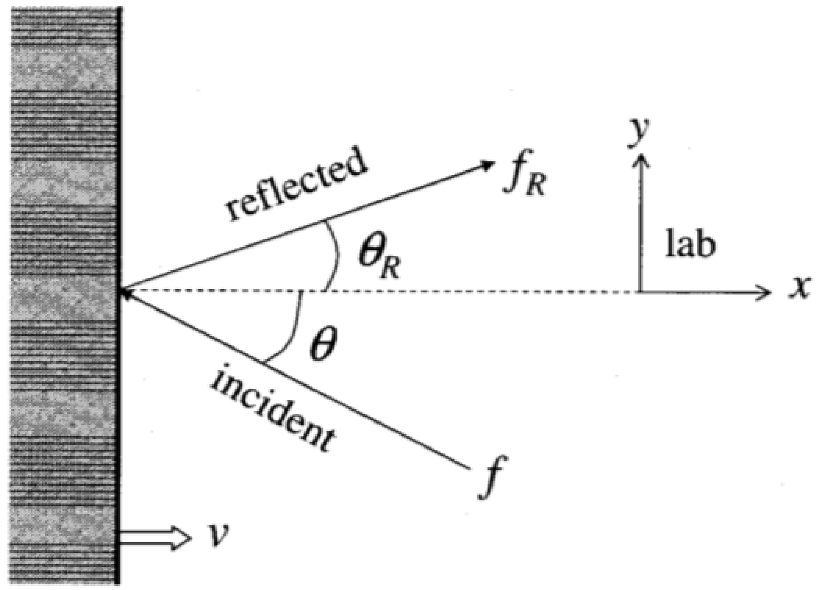
A. Roughly calculate the radius of a neutron star.

A neutron star will often accrete matter onto its hard surface from a companion star. Assume this accretion flow is spherically symmetric and is falling onto the neutron star surface at a rate of approximately $dm/dt = 10^{-9}$ solar masses per year.

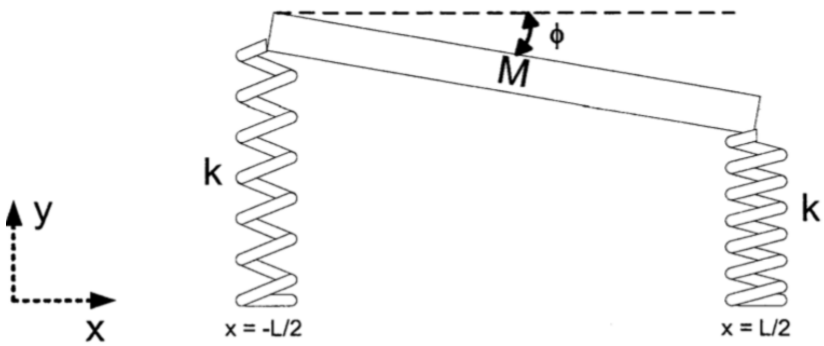
B. Estimate the total accretion luminosity (power) emitted by a neutron star at the given accretion rate.

C. Assuming the neutron star radiates like a black body, what is the typical energy of the emitted photons? In what wavelength band (e.g. radio? visible?) does this radiation predominantly occur?

5. A monochromatic beam of light is incident on a flat mirror. With respect to the laboratory, the mirror is traveling at relativistic speed v in the $+x$ direction. (The plane of the mirror is perpendicular to the x-axis.) In the lab frame, the incident light beam has frequency f and travels at angle θ with respect to the x-axis. Find the frequency f_R and the angle θ_R of the reflected light beam as measured in the lab frame.



6. A thin bar with uniform density of length L and mass M is supported by two massless springs. The springs have identical spring constants k and unloaded lengths l. The centre of mass of the bar is constrained to move only vertically with displacement $y(t)$, but the bar can rotate in the xy plane with angle $\phi(t)$.



A. Determine the Lagrangian of the system for small displacements, taking gravity into account.

B. Solve the Euler-Lagrange equations of motion to determine the frequencies and eigenvectors for the normal modes of the system.

C. Suppose at $t = 0$ the end of the bar at $x = L/2$ is depressed by a *small* amount d, while the other end is held at its equilibrium position. (In other words, $y_1(t = 0) = l$ and $y_2(t = 0) = l - d$). If all initial generalized velocities are zero, find expressions for $y(t)$ and $\phi(t)$.

7. A hydrogen atom in its ground state has a wavefunction given by $\Psi_0(r,\theta,\phi) \propto e^{-r/a_0}$, where $a_0 = 5.3\times10^{-11}$ m is the Bohr radius. When placed in an external electric field $\vec{E} = E_0\hat{z}$, the new wavefunction, to lowest order in perturbation theory, becomes:

$$\Psi(r,\theta,\phi) \propto \Psi_0 \left[1 - \left(\frac{E_0}{A}\right) \left(\left(\frac{r}{a_0}\right) + \frac{1}{2}\left(\frac{r}{a_0}\right)^2 \right) \cos\theta \right]$$

Here A is a parameter characterizing the unperturbed atom.

Use this wavefunction to calculate the electric dipole moment of the atom in the external field, to first order in E_0.

Hint:

$$\int_0^\infty dr\ e^{-r} r^n = n!$$

8. An electron spin is oriented upward along $+\hat{z}$, and subject to a magnetic field B_x along $+\hat{x}$. The precession period of the spin around the magnetic field direction is T. The operator S_z is measured repeatedly, N times within a time interval $T/2$ (assume the measurements are evenly spaced in time). What is the probability that all N measurements returned $+\hbar/2$, in the limit of large N? (Express your answer in terms of N.)

Autumn 2018 Physics Qualifying Exam
for Advancement to Candidacy
Day 2: August 31, 2018

If you are in the Ph.D. in astronomy program, don't write this exam! Ask a proctor for the astronomy version of today's exam!

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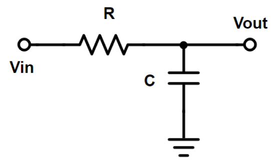
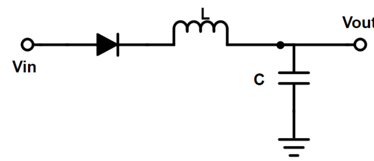
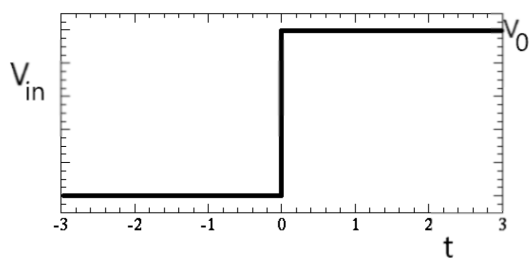
9. Do an order of magnitude estimate of the amount of energy stored in the magnetic field surrounding the Earth. (The field strength on the Earth's surface is $\sim 5 \times 10^{-5}$ T.) Compare this to an order of magnitude estimate of the amount of electrical energy used in the greater Vancouver area each year.

10. The Sun is in mechanical equilibrium between gravitational and pressure forces. Make an order of magnitude estimate of the temperature T of the centre of the Sun.

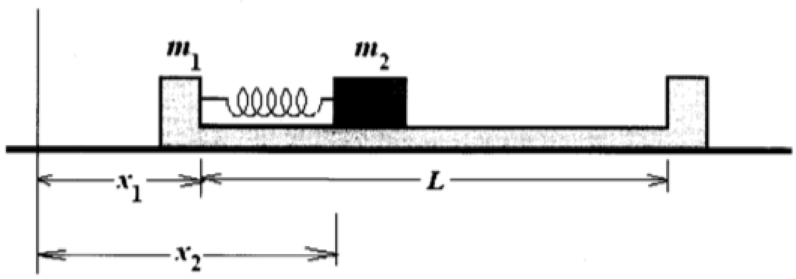






11.
You are exploring two schemes of charging a capacitor C through a resistor R (scheme A on the left above) or through a series of diode and inductor L (scheme B on the right above). In both schemes an input voltage V_{in} is switched from 0 V to some fixed, positive DC voltage V_0. Calculate the final voltage of the charged capacitor and the energy 'lost' to components other than the capacitor under the two schemes. Consider the diode to be ideal in that it has a 0 V forward voltage drop and infinite reverse resistance.

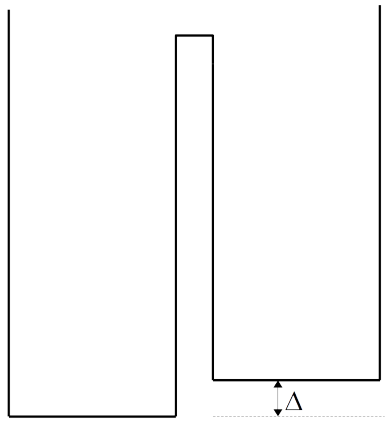
12. A block of mass m_2 slides inside a cavity of length L inside a second block with mass m_1, which rests on a horizontal table . The masses m_1 and m_2 are connected by a massless spring with spring constant k and equilibrium length $l \ll L$. Intially both blocks are at rest and located at $x_1 = 0$ and $x_2 = l - \Delta l$, where Δl is the initial compression of the spring.



A. If the mass m_1 slides without friction on the table and m_2 slides without friction on the second block, find the positions $x_1(t)$ and $x_2(t)$ as a function of time.

B. If mass m_1 exerts a frictional force on m_2 proportional to their relative velocity, $F_{1 \text{ on } 2} = -\sigma(\dot{x}_2 - \dot{x}_1)$, again determine the resulting motion of the two masses. (Continue to assume that m_1 slides without friction on the table, and also assume that σ is small.)

13. Describe what happens to an atom when it approaches a conducting sphere with a charge of $+1$ coulomb. How does the force on the atom depend on the distance from centre of the sphere? If the force is F at distance y from the centre of the sphere, what is the force when the distance is $2y$?



14. Consider a symmetric double well potential with a barrier much higher than the oscillation frequency at the bottom. (See above figure and initially assume $\Delta = 0$). Initially, a particle is in the left (L) side of the well. An experimentalist observes that the probability of finding the particle in the right hand side increases until it reaches a maximum value X at time T.

A. What is the value of X? With this information, can you predict what happens to the probability after that?

B. Describe qualitatively what happens to your prediction if there is a tilt in the double well potential so that L is Δ lower in energy than R? (The tilt is much smaller than the barrier height.) Does X depend on the tilt Δ?

15. A rubber band is stretched between two posts 10 cm apart; the tension in the rubber band is 10 N and the temperature is 25°C. Then, the temperature rises from 25°C up to 30°C, and as a result the tension in the band increases to 11 N. Next, the rubber band is stretched by an additional 1 cm. The rubber band stays at 30°C during this process by exchanging heat with the surrounding air. How much heat is exchanged, and is it released or absorbed by the rubber band? Assume the thermal expansion between the posts during the temperature rise is negligible. Hint: start by writing down a differential of internal energy U in terms of tension, length, temperature, and entropy, then work out a Maxwell relation to solve the problem.

16. In this problem we will consider the rotational kinetic energy of a hydrogen (H_2) molecule.

A. If the interatomic distance is R, what is the energy of the lowest possible excited rotational state of H_2?

B. Suppose that the protons' spins are parallel. What are the allowed values of the orbital angular momentum quantum number j of the molecule? (You should neglect the electrons' degrees of freedom here.)

C. Write down formulas for the partition function and the average rotational kinetic energy of the molecule if the protons' spins are parallel and the molecule is in thermal equilibrium with a thermal bath of temperature T.