PhD(Astro) Qualifying examination - 2018

13:30 - 17:30, 31 August 2018

Do not open the exam until instructed to do so
but you may read this cover sheet

Instructions:

A one-page (8.5×11 inch) hand-written double-sided sheet of notes is allowed.

A scientific calculator is allowed and expected.

Put your student number on upper right corner of your exam booklet. Do not write your name on the booklet. This will allow us to grade the exams anonymously. We'll match your name to your student number after the exams have been graded.

All answers must be written in the exam booklets. If you use more than one exam booklet, be sure to write your student number on each.

There are 8 questions to choose from. You will only select and provide answers for 5 of them; you may not attempt any portion of the other 3 questions. The five questions you choose all have equal value, and thus you may wish to time budget about 45 minutes per question.

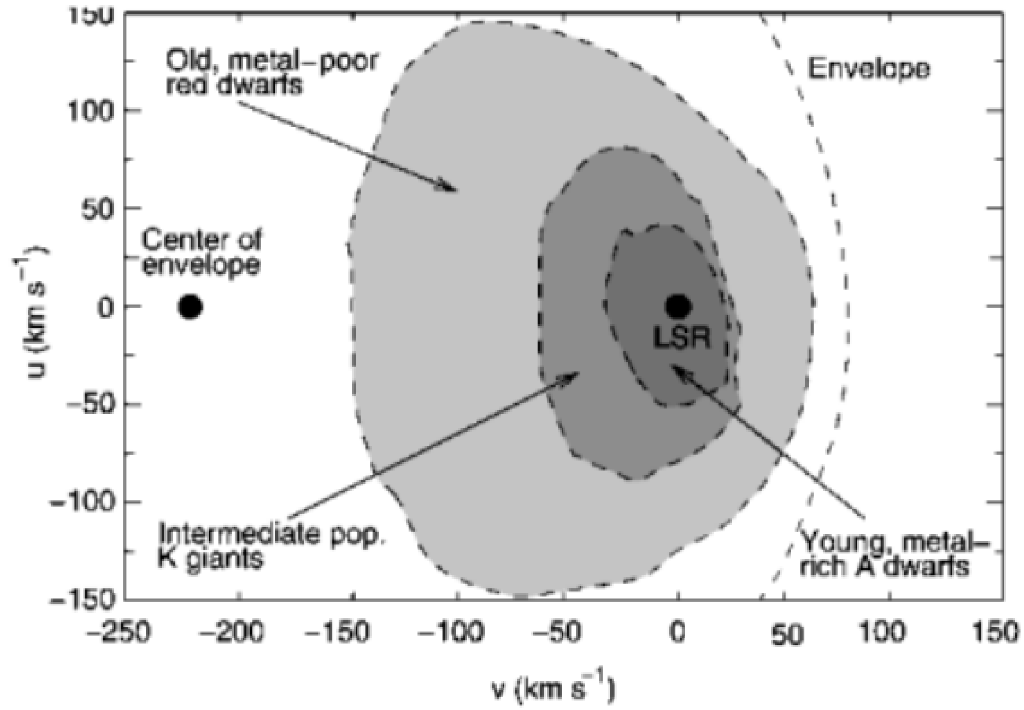
On the front of your exam booklet you should clearly indicate which 5 questions you wish graded if you have attempted more than 5 questions. If not, the first 5 questions started in your exam booklets will be graded.

Start every question on a new page.

Please return this examination with your exam booklet.

Here are some constants that might be useful:

astronomical unit	AU	1.496×10^{11} m
parsec	pc	3.086×10^{16} m
year	yr	3.156×10^{7} s
solar mass	$M_\odot$	1.989×10^{30} kg
proton mass	m_p	1.673×10^{-27} kg
electron mass	m_e	9.109×10^{-31} kg
electron volt	eV	1.602×10^{-19} J
Boltzmann's constant	k	1.381×10^{-23} J/K
Planck's constant	h	6.626×10^{-34} Js
Rydberg constant	R	13.606 eV
Speed of light	c	2.998×10^{8} ms^{-1}
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}



1. The attached figure shows contours containing most of the the members of different stellar populations, relative to the LSR. [Recall that the u/v speeds are along the directions (to the galactic center)/(perpendicular to the x axis along direction of rotation), respectively.] The 'envelope' encompasses all stars of the Galaxy.

   (a) (10 points) Explain as many patterns or aspect of this figure as you can, in terms of an understanding of the structure of the galaxy and its history of star formation.

   (b) (10 points) Assume that the recently discovered interstellar interloper 1997 1I/'Oumuamua is an ejected fragment from planet formation around a roughly solar-mass star. It was discovered at about 1.1 au heliocentric distance, moving radially away from both the Sun and Earth at the time. Using information from the figure and your knowledge of dynamics, compute the heliocentric velocity vector of 1I at the instant of detection, in km/s. Then compute the geocentric vector and determine the sky rate of motion relative to the stellar background at detection, expressing your answer in both arcseconds per hour and in degrees per day).

2. White dwarf stars are supported against collapse by the pressure of their degenerate electrons. Their luminosity (energy/time) is provided by the ions that are not degenerate and can lose energy and cool. This energy is then radiated away by the star.

   Consider a 0.5 solar mass (1 solar mass = 2 x 1030 kg) white dwarf star consisting entirely of carbon atoms. Such a star has a radius of 6000 km. After 1 billion years of cooling this white dwarf will have a surface temperature of 8000 K. Its core temperature at this time is 2 x 107 K.

   (a) (10 points) What is the luminosity of the white dwarf at this time?

(b) (10 points) After another billion years of cooling, the surface temperature drops to 6000 K. Estimate the core temperature of the white dwarf at this time.

3. The plate scale of a telescope relates the angular size of an astronomical object to the size of its image on the telescope's focal plane.

   The UVIS channel of the Wide Field Camera 3 (WFC3) on the Hubble Space Telescope has a plate scale of $0.04''$/pixel (arcsec/pixel), and its square pixels are $15\,\mu$m on a side.

   (a) (4 points) What is the plate scale in arcsec/mm?

   (b) (4 points) What is the effective focal length of WFC3?

   (c) (4 points) WFC3 can observe wavelengths from 200 nm to 1000 nm. The diameter of the Hubble primary mirror is 2.4 m. What is the diffraction-limited Point Spread Function (PSF) width of the Hubble at 200 nm and 1000 nm?

   (d) (4 points) How do the sizes of these PSFs compare to the plate scale in arcsec/pixel – that is, is the PSF well-sampled by the camera at all wavelengths? If not, then why might this plate scale have been chosen?

   (e) (4 points) The WFC3 CCD is roughly 4096 by 4096 pixels square. What is its field of view in square arcsec? How many pointings would it take to cover 1 square degree of sky?

4. (20 points) Consider that you have N atoms in a bunch. At what value of N does gravity begin to dominate over other forces in determining the structure of the bunch. What is the mass of the bunch where gravity just begins to dominate? Is this mass reasonable? Please give your assumptions in your analysis.

5. Consider a geometrically-thin protoplanetary disc in equilibrium about a star of mass M. Use cylindrical coordinates to describe the system such that R is the distance from the star, as measured along the disc's midplane, and z is the perpendicular distance away from the midplane (i.e., the disc altitude).

   (a) (5 points) Starting with the gravitational potential, show that the vertical component of the stellar gravitational acceleration is well approximated as $g_z = \Omega^2 z$ for any given R, where $\Omega^2 = GM/R^3$

   (b) (10 points) Assuming that the disc is vertically isothermal with temperature T, what is the vertical density profile of the disc? Let the gas have a mean molecular weight of μ.

   (c) (5 points) Protoplanetary discs are often characterized by their scale height ratio H/R, where H is a representative length scale for the given density profile. Using your density profile from (b), identify an appropriate H and then calculate the temperature of the disc for $H/R \sim 0.07$. Let $R \sim 3$ AU and $M \sim M_\odot$. Use a reasonable value for μ. It might be helpful to recall that the circular Keplerian orbital speed at 1 AU is about 30 km s^{-1}. The gas constant is 8.314 J K^{-1}mol^{-1}.

6. In the early Universe, matter and radiation were kept in balance via reactions such as $e^- + e^+ \rightarrow \gamma + \gamma$.

   (a) (10 points) Show that the single-photon process $e^- + e^+ \rightarrow \gamma$ is not permitted. (Hint: Use the properties of vectors k and p that describe the four-momentum of a photon and electron respectively.)

(b) (10 points) As the Universe expanded and cooled, electron-positron pair production ceased. At roughly what temperature did that happen. What was the energy density in the Universe at that time?

7. (20 points) In this problem, you will explicitly derive the relationship between scale factor and time in a critical density universe, treating it as a Newtonian problem. The (positive) kinetic energy of a galaxy a distance r from us is equal and opposite to the (negative) gravitational potential energy due to the gravitational pull of the mass within the radius r. Using this fact, write down a differential equation for the relation between radius and time, and solve it. Use this to get an expression for the present age of the universe, expressed in terms of the Hubble Constant.

8. (20 points) Suppose the Galaxy's mass (let's say $8 \times 10^{10} M_\odot$) is all located at the centre of the Galaxy and that a star was formed in a circular orbit 8 kpc from the centre at the time the Galaxy formed. Suppose that the Galaxy has a central black hole which is turning mass into radiation at the rate of $1 M_\odot \text{ yr}^{-1}$ and the Universe is 1.3×10^{10} years old. How far from the centre of the Galaxy will the star be today? How does this increase in the star's distance compare with the expansion of the Universe? (Think carefully about what is conserved in this problem, and state your assumptions clearly.)