PhD(Astro) Qualifying examination - 2019

13:00 - 17:00, 30 August 2019

Do not open the exam until instructed to do so
but you may read this cover sheet

Instructions:

A one-page (8.5×11 inch) double-sided sheet of notes is allowed.

A scientific calculator is allowed and expected.

Put your student number on upper right corner of your exam booklet. Do not write your name on the booklet. This will allow us to grade the exams anonymously. We'll match your name to your student number after the exams have been graded.

All answers must be written in the exam booklets. If you use more than one exam booklet, be sure to write your student number on each.

There are 8 questions to choose from. You will only select and provide answers for 5 of them; you may not attempt any portion of the other 3 questions. The five questions you choose all have equal value, and thus you may wish to time budget about 45 minutes per question.

On the front of your exam booklet you should clearly indicate which 5 questions you wish graded if you have attempted more than 5 questions. If not, the first 5 questions started in your exam booklets will be graded.

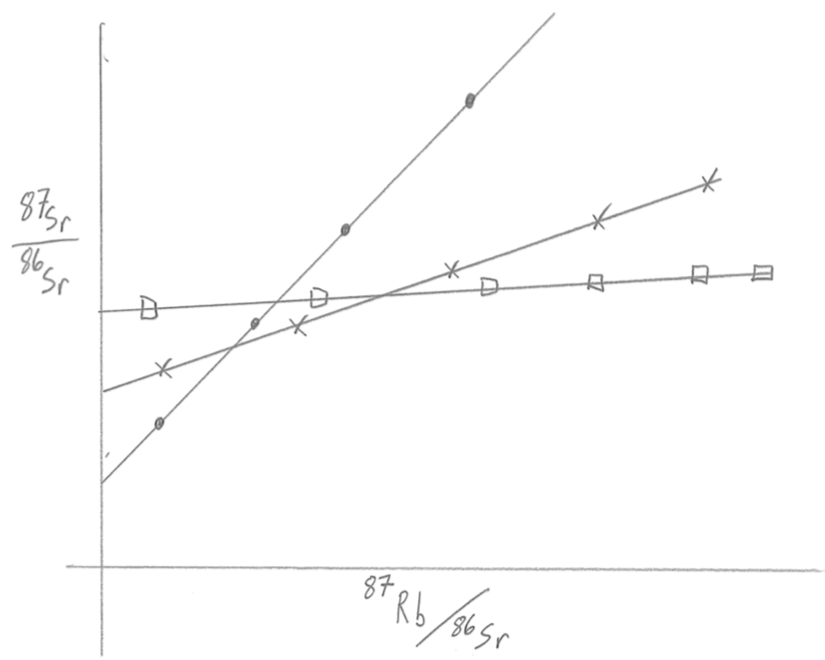
Start every question on a new page.

Please return this examination with your exam booklet.

Here are some constants that might be useful:

astronomical unit	AU	1.496×10^{11} m
parsec	pc	3.086×10^{16} m
year	yr	3.156×10^{7} s
solar mass	$M_\odot$	1.989×10^{30} kg
proton mass	m_p	1.673×10^{-27} kg
electron mass	m_e	9.109×10^{-31} kg
electron volt	eV	1.602×10^{-19} J
Boltzmann's constant	k	1.381×10^{-23} J K^{-1}
Planck's constant	h	6.626×10^{-34} Js
Rydberg constant	R	13.606 eV
Speed of light	c	2.998×10^{8} ms^{-1}
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Radiation constant	a	7.566×10^{-16} J m^{-3} K^{-4}
H-α wavelength		656.3 nm
Ly-α wavelength		121.6 nm

1. The figure below is a cartoon depiction of strontium-rubidium isotope measurements for three different rocks, shown by three lines. The results of measurements from different regions within the same rock are indicated by points for one rock, squares for another and X's for the third. The parent isotope in this system is ^{87}Rb, which decays to the daughter isotope ^{87}Sr at a rate characterized by a half-life of $t_\lambda = 48$ Gyr. The isotopes ^{87}Sr and ^{86}Sr are stable.



(a) (3 points) Rank the rocks from oldest to youngest. If you do not have enough information or there is ambiguity, then point it out.

(b) (3 points) Rank the rocks from highest to lowest ^{87}Sr/^{86}Sr abundance ratios at the time of each rock's formation. If you do not have enough information or there is ambiguity, then point it out.

(c) (6 points) Assuming that all of the decayed ^{87}Rb goes into ^{87}Sr, we can write

$$\left.\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right|_t = \left.\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right|_0 + \left.\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right|_0 \left[1 - \exp\left(-t/t_d\right)\right].$$

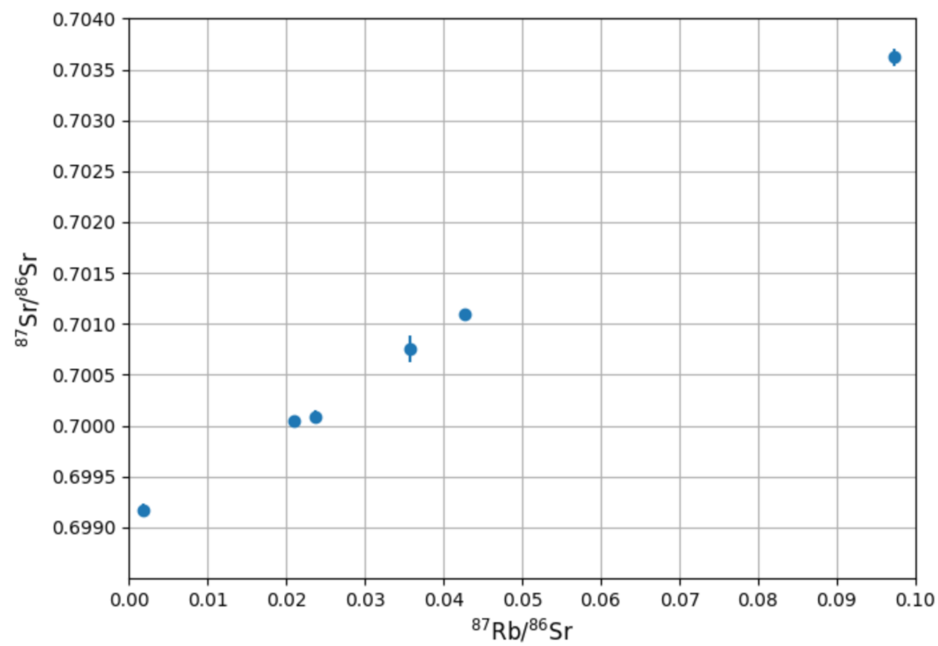
Here, t_d is the decay timescale and the naughts represent the abundance ratios at the time of the rock's formation. The decay timescale can be related to the half life through a change of base. The time t refers to the time since the rock's formation. The current abundance ratio of rubidium to strontium at a given time t is

$$\left.\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right|_t = \left.\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right|_0 \exp\left(-t/t_d\right).$$

Show that abundance ratio measurements of ^{87}Sr/^{86}Sr and ^{87}Rb/^{86}Sr for a given rock should form a line (its isochrone) on the plot above. Assume that all components of the rock (e.g., different minerals and sections) all formed at the same time from the same well-mixed reservoir of material.

(d) (4 points) Now consider the strontium-rubidium measurements shown in the figure below, which is from a Solar System rock. What is the age of the rock?

(e) (4 points) What is the initial $\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}$ abundance?



2. The state occupation probability for Fermi-Dirac particles as a function of the energy ϵ of the state and the chemical potential μ is

$$f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_B T} + 1}.$$

(a) (4 points) Show that in the low temperature limit, $k_B T \ll \mu$, this probability is 0 at high energy and 1 at low energy.

(b) (5 points) Sketch a graph of $f(\epsilon)$ for $k_B T \simeq \mu/4$. Label your graph and show where $\epsilon = \mu$.

(c) (5 points) The number of quantum states below some threshold energy ϵ_F for a particle of mass m in a volume V is

$$N = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\epsilon_F\right)^{3/2}.$$

Explain why your answer to part (a) implies that ϵ_F is the chemical potential associated with N fermions in volume V. Solve for ϵ_F as a function of the number density of particles.

(d) (6 points) A neutron star is a solar mass of neutrons supported against gravitational collapse by its own Fermi energy. Compared to the neutron density ρ_N, what electron density within a neutron star has the same Fermi energy as neutrons do? (The electron mass is about 1/2000 that of the neutron.) Comment.

3. A thermally-emitting HII region has a flat spectrum, such that $F_\nu = 10$ mJy over a wide range of frequencies. Recall that 1 Jy $= 10^{-26}$ W m^{-2} Hz^{-1}.

   (a) (4 points) What is the most likely emission mechanism, and why?

   (b) (4 points) Give an expression for F_λ in terms of F_ν, and estimate F_λ at 500 nm in units of W m^{-2} μm^{-1}.

   (c) (4 points) The HII regions is circular with a radius of 1 arcminute. What is the solid angle Ω occupied by the source?

   (d) (4 points) For simplicity, you may assume that the specific intensity I_ν is constant across the HII region. What is I_ν?

   (e) (4 points) The HII region is observed with the 300m-diameter Arecibo telescope for 10 minutes in the frequency range $420 - 440$ MHz. How much energy is collected from the source during this time?

4. Dark matter likely pervades the solar neighbourhood of the galaxy. Assume that this the dark matter is a uniform mass density that has no effect other than it's gravitational interactions.

   (a) (16 points) Explain the dynamics of how one would use orbits of planets or small bodies to detect the presence of the dark matter. For full marks on this portion you should explain how one would compute the magnitude for at least some of the effect(s), referencing relevant equations.

   (b) (4 points) Estimate the order of magnitude of the effect using the most sensitive case you think existing data may have access to.

5. (20 points) An interstellar cloud of thickness L is comprised solely of atomic hydrogen at a temperature T and number density n_H. We are interested in how opaque the cloud is to H_α photons. Assuming that the absorption optical depth is proportional to the fraction of the atoms in the $n = 2$ state of hydrogen, what is the ratio of the optical depth of such a cloud at $T = 1000$ K and $T = 100$ K? [Hint: You don't need the H_α absorption cross section to solve this.]

6. The rotation curves of spiral galaxies are flat as far as they can be measured, implying the existence of a dark matter halo. It is not known how far this dark matter halo extends. Imagine that the dark matter halo of the Milky Way extends half-way to the Andromeda Galaxy (which is itself about 780 kpc away), so that the halos of the two galaxies just touch each other. We assume that the rotation curve remains flat to this point, so that the rotation speed at that radius is equal to that at our radius.

   (a) (6 points) What is the total mass you infer for the Milky Way, in solar masses?

   (b) (9 points) The current mean distance between massive galaxies in the universe is about 3 Mpc. Assuming that they all have the same mass that you just calculated above, and they represent all the mass of the universe, what is the mean density of the universe? What is the value of the cosmological density parameter Ω_{matter}?

   (c) (5 points) What volume of space at the density you calculated in part (b) would contain the mass of the Sun? Express your answer in terms of the side of a cube of this volume, in pc.

7. Consider an elliptical galaxy in equilibrium. Suppose you decrease the velocity of all of the stars by 10% and then allow the system to relax to a new equilibrium.

   (a) (12 points) After the galaxy has reached a new equilibrium, by what factor has the radius of the stellar system changed?

   (b) (8 points) By what factor has velocity dispersion changed?

8. Consider a flat radiation-only universe. This describes the early evolution of our own universe. $\Omega_{r0} = 1$ and $\Omega_{m0} = \Omega_\Lambda = \Omega_k = 0$.

   (a) (10 points) Find the evolution of the scale factor with time.

   (b) (10 points) Find the comoving size of the past light cone.